Group analyses

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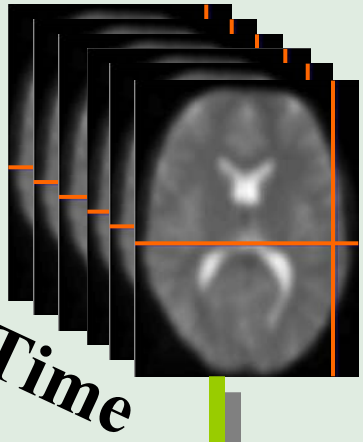
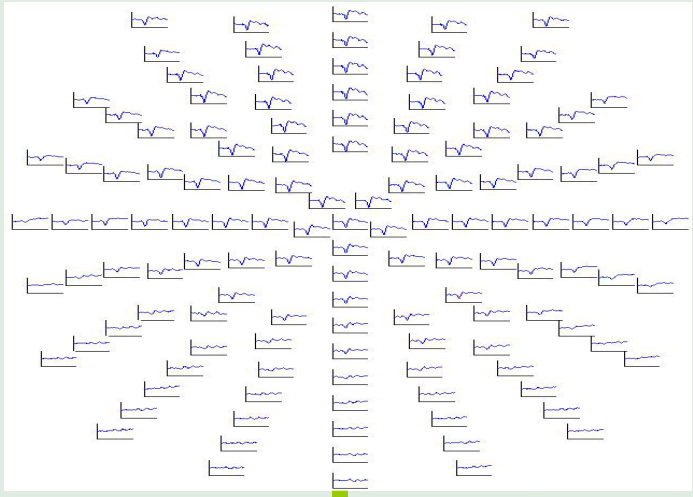
Data

