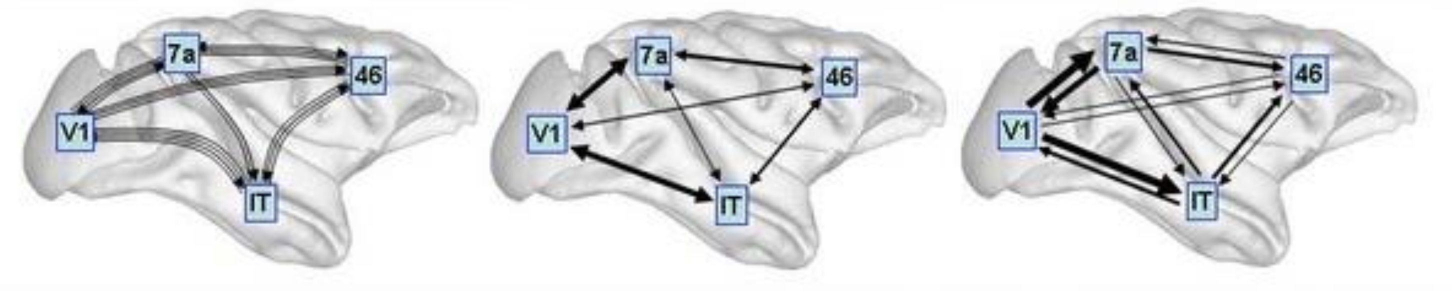
Wellcome Centre for Human Neuroimaging
12 Queen Square Institute of Neurology, London,UK, WC1N 3AR

Principles of Dynamic Causal Modelling

SPM course for EEG/MEG, May 2019

Amirhossein Jafarian

Structural connectivity Functional connectivity Effective connectivity



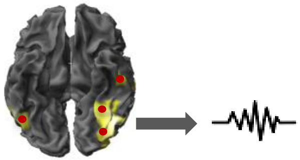
- ***Structural* connectivity**
  presence of axonal connections
- ***Functional* connectivity**
  statistical dependencies between regional time series (correaltion, choherence, etc)
- ***Effective* connectivity**
  causal influences between neuronal populations, and experimental contexts!

O. Sporns 2007, *Scholarpedia*

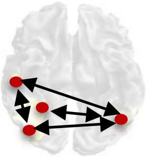
Dynamic causal modelling :generic pipeline

(a) Experimental design

(b) Neuroimaging

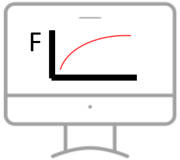


(c) Feature selection



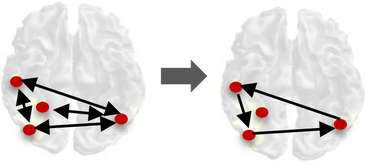
(d) Model specification

Generative models & network structure



(e) Model identification

Variational Laplace



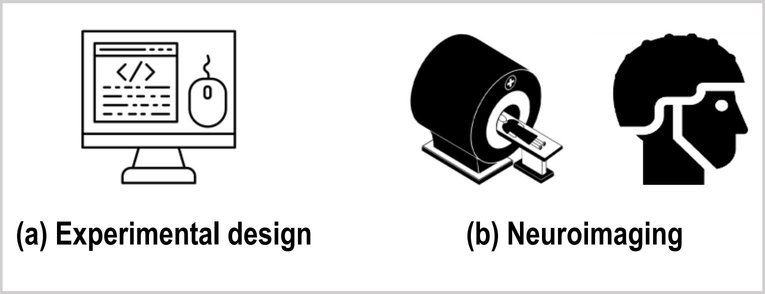
(f) Structure learning

e.g., Bayesian model comparison

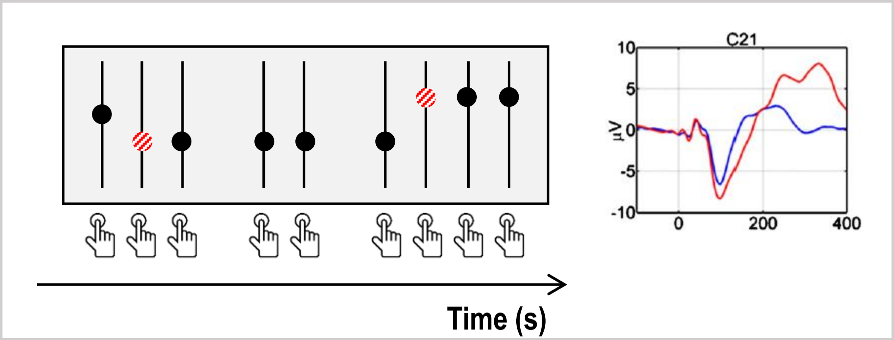
Jafarian et al, 2019

Experimental design

UCL



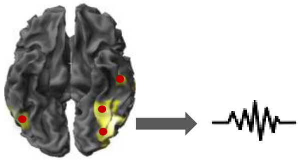
A DCM study begins by articulating hypotheses about brain function and designing an experiment (e.g. factorial design) to test them.



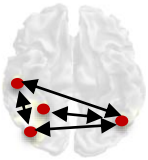
Garrido et al. 2009
Jafarian et al, 2019

Dynamic causal modelling

We select features in the collected data that are important (i.e., informative) from a modelling standpoint!

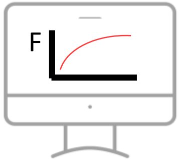


(c) Feature selection



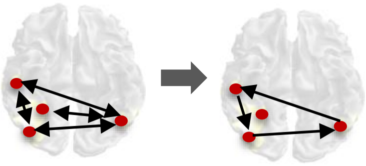
(d) Model specification

generative Models & network structure



(e) Model identification

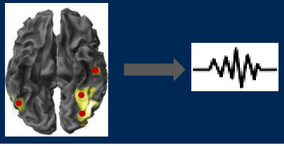
Variational Laplace



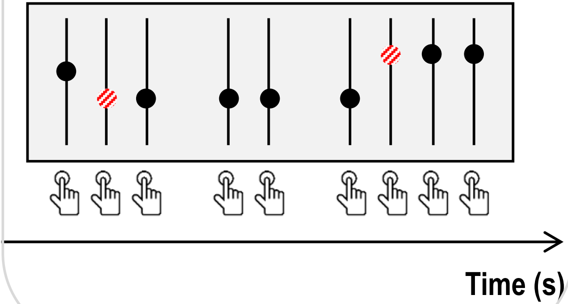
(f) Dynamic causal modelling

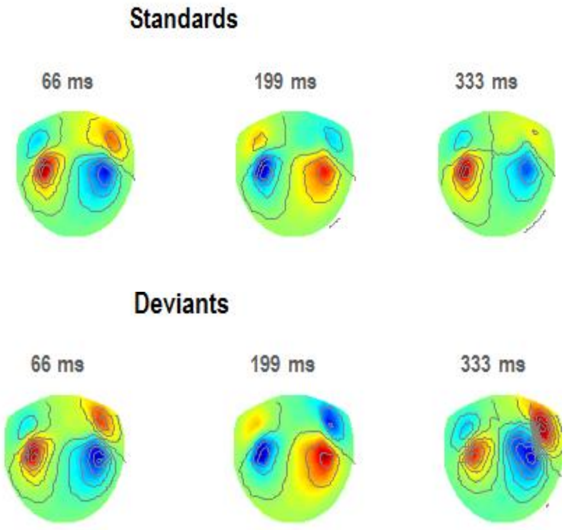
Bayesian model comparison

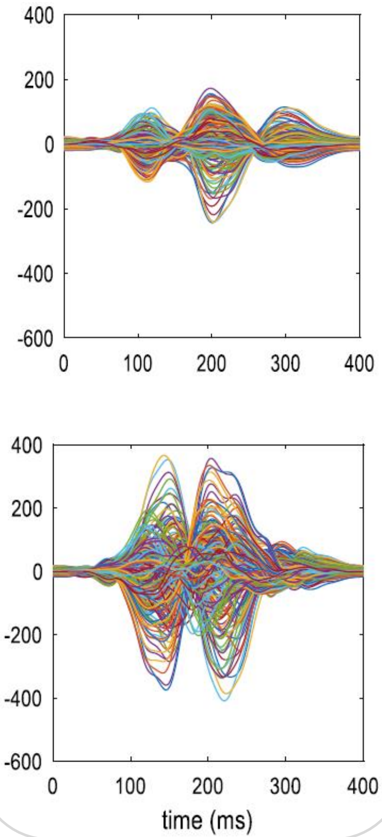
EEG/MEG data feature: event-related potential



9 to 36 tones
Duration: 700ms
Interval: 400-2000ms
Deviants: 0 to 6

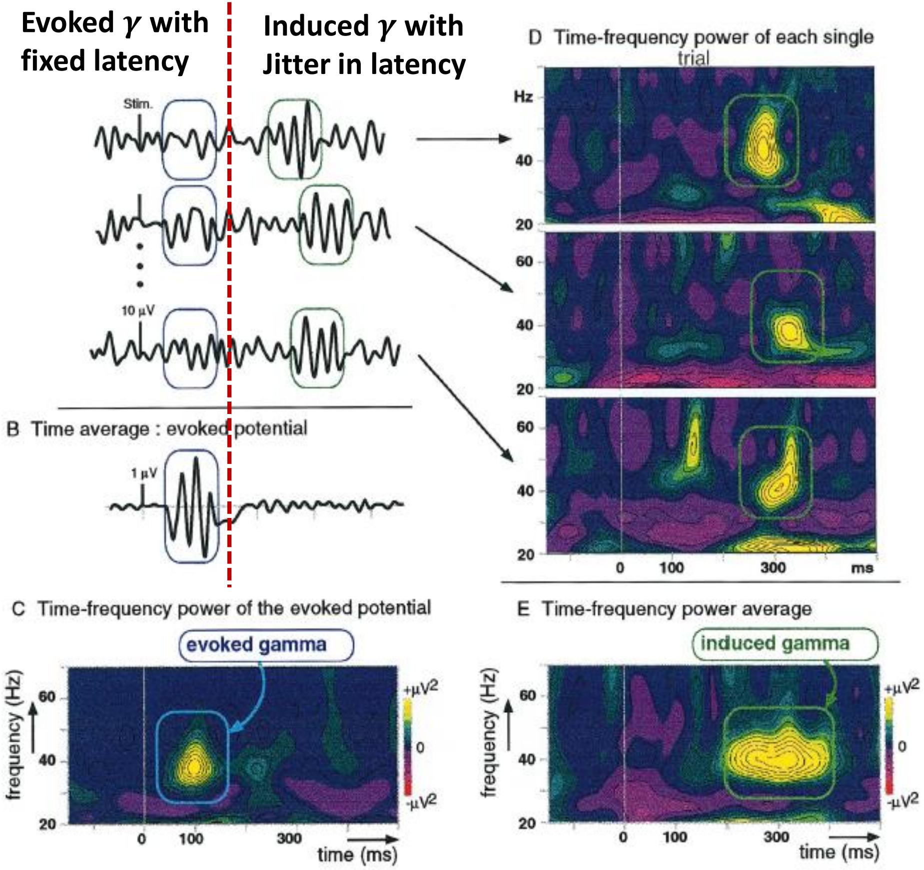




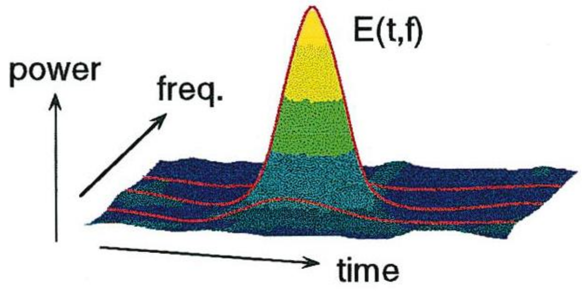


Jafarian et al, 2019

EEG/MEG data feature: time-frequency features

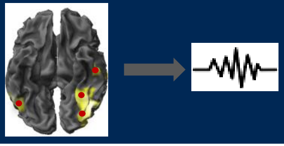


- Evoked oscillations are phase locked to the stimulus, whereas induced oscillations are otherwise!
- Example of induced response: voluntary finger movement which is not phased locked to the onset of the task.