fMRI, single subject



fMRI, multi-subject

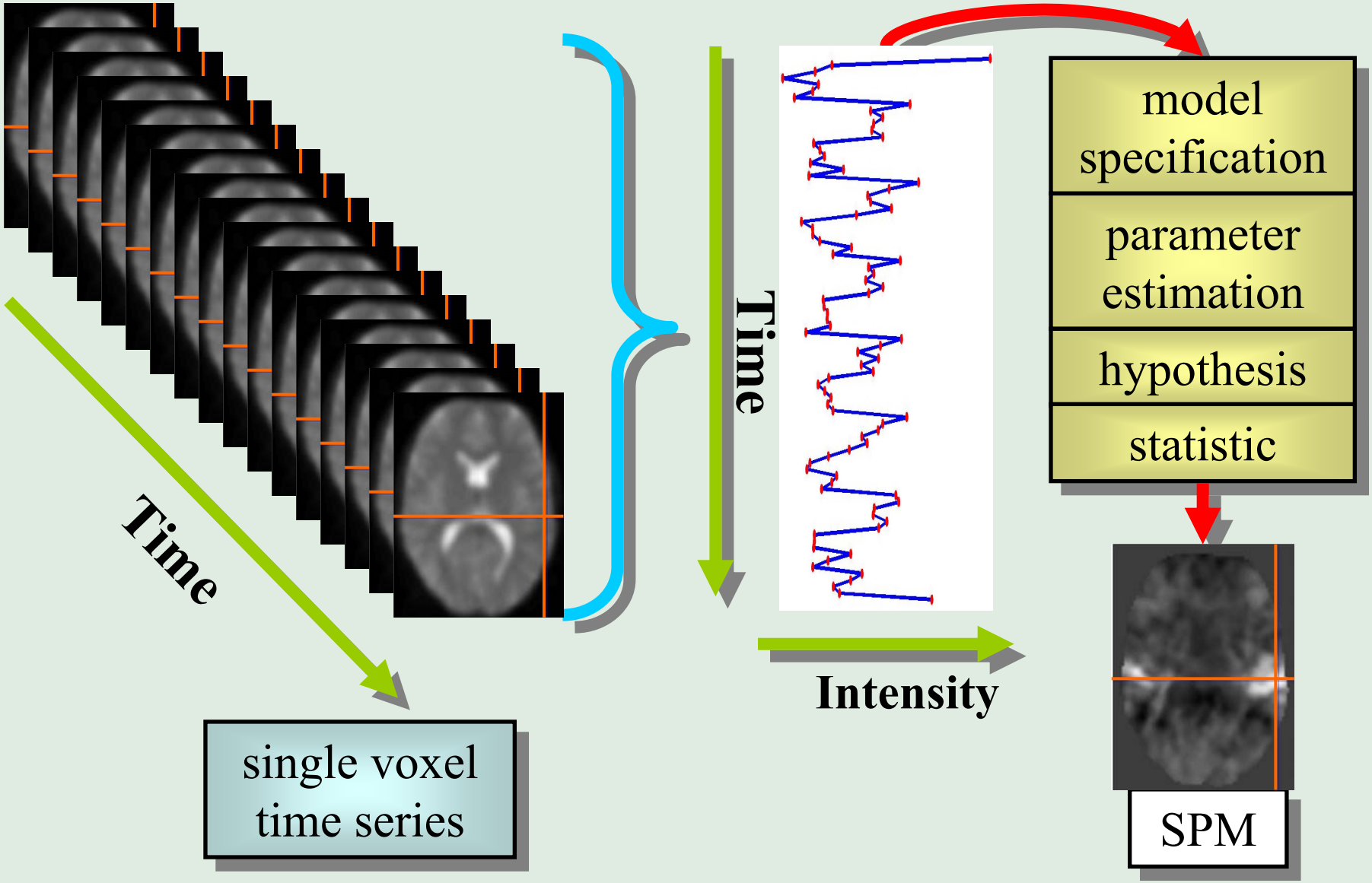
EEG/MEG, single subject



ERP/ERF, multi-subject

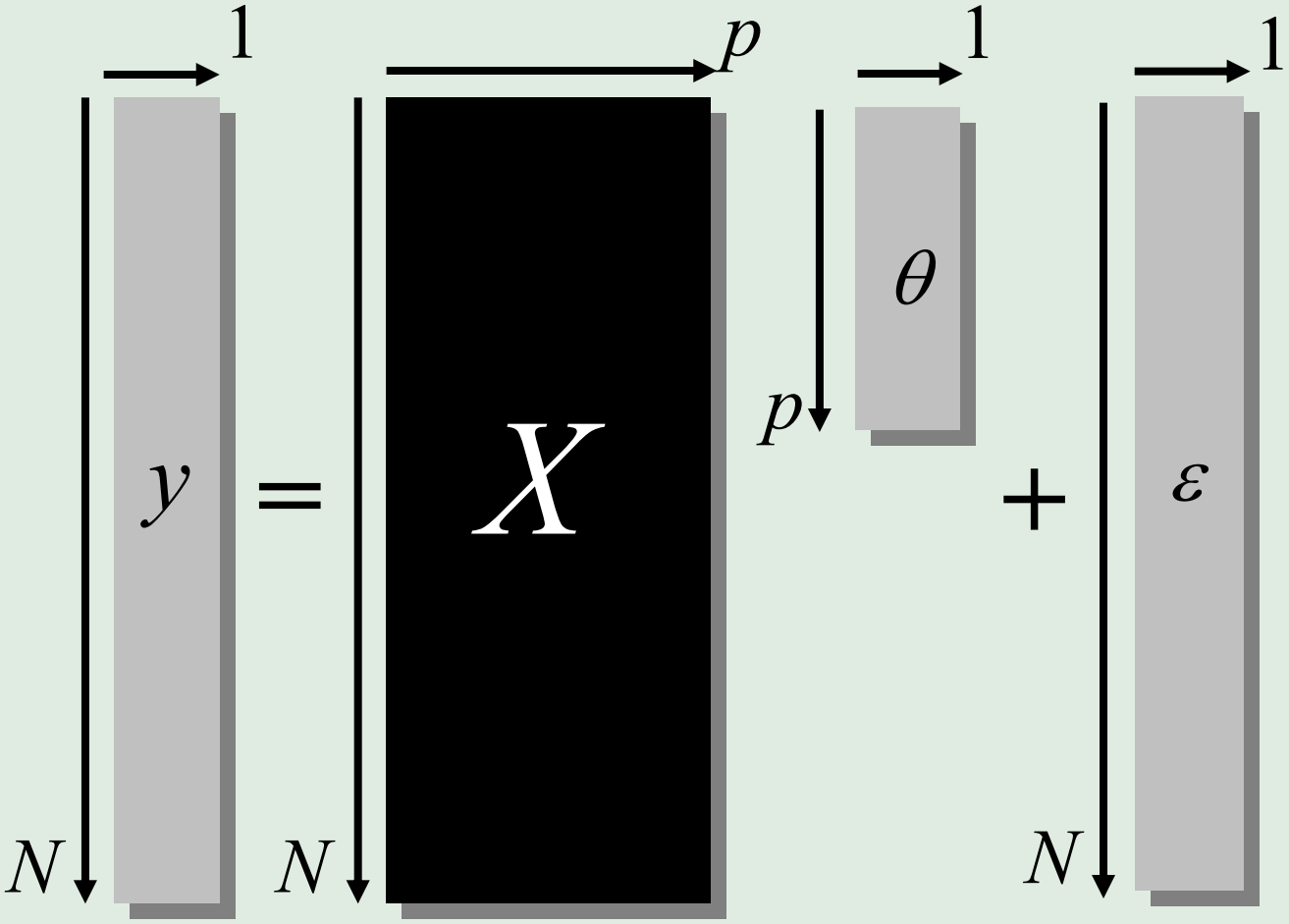
Hierarchical model for all imaging data!

Reminder: voxel by voxel



General Linear Model

$$y = X\theta + \varepsilon$$



Error Covariance

$$C_\varepsilon = \sum_k \lambda_k Q_k$$

N: number of scans
p: number of regressors

Model is specified by
1. Design matrix X
2. Assumptions about ε

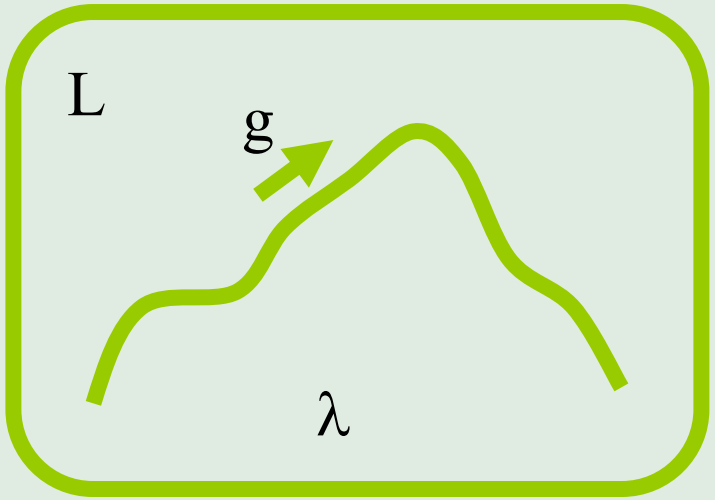
Estimation

$$\underset{N\times 1}{y} = \underset{N\times p}{X}\ \underset{p\times 1}{\theta} + \underset{N\times 1}{\varepsilon}$$

1. ReML-algorithm

$$C_\varepsilon = \sum_k \lambda_k Q_k$$

Maximise $L = \ln p(y \mid \lambda) = \ln \int p(y \mid \theta, \lambda) d\theta$



$$g = \frac{dL}{d\lambda}$$

$$J = \frac{d^2 L}{d\lambda^2}$$

$$\lambda = \lambda + J^{-1} g$$

2. Weighted Least Squares

$$\theta = (X^T C_e^{-1} X^T) X^T C_e^{-1} y$$

Friston et al. 2002, Neuroimage

Hierarchical model

Hierarchical model

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$
$$\vdots$$
$$\theta^{(n-1)} = X^{(n)}\theta^{(n)} + \varepsilon^{(n)}$$