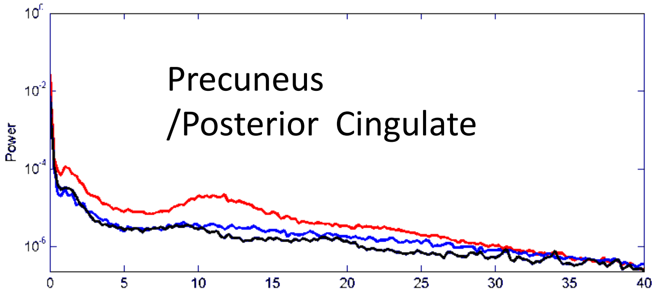
Trends Cogn Sci. 1999 Apr;3(4):151-162

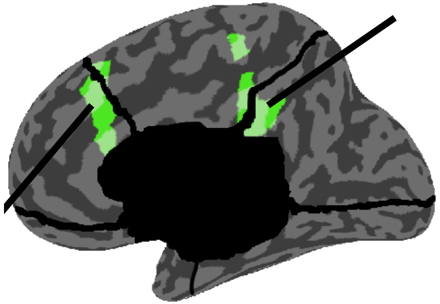
EEG/MEG data feature: (cross)power spectra

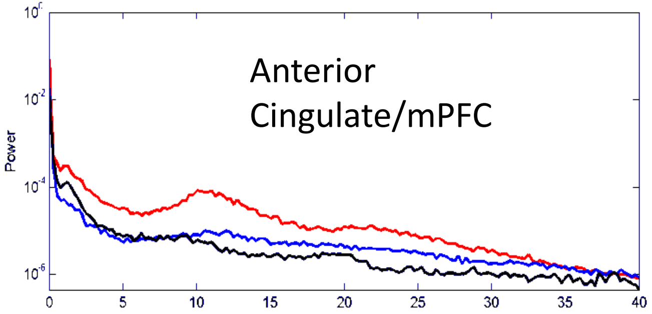


Anesthesia-induced loss of consciousness:

Condition A: Wake
Condition B: Mild sedation: responsive to command
Conditioin C: Deep sedation: loss of consciousness



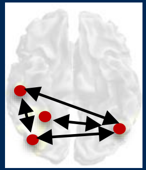




Increased fast activity (e.g., beta to gamma power range) caused by infusion of propofol (vs wake)
Increased slow activity (from delta to alpha) power when consciousness is lost

Murphy et al. 2011

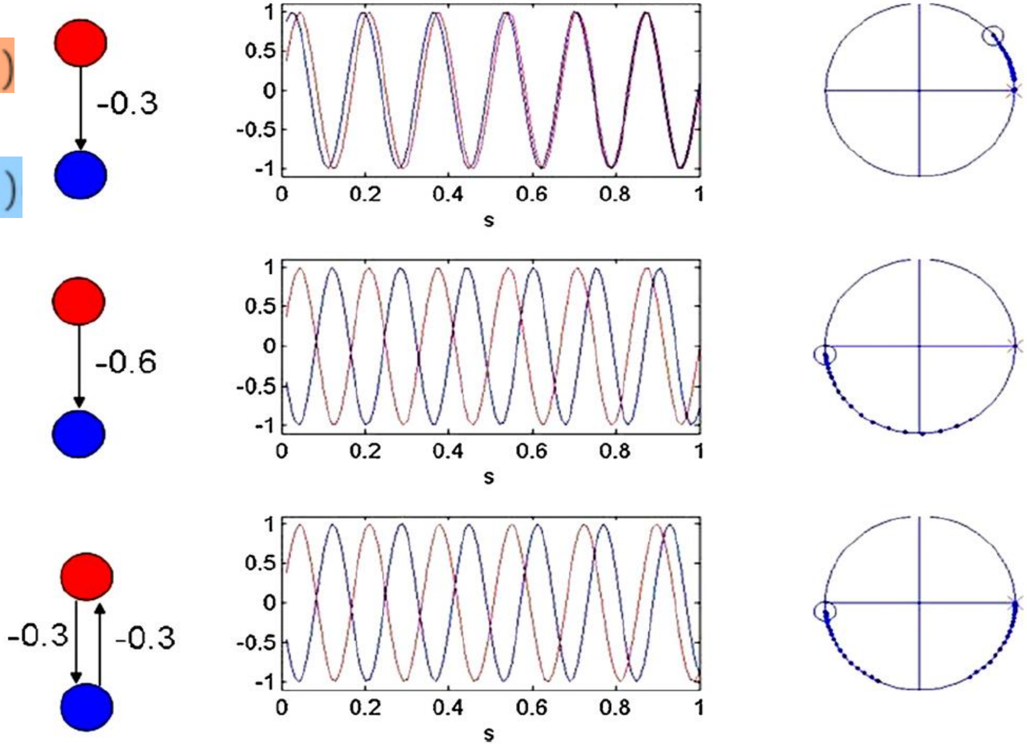
Model specification:
Phenomenological: Model of Phase Coupling



$$\dot{\phi}_1 = f + a_{12}\sin(\phi_1 - \phi_2)$$

$$\dot{\phi}_2 = f + a_{21}\sin(\phi_2 - \phi_1)$$

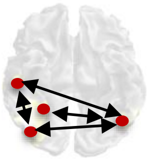
phase coupling between regions induced Synchronization!



Penny, et al 2009

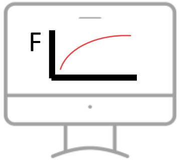
Dynamic Causal Modelling

The hypotheses are then formally expressed in terms of biologically informed model architectures, each describing possible interactions between experimental inputs and neuronal dynamics.



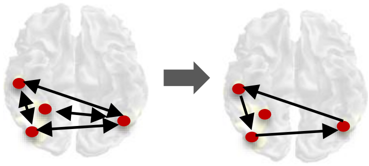
(d) Model specification

Generative models & network structure



(e) Model identification

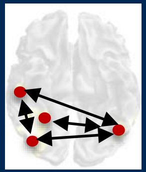
Variational Laplace



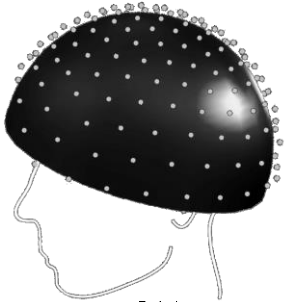
(f) Dynamic causal modelling

Bayesian model comparison

Model specification: model of brain region



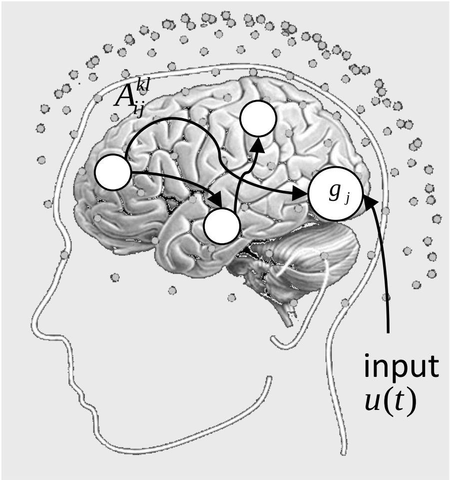
- Physiological : biologically inspired model that can emulate different temporal, spectral and spatial features of E/MEG data.
- Phenomenological: quantitative models that governs some aspects of E/MEG dynamics.



$d(t)$

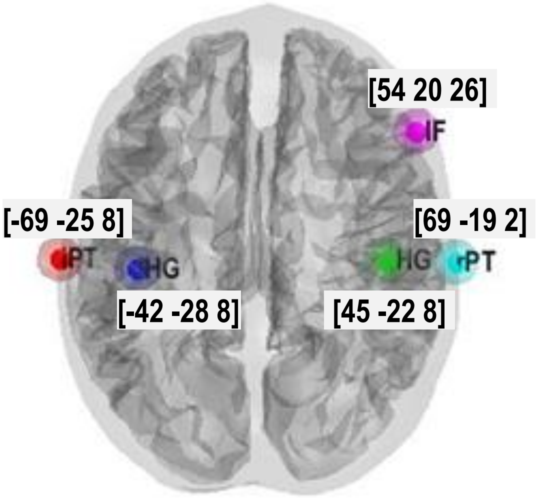
Data in channel space

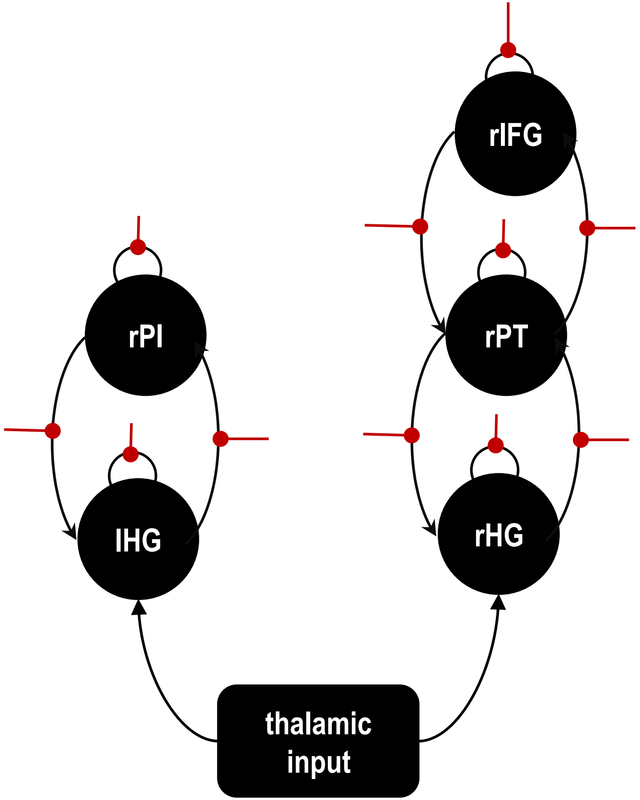
Inversion of electromagnetic model *L*



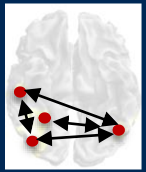
Chen et al 2008

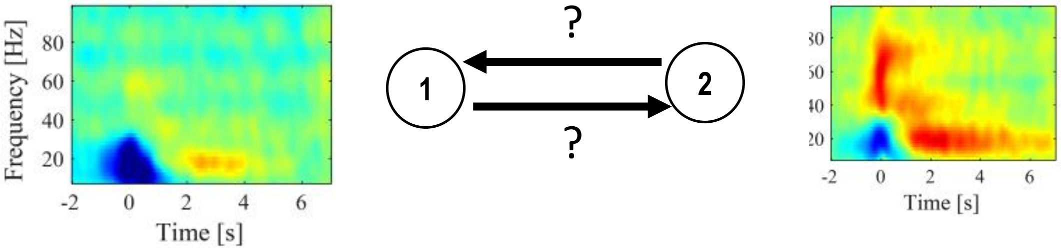
Model specification:





Model specification: phenomenological

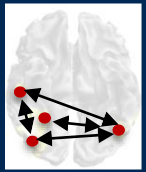


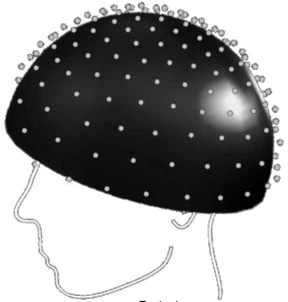


Modelling dynamic changes in power spectral density caused by external input and/or coupling strengths. (*e.g., beta activity in region 1 leads to a gamma increase in region 2)*

van Wijk et al, 2013,2018

Model specification: phenomenological





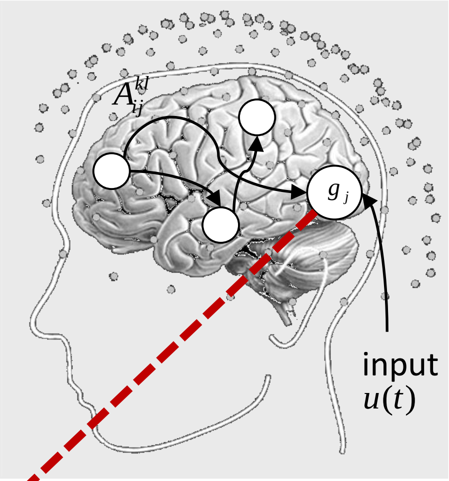
$d(t)$

Data in channel space

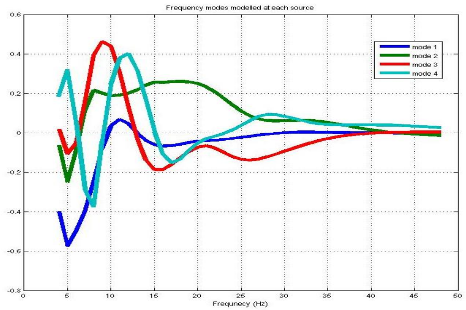
Inversion of electromagnetic model L

$$x(t) = L^{-} d(t)$$

$$\tilde{g}_j^k(t) = U^T \left| FT(x_j(t)) \right|^2 = U^T \begin{bmatrix} \tilde{g}_j(\omega_1, t) \\ \vdots \\ \tilde{g}_j(\omega_{K'}, t) \end{bmatrix}$$



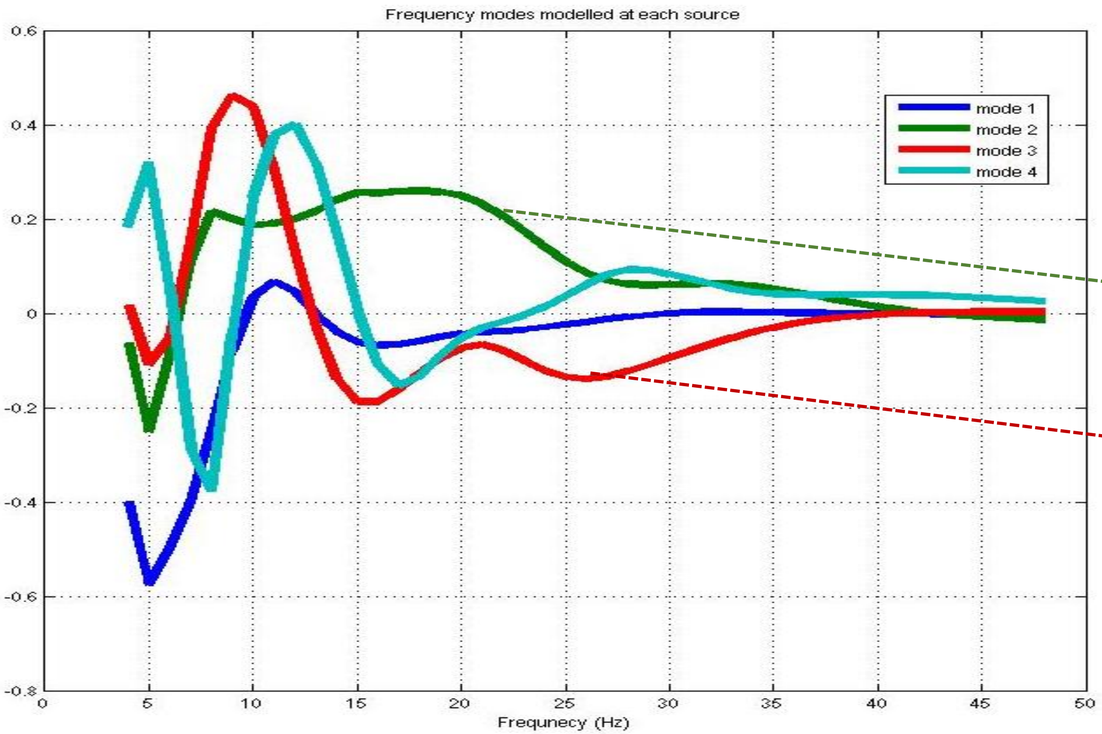
K frequency modes in j-th source



Chen et al 2008

Bi linear model of changes power spectrum

$$\tau\dot{g}(t) = (A + \sum uB)g(t) + CU$$



$$\begin{bmatrix} g_j(\omega_1, t) \\ \vdots \\ g_j(\omega_K, t) \end{bmatrix}$$

TF analysis of one brain region decomposed into several modes

van Wijk ,2018
Chen et al, 2008

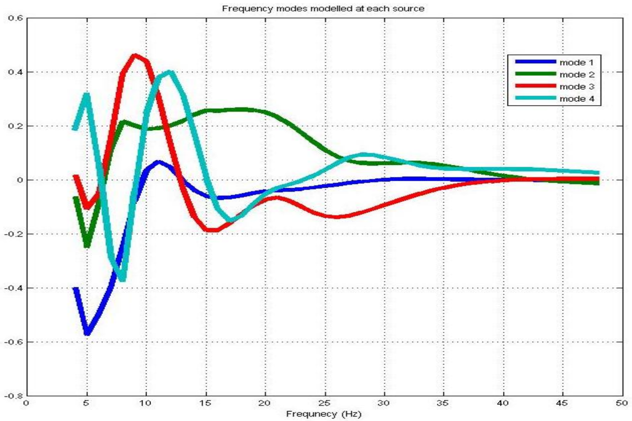
Bi linear model of changes power spectrum

$$\tau\dot{g}(t) = (A + \sum uB)g(t) + CU$$

Intrinsic (within-source) coupling

$$\tau\dot{g}(\omega,t) = \tau\begin{bmatrix}\dot{g}_1\\ \vdots\\ \dot{g}_J\end{bmatrix} = \left\{\begin{bmatrix}A_{11} & \cdots & A_{1J}\\ \vdots & \ddots & \vdots\\ A_{J1} & \dots & A_{JJ}\end{bmatrix} + \sum U(t)\begin{bmatrix}B_{11} & \cdots & B_{1J}\\ \vdots & \ddots & \vdots\\ B_{J1} & \dots & B_{JJ}\end{bmatrix}\right\}g(\omega,t) + \begin{bmatrix}C_1\\ \vdots\\ C_J\end{bmatrix}u(t)$$

Extrinsic (between-source) coupling



Linear (within-frequency) coupling

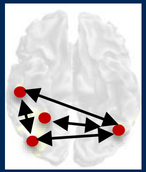
$$A_{ij} = \begin{bmatrix}A_{ij}^{11} & \cdots & A_{ij}^{1K}\\ \vdots & \ddots & \vdots\\ A_{ij}^{K1} & \dots & A_{ij}^{KK}\end{bmatrix}$$

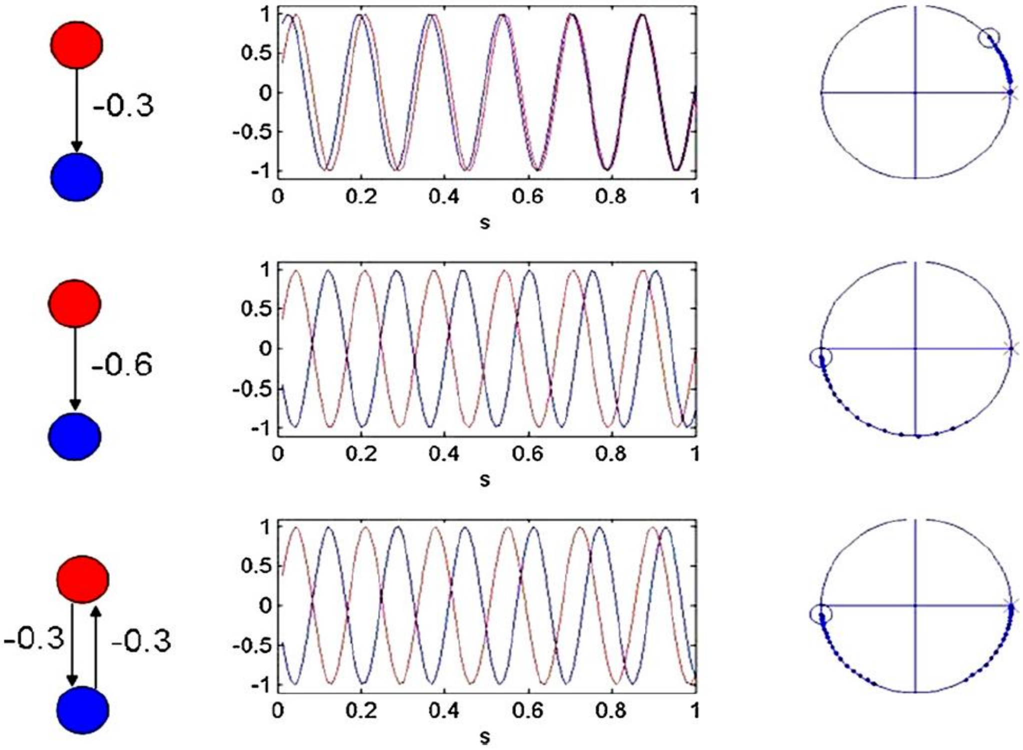
How frequency K in region j affects frequency 1 in region i

Nonlinear (between-frequency) coupling

van Wijk ,2018

Model specification:
Phenomenological: Model of Phase Coupling





phase coupling between regions induced Synchronization!