Multiple variance components at each level

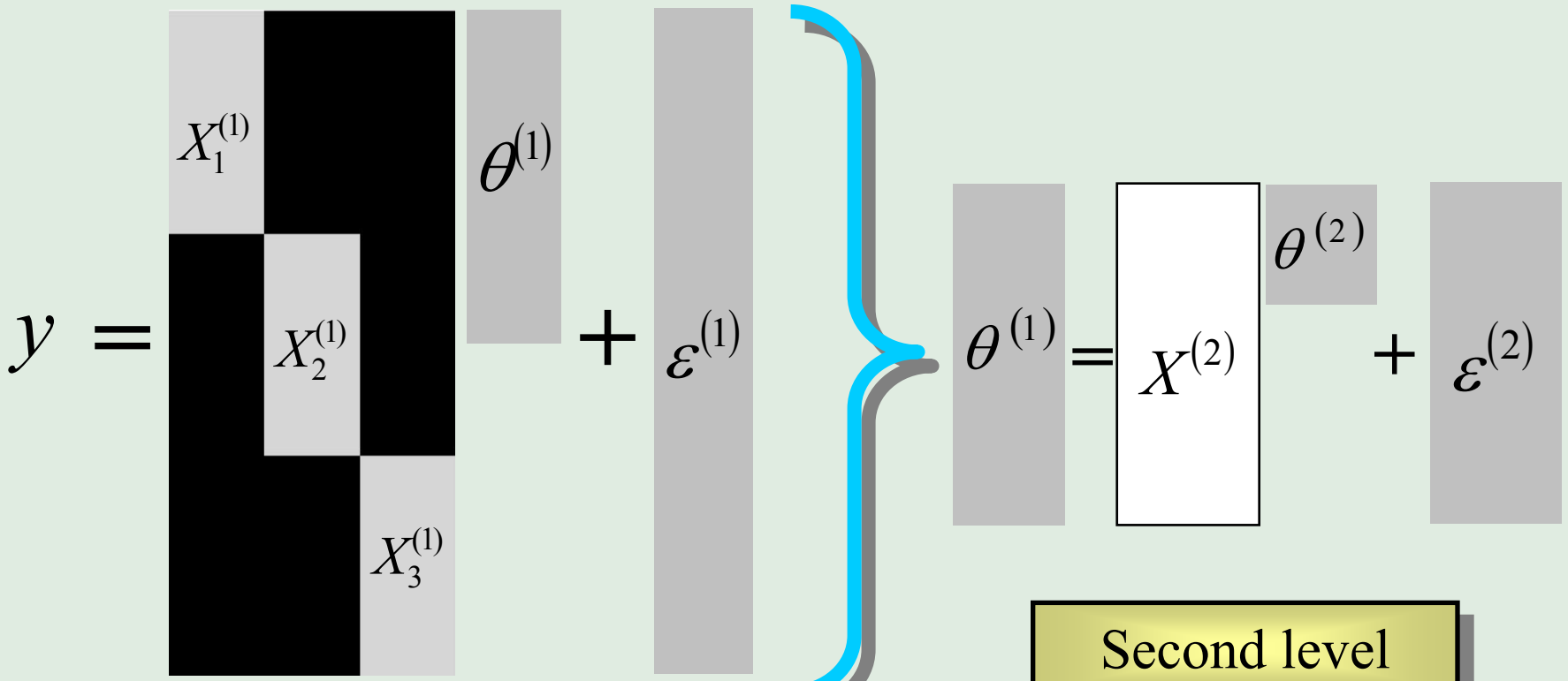
$$C_{\varepsilon}^{(i)} = \sum_{k} \lambda_k^{(i)} Q_k^{(i)}$$

At each level, distribution of parameters is given by level above.

What we don't know: distribution of parameters and variance parameters.

Example: Two level model

$$y = X^{(1)}\theta^{(1)} + \varepsilon^{(1)}$$
$$\theta^{(1)} = X^{(2)}\theta^{(2)} + \varepsilon^{(2)}$$



Estimation

Hierarchical model

$$
\begin{aligned}
y &= X^{(1)}\theta^{(1)} + \varepsilon^{(1)} \\
\theta^{(1)} &= X^{(2)}\theta^{(2)} + \varepsilon^{(2)} \\
&\vdots \\
\theta^{(n-1)} &= X^{(n)}\theta^{(n)} + \varepsilon^{(n)}
\end{aligned}
$$

Single-level model

$$
\begin{aligned}
y = \; & \varepsilon^{(1)} + X^{(1)}\varepsilon^{(2)} + \\
& \ldots + \\
& X^{(1)} \ldots X^{(n-1)}\varepsilon^{(n)} + \\
& X^{(1)} \ldots X^{(n)}\theta^{(n)} \\
= \; & X\theta + e
\end{aligned}
$$

Group analysis in practice

Many 2-level models are just too big to compute.

And even if, it takes a long time!

Is there a fast approximation?

Summary Statistics approach

First level

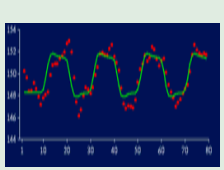
Second level

$$t = \frac{c^T \hat{\alpha}}{\sqrt{V\hat{a}r(c^T \hat{\alpha})}}$$

Data

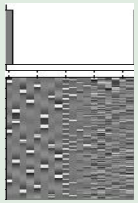
Design Matrix

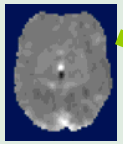
Contrast Images

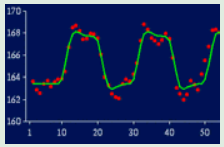


$\hat{\alpha}_1$

$\hat{\sigma}_1^2$

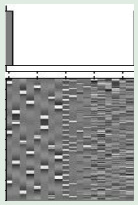


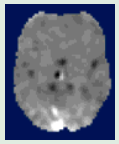




$\hat{\alpha}_2$

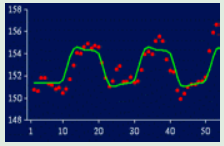
$\hat{\sigma}_2^2$

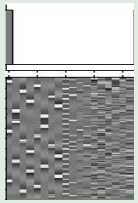


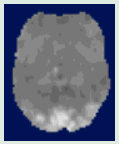


$\hat{\alpha}_{11}$

$\hat{\sigma}_{11}^2$

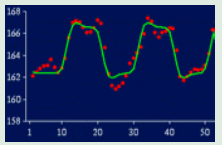


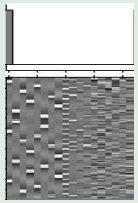


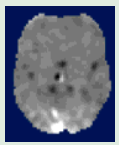


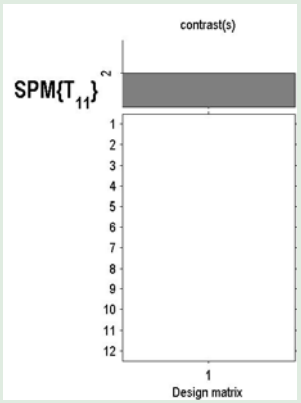
$\hat{\alpha}_{12}$

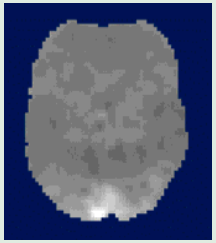
$\hat{\sigma}_{12}^2$



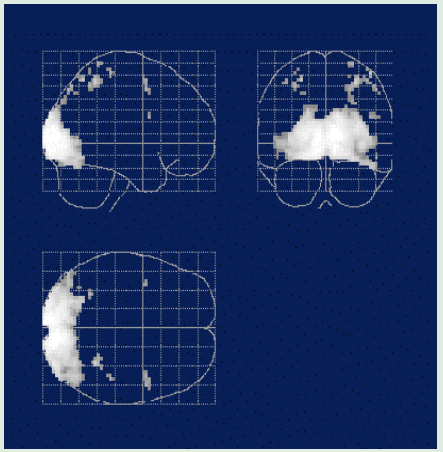








SPM(t)



One-sample
t-test @ 2nd level

Validity of approach

The summary stats approach is exact if for each session/subject:

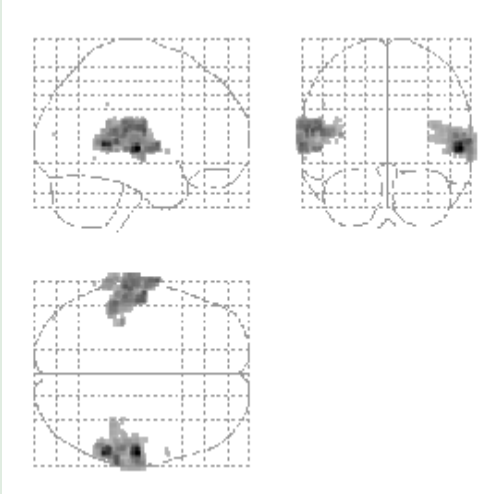
Within-session covariance the same

First-level design the same

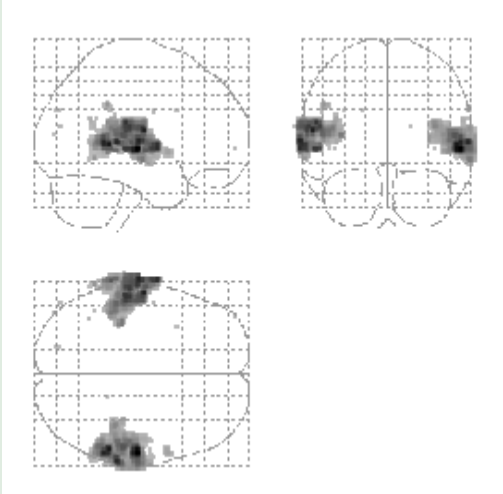
All other cases: Summary stats approach seems to be robust against typical violations.