Parameter of interest:

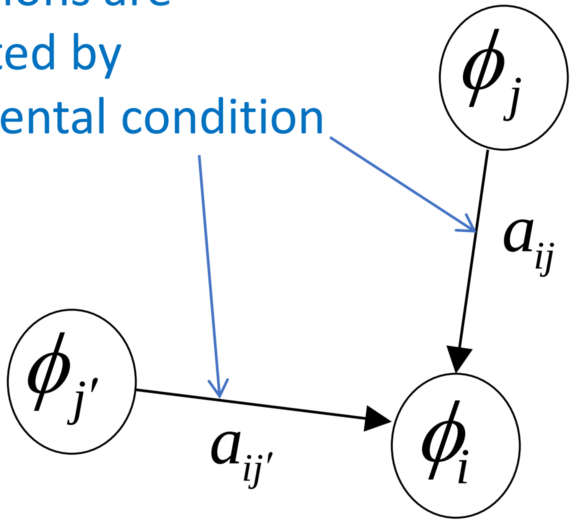
(frequency-dependent) coupling values

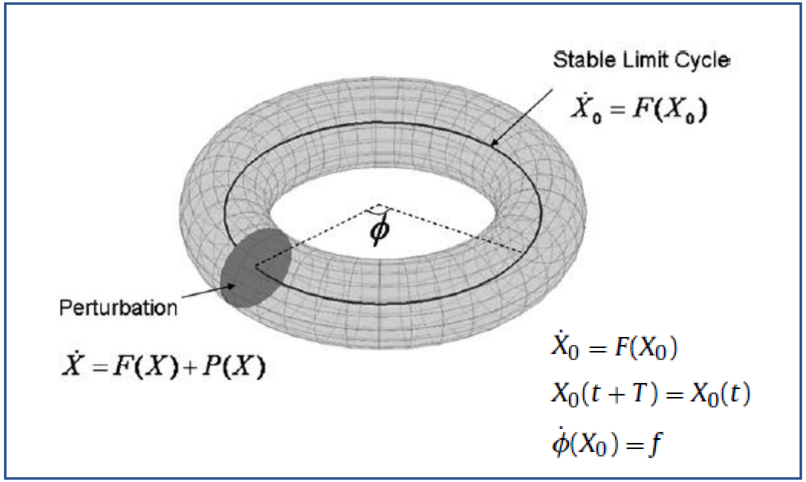
$$\dot{\phi}_1 = f + a_{12}\sin(\phi_1 - \phi_2)$$

$$\dot{\phi}_2 = f + a_{21}\sin(\phi_2 - \phi_1)$$

Penny, et al 2009

Connections are modulated by experimental condition



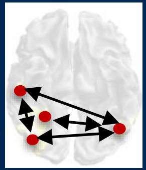


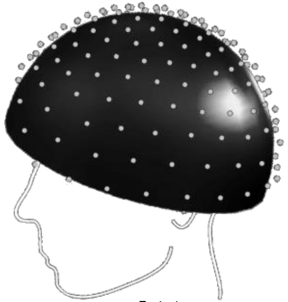
$$\dot{\phi}_i = f_i + \sum a_{i,j} \sin(\phi_i - \phi_j)$$

f_i is intrinsic frequency in i^{th} region;
ϕ_i is the phase in the in i^{th} region and effective connectivity is denoted by a_{ij}.

Penny, et al 2009
van Wijk ,2018

Model specification:
biophysical models

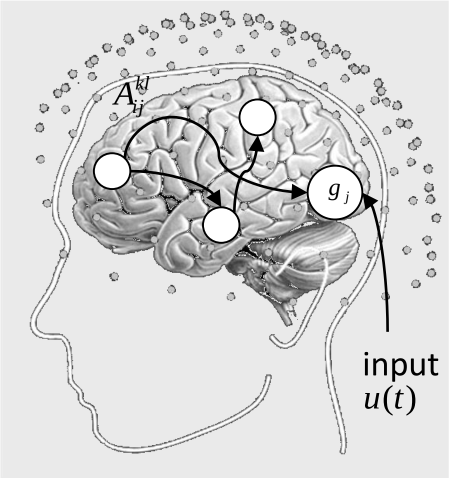




$d(t)$

Data in channel space

Inversion of electromagnetic model L

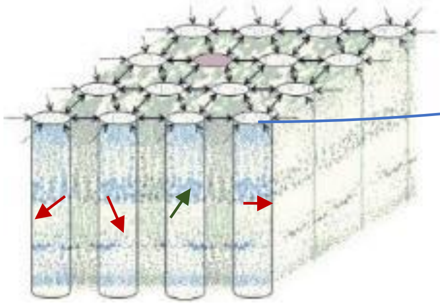


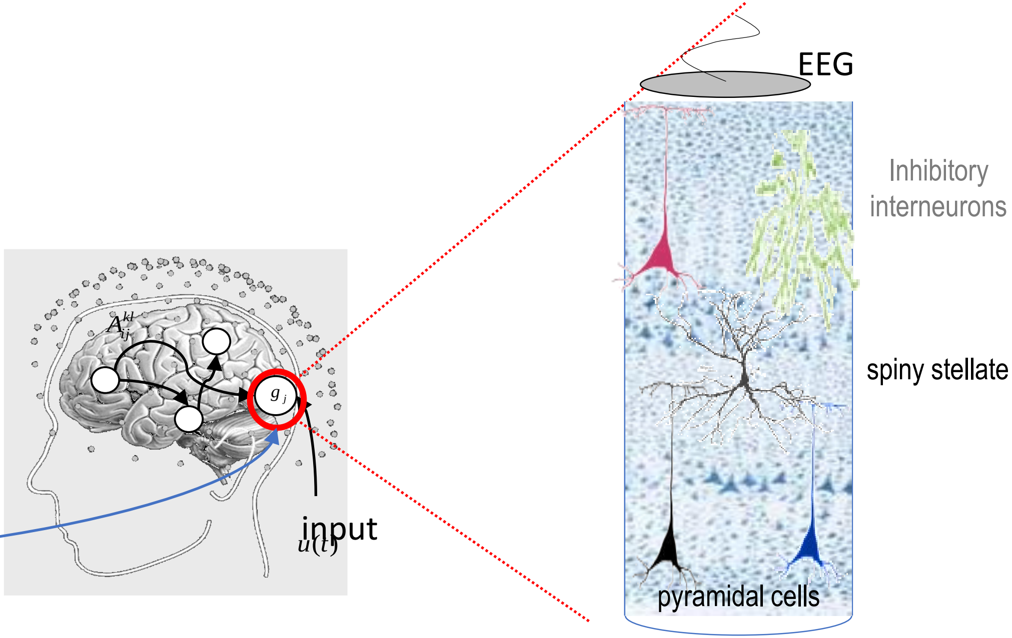
Model specification:

biophysical model: cortical column

Mountcastle

Hubel and Wiesel

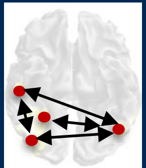


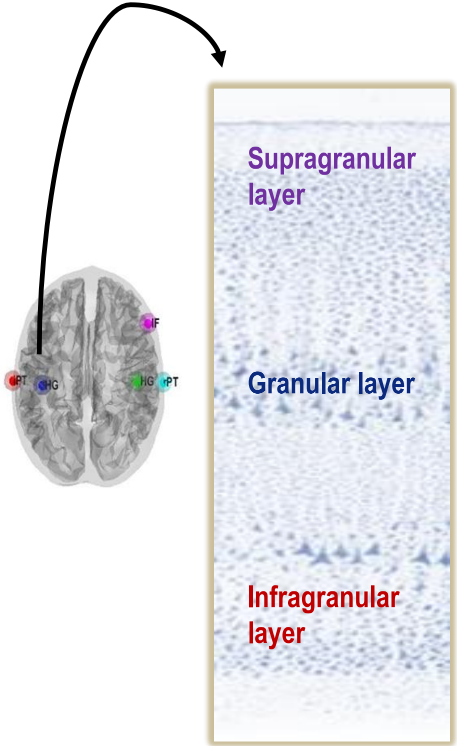


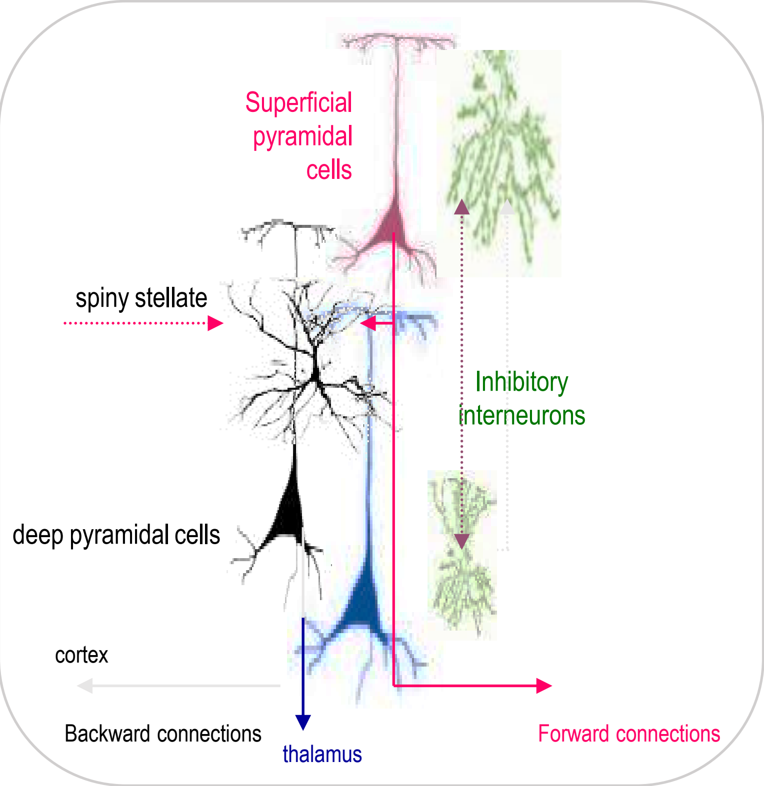
≈10,000 neurons

Mountcastle 1957, Hubel and Wiesel 1959

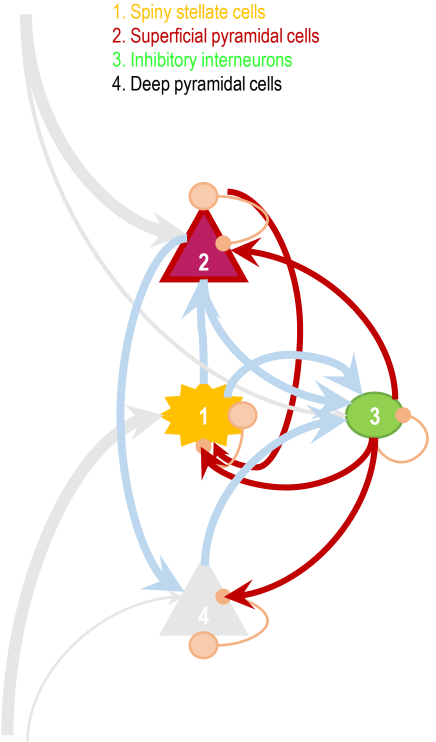
Model specification: physiological (neural mass model)



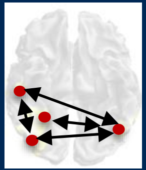


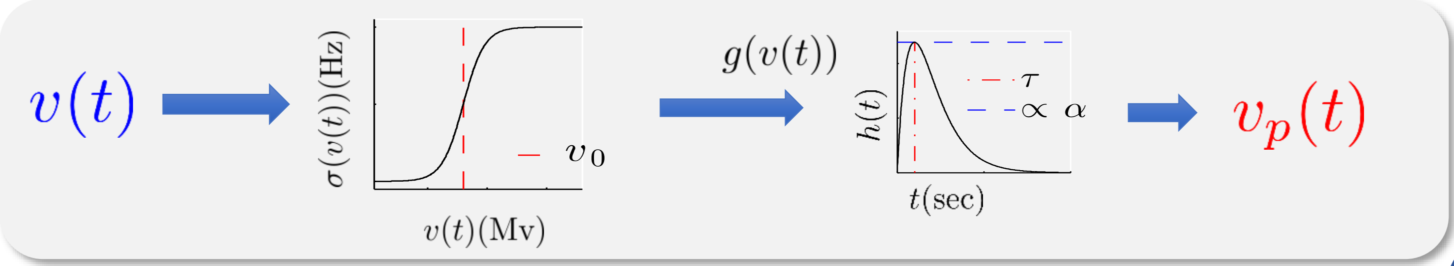


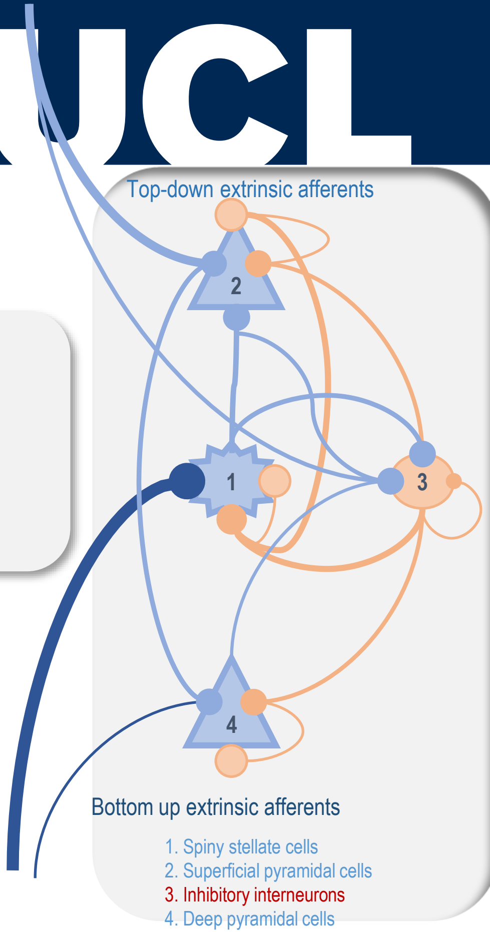
Abstract model



Model specification:
Physiological (neural mass model)





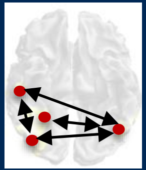


Dynamics of each neuronal population is described by two conversion operators

1- Mean potential to mean firing rates

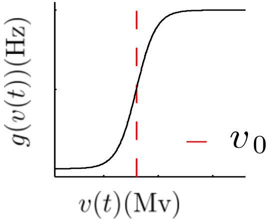
2- Firing rates to potential conversion

Model specification:
physiological (neural mass model)



Mean firing rate (sigmoid function)

$v_{in}(t)$



$g(v_{in}(t))$

It is hoped that the relative simplicity of the model may serve as a basis for a better understanding of the functional significance of cortical complexity.

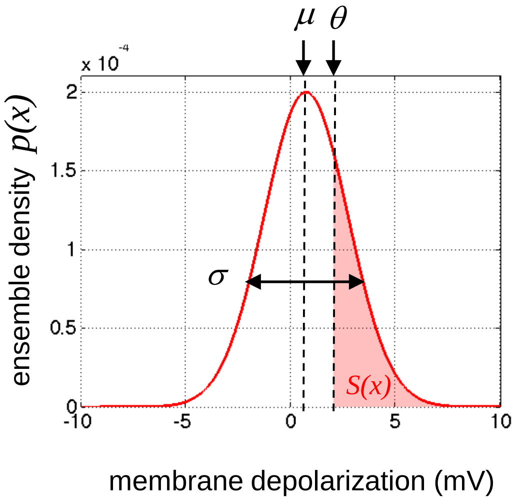
(Hugh Wilson and Jack Cowan, 1973)

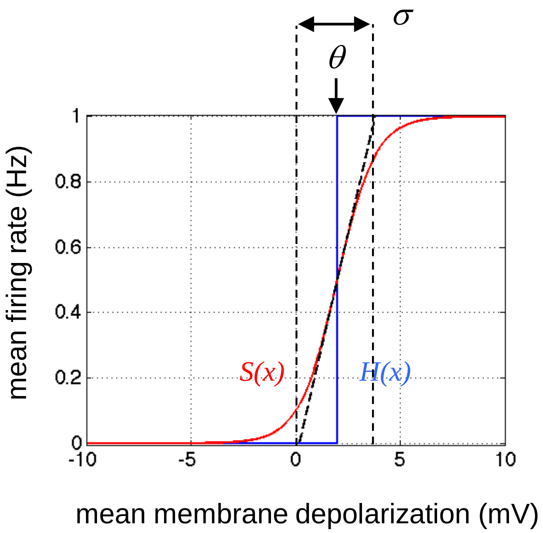
Model specification:
physiological (neural mass model)

$x_j(t)$: post-synaptic potential of j^{th} neuron within its ensemble

$$\frac{1}{N-1}\sum_{j'\neq j} H\left(x_{j'}(t)-\theta\right) \xrightarrow{N\to\infty} \int H\left(x(t)-\theta\right) p\left(x(t)\right)dx$$

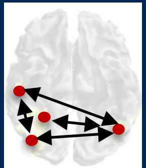
$\approx S(\mu)$ mean-field firing rate





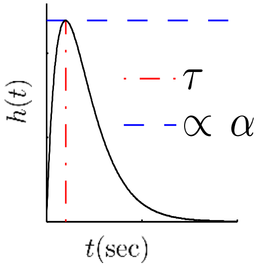
Daunizeau J

Model specification:
physiological (neural mass model)