Auditory Data

Summary statistics



Hierarchical Model



Friston et al. (2004) Mixed effects and fMRI studies, Neuroimage

Multiple contrasts per subject

Stimuli: Auditory Presentation (SOA = 4 secs) of words

Motion	Sound	Visual	Action
"jump"	"click"	"pink"	"turn"

Subjects: (i) 12 control subjects

Scanning: fMRI, 250 scans per subject, block design

Question: What regions are affected by the semantic content of the words?

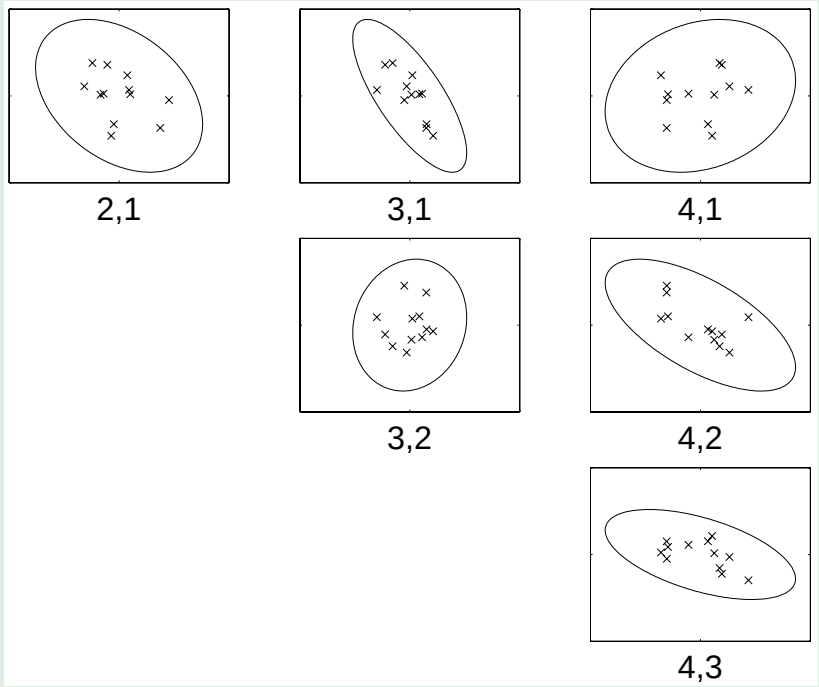
U. Noppeney et al.

ANOVA

1st level:

1.Motion ? 2.Sound ? 3.Visual ? 4.Action

2nd level:

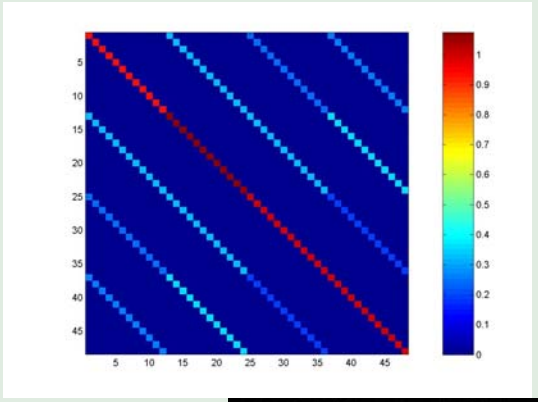


ANOVA

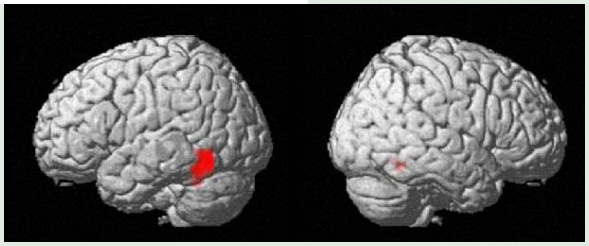
1st level:

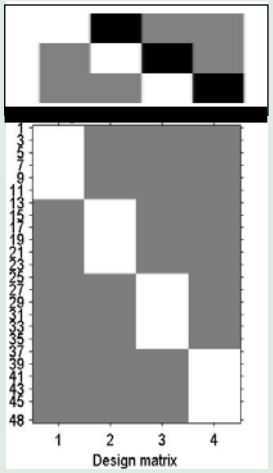
Motion | Sound | Visual | Action

2nd level:



V





$$c^T = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

X

Summary

Linear hierarchical models are general enough for typical multi-subject imaging data (PET, fMRI, EEG/MEG).

Summary statistics are robust approximation for group analysis.

Also accomodates multiple contrasts per subject.