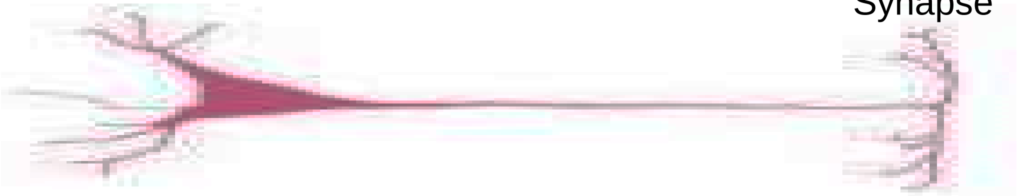
W. Freeman

$$g(v_{in}(t), v_0)$$

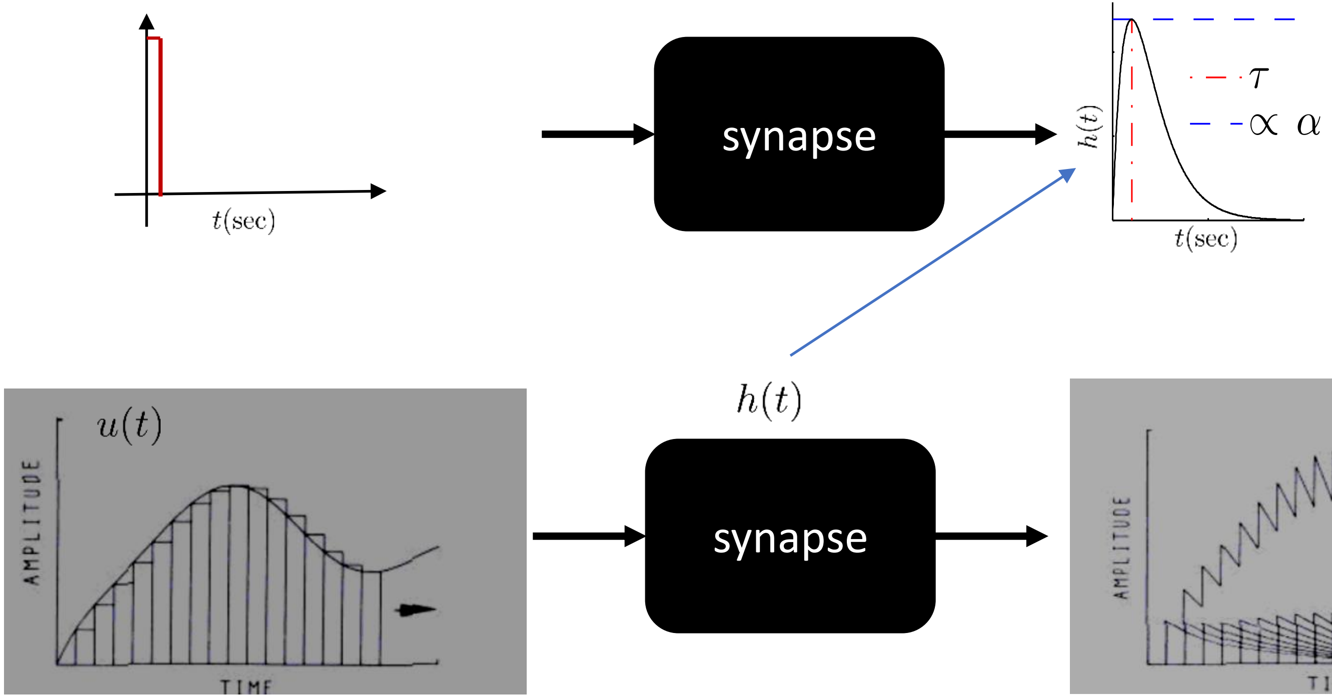


$$v_{out}(t)$$

$$v_{out}(t) = g(v_{in}(t)) * h(t)$$



Freeman 1975



$$h(t) = \alpha \frac{-t}{\tau} e^{-\frac{t}{\tau}}$$

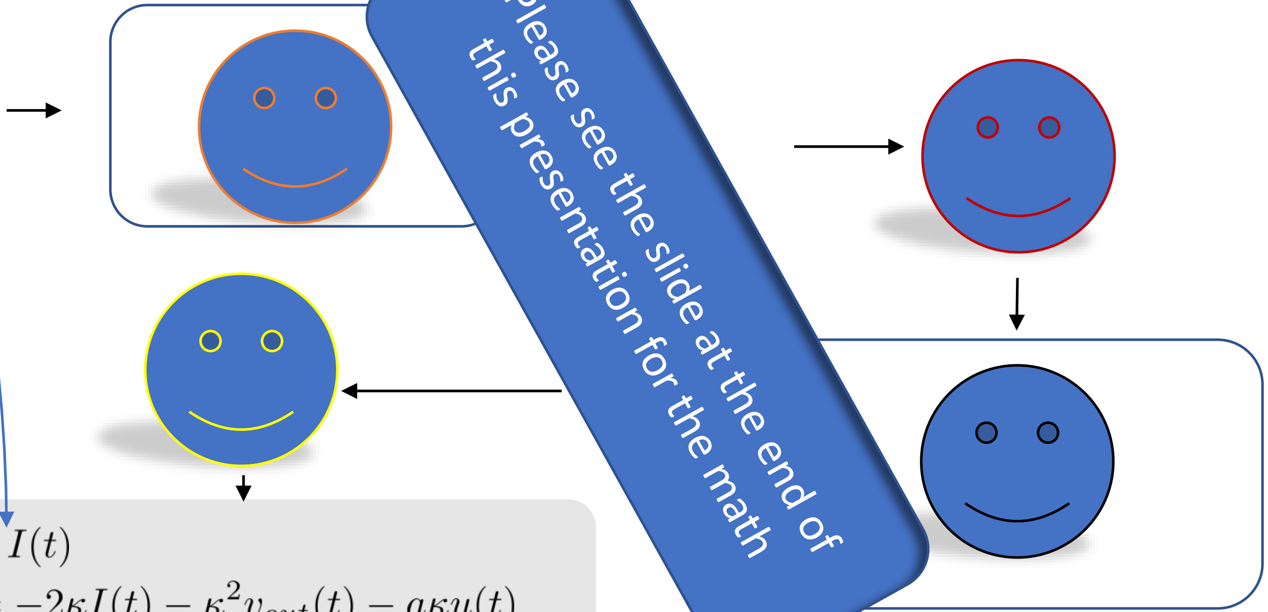
$$v_{out}(t) = u(t) * h(t) = \int h(t-k)u(k)\mathrm{d}k$$

Freeman 1975

Model specification:
Physiological (neural mass model)

$$v_{out}(t) = u(t) * h(t)$$

$$h(t) = a\frac{-t}{\kappa}e^{-\frac{t}{\kappa}}$$

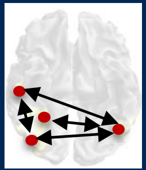


Please see the slide at the end of this presentation for the math

$$v'_{out} = I(t)$$
$$I'(t) = -2\kappa I(t) - \kappa^2 v_{out}(t) - a\kappa u(t)$$

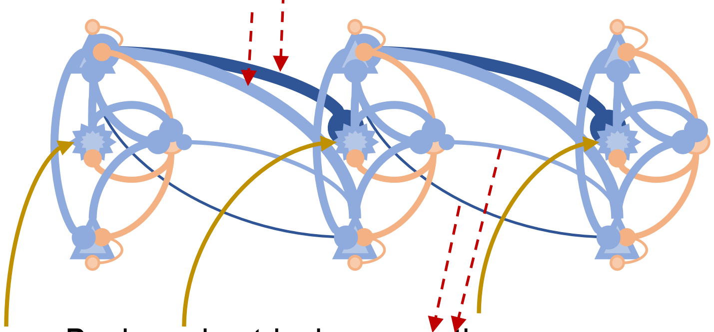
Parameter a is called intrinsic connectivity and κ is rate constant

Model specification: multi-region interconnection

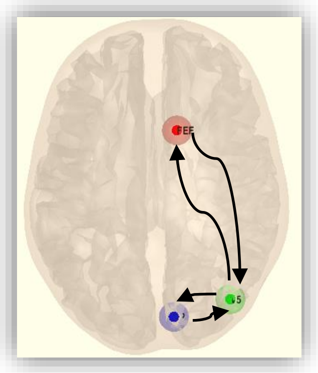


V1

Forward extrinsic connection



Backward extrinsic connection



Intrinsic (connectivity) parameters

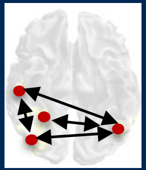
$$a = 512 \cdot \begin{bmatrix} -4 & -2 & -2 & 0 \\ 2 & -4 & -1 & 0 \\ 2 & 1 & -2 & 1 \\ 0 & 1 & -1 & -2 \end{bmatrix} \qquad \kappa = 16 \cdot \begin{bmatrix} 8 & 4 & 1 & 2 \end{bmatrix}$$

Extrinsic (connectivity) parameters

$$\underbrace{A^{(1,2)} = 512 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}}_{\text{Superficial pyramidal} \to \text{spiny stellate}} \quad \underbrace{A^{(4,2)} = 512 \cdot \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\text{Superficial pyramidal} \to \text{deep pyramidal}} \quad \underbrace{A^{(2,4)} = 512 \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Deep pyramidal} \to \text{superficial pyramidal}} \quad \underbrace{A^{(3,4)} = 512 \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{Deep pyramidal} \to \text{inhibitory interneurons}}$$

Friston et al. 2017

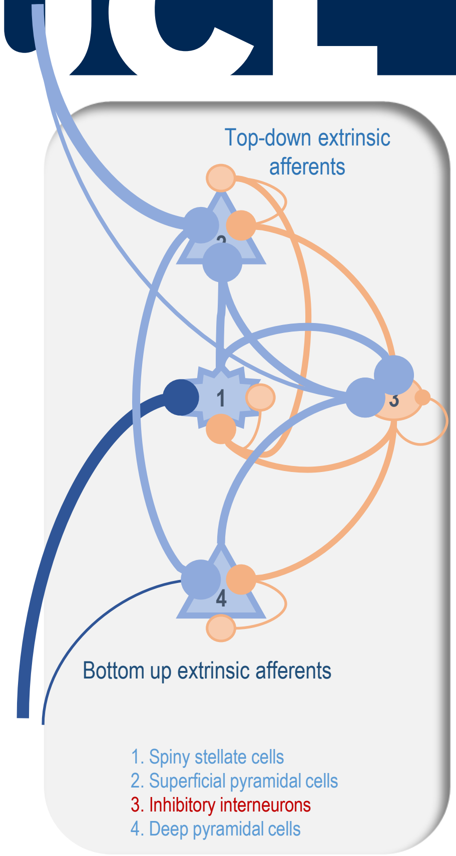
Model specification:



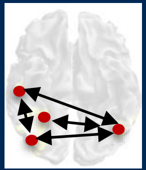
DCM: model equation

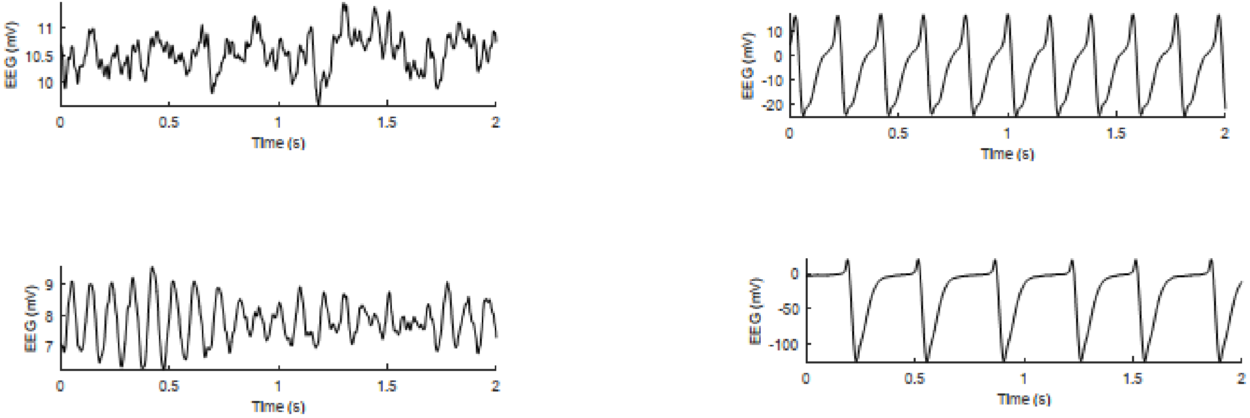
Neural state equations $\dot{x} = f(x, u, \theta_1)$

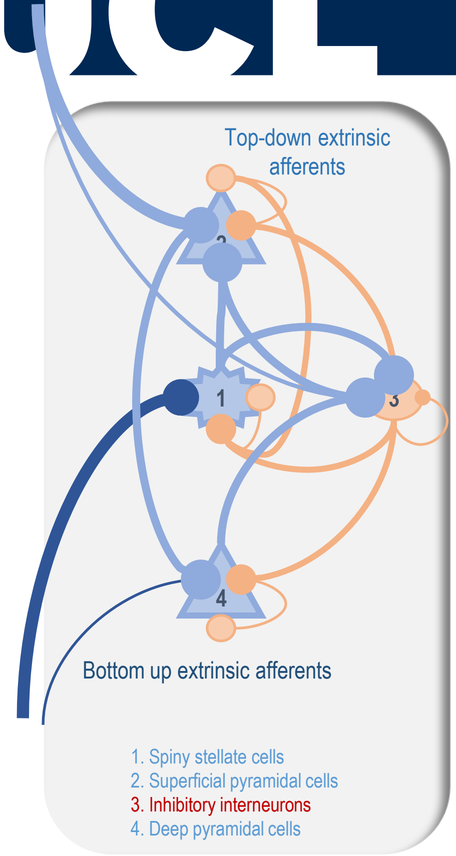
Observation function $y = g(x, \theta_2) + \epsilon$



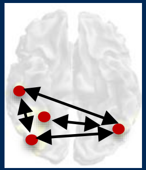
Model specification:
forward simulation







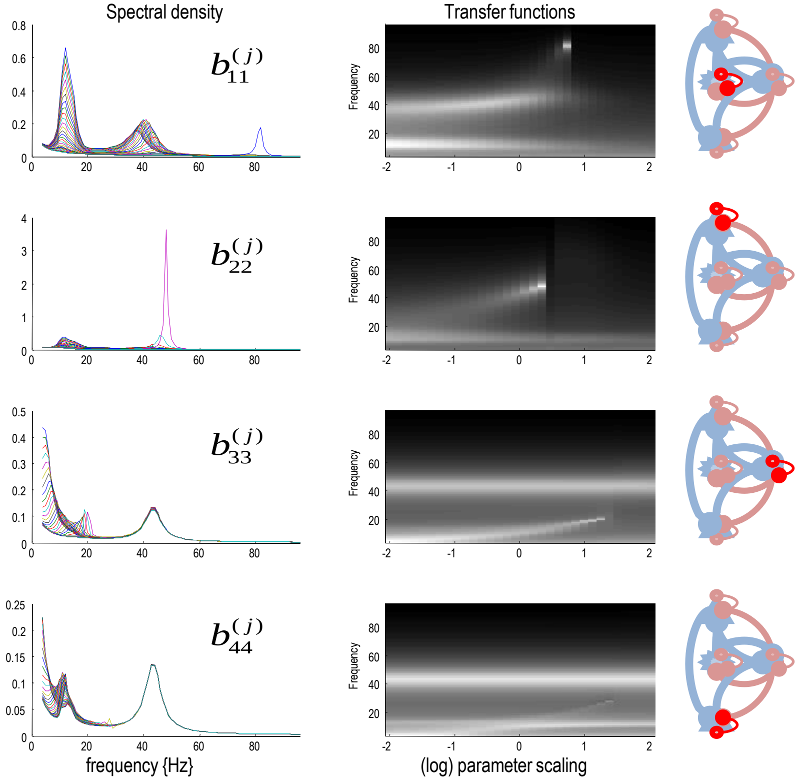
Model specification:
forward simulation

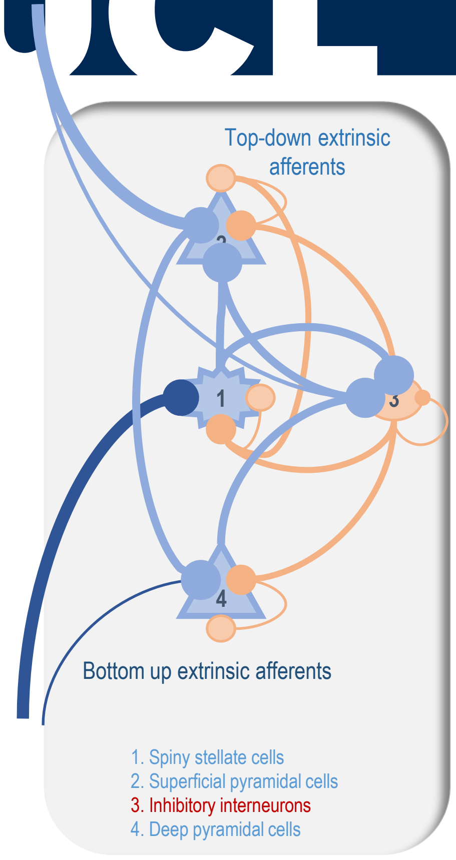


Increasing the self-inhibition of spiny stellate cells rapidly suppresses alpha activity and increases the frequency of gamma activity until a bifurcation at a peak gamma activity of about 80 Hz.

This phase transition is seen even earlier as the self-inhibition of superficial pyramidal cells increases, with a peak gamma of about 42 Hz.

The effects of increasing self-inhibition of inhibitory interneurons and deep pyramidalcells are to suppress alpha activity and convert it into fast activity.



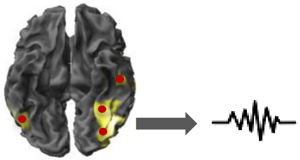


Friston et al. 2017

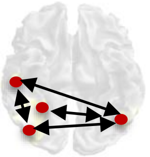
Dynamic causal modelling

(a) Experimental design

(b) Neuroimaging

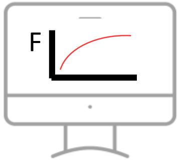


(c) Feature selection



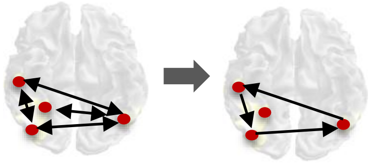
(d) Model specification

generative Models & network structure



(e) Model identification

Variational Laplace

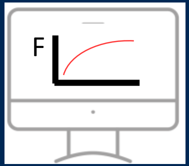


(f) Structure learning

Bayesian model comparison

Bayesian model reduction

Model identification:



- Forward simulation of the model : given a generative model (e.g., NMM) , we change parameters and observe simulated brain activity. This is called prediction of the model for a given set of parameters.

- Inverse problem: given measured electrophysiological data, we would like to know what sorts of (biological) parameters (forward/backward connections, synaptic efficacy) are likely to produce the measurement data. We would like to know which model among many, better explains the data.

Model identification:

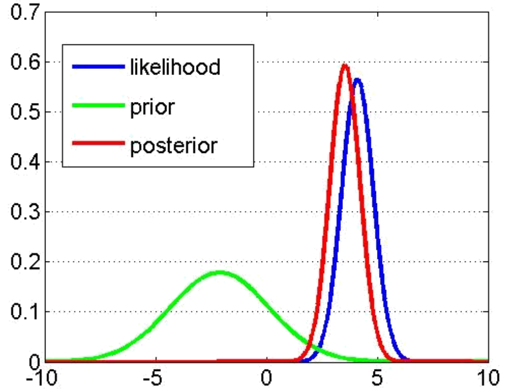
DCM: model structure

Priors on all parameters $N(\mu, \Sigma)$

Neural state equations $\dot{x} = f(x, u, \theta_1)$

Observation function $y = g(x, \theta_2) + \epsilon$

→ Likelihood $p(y|\theta, m)$



DCM: Bayesian inference (expectation-maximization)

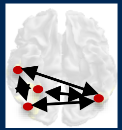
Posterior parameter estimates $N(\mu, \Sigma)$

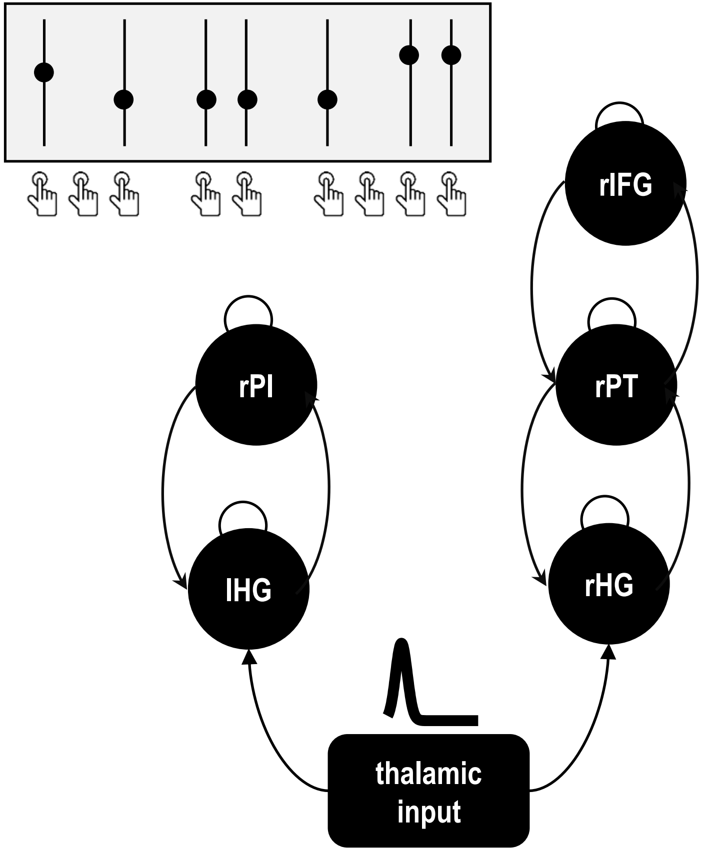
Model evidence
or 'Free Energy'

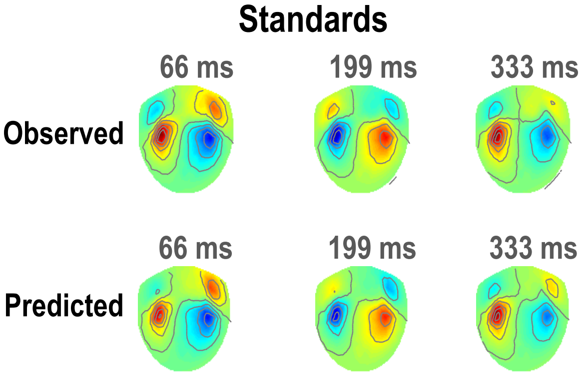
$$p(y|m) = \int p(y|\theta, m)p(\theta|m)d\theta$$

"Accuracy - Complexity"

Model inversion:

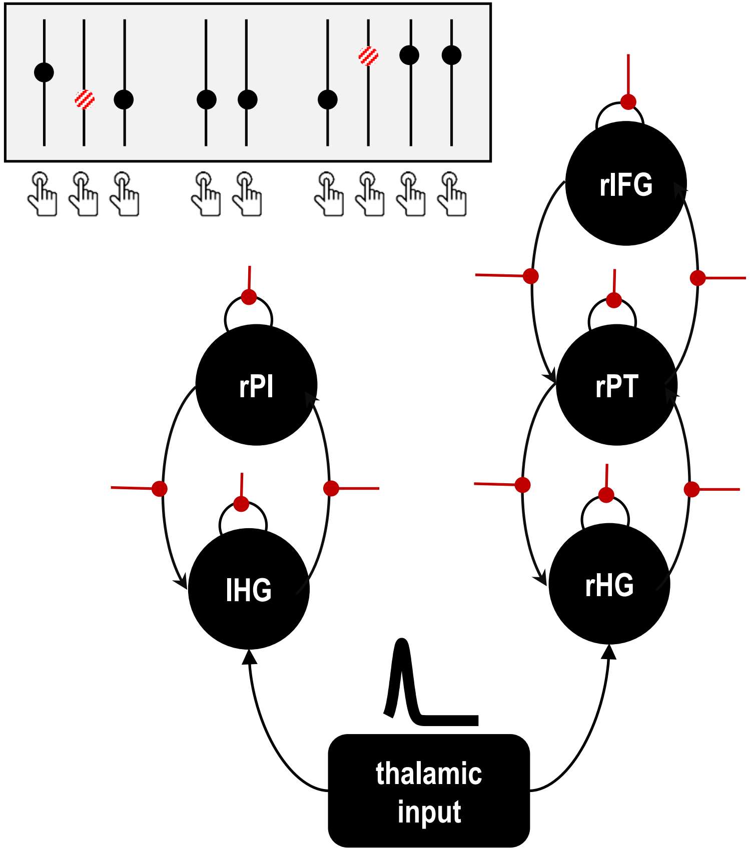


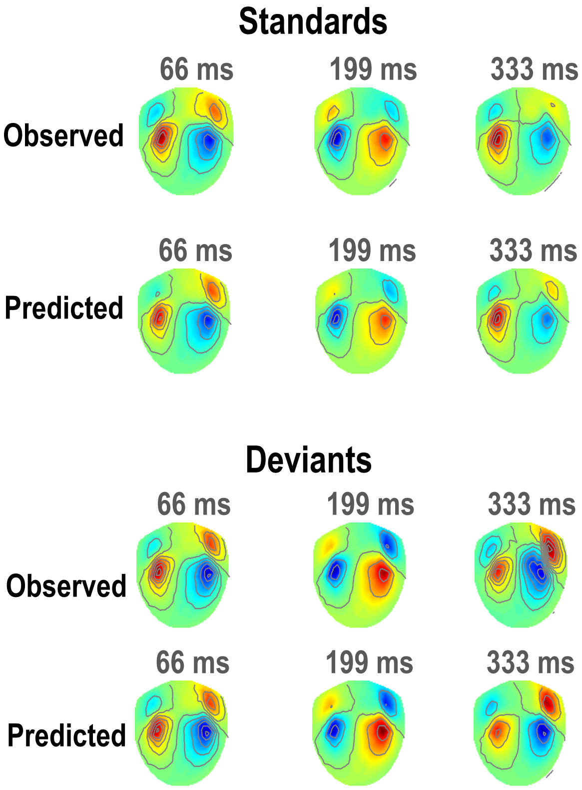




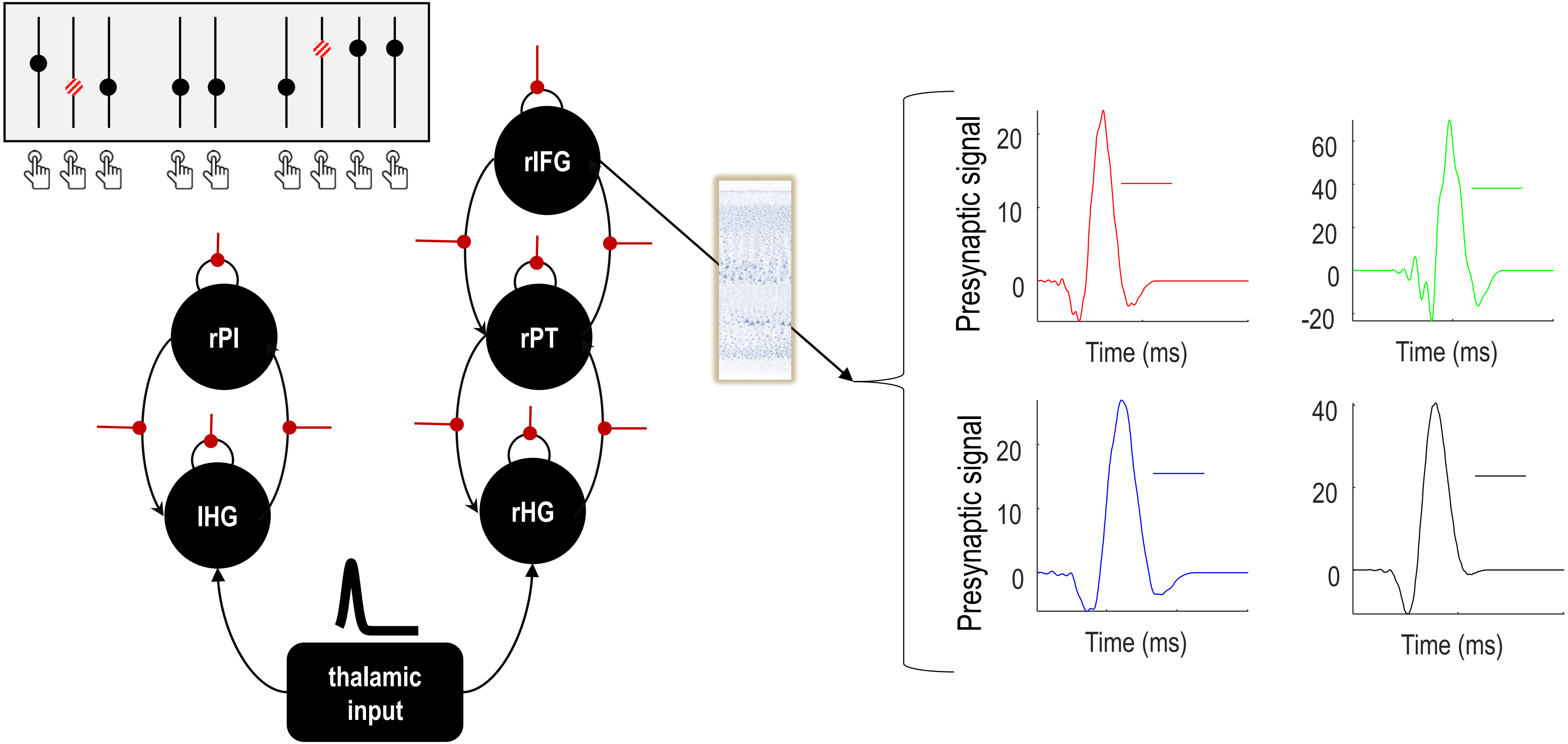
What should be added to the estimated parameters of the neuronal model that generates the standard response to produce the deviant response?

Model inversion: condition specific inversion





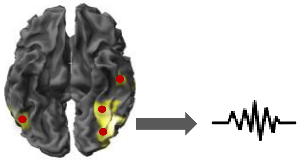
Model specification:
prediction of the model given the estimated parameters



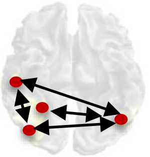
Dynamic causal modelling

(a) Experimental design

(b) Neuroimaging

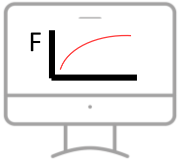


(c) Feature selection



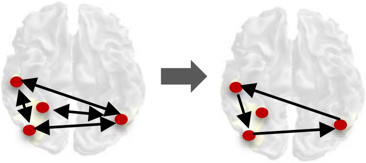
(d) Model specification

generative Models & network structure



(e) Model identification

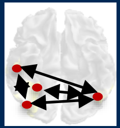
Variational Laplace



(f) Structure learning

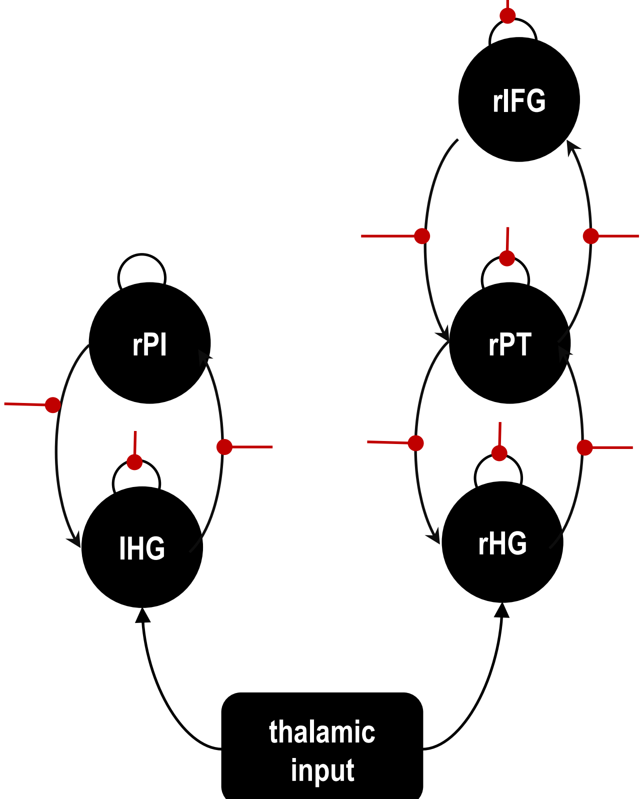
Bayesian model comparison

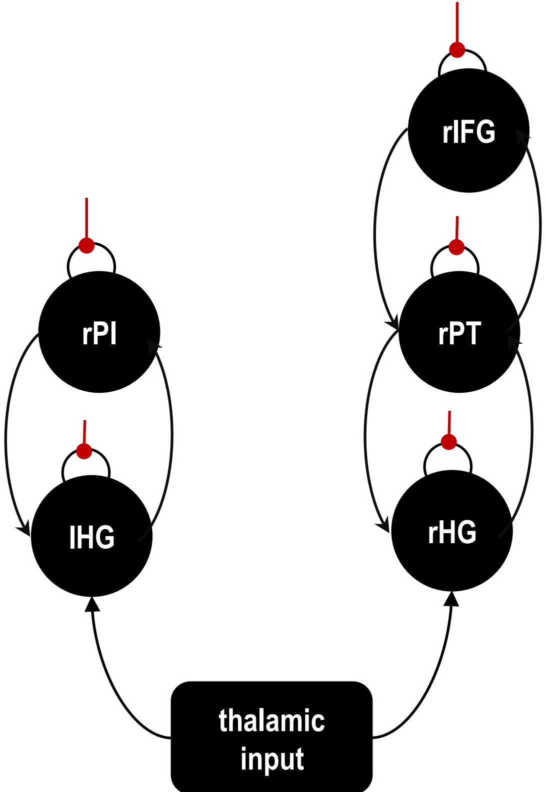
Model specification: prediction of the model



Which model better explains the underlying dynamics in the data ?

Will be explained in the Bayesian model comparison Lecture





Refernce

1- Garrido, M. I., Kilner, J. M., Stephan, K. E., & Friston, K. J. (2009). **The mismatch negativity: a review of underlying mechanisms**. Clinical neurophysiology, 120(3), 453-463.

2- Tallon-Baudry, C., & Bertrand, O. (1999). **Oscillatory gamma activity in humans and its role in object representation**. Trends in cognitive sciences, 3(4), 151-162.

3- David, O., Kilner, J. M., & Friston, K. J. (2006). **Mechanisms of evoked and induced responses in MEG/EEG**. Neuroimage, 31(4), 1580-1591.

4- Chen, C. C., Kiebel, S. J., & Friston, K. J. (2008). **Dynamic causal modelling of induced responses**. Neuroimage, 41(4), 1293-1312.

5- van Wijk, B. C., Cagnan, H., Litvak, V., Kühn, A. A., & Friston, K. J. (2018). **Generic dynamic causal modelling: An illustrative application to Parkinson's disease**. NeuroImage, 181, 818-830.

6- van Wijk, B. C., Litvak, V., Friston, K. J., & Daffertshofer, A. (2013). **Nonlinear coupling between occipital and motor cortex during motor imagery: a dynamic causal modeling study**. Neuroimage, 71, 104-113.

7- van Wijk, B. C.,. (2018).Principal of DCM, SPM course for M/EEG 2018.

8- Mountcastle VB (1957). "**Modality and topographic properties of single neurons of cat's somatic sensory cortex**". Journal of Neurophysiology. 20 (4): 408–34.

9- Jafarian A., Zeidman P., Litvak V. , Friston K.J. (2019) **Structure Learning in Coupled Dynamical Systems and Dynamic Causal Modelli**ng, arXiv preprint arXiv:1904.03093

10- Jafarian A., Litvak V., Cagnan H., Friston K.J., Zeidman P. (2019) **Neurovascular coupling: insights from multi-modal dynamic causal modelling of fMRI and MEG**, arXiv preprint arXiv:1903.07478

11- Penny, W. D., Litvak, V., Fuentemilla, L., Duzel, E., & Friston, K. (2009). **Dynamic causal models for phase coupling. Journal of neuroscience methods**, 183(1), 19-30.

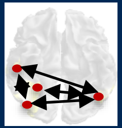
12- Friston, K. J., Preller, K. H., Mathys, C., Cagnan, H., Heinzle, J., Razi, A., & Zeidman, P. (2017). **Dynamic causal modelling revisited**. Neuroimage.

13- Wilson, H. R., & Cowan, J. D. (1973). **A mathematical theory of the functional dynamics of cortical and thalamic nervous tissue**. Kybernetik, 13(2), 55-80.

14- Freeman, W. J. (1975**). Mass action in the nervous system** (Vol. 2004). Academic Press, New York.

15- Daunizeau J, **Dynamic Causal Modelling for EEG/MEG**: principles

Model specification: prediction of the model

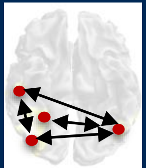


Many thanks for your attention.

Please get in touch using SPM mailing list should you have any questions.

Appendix

From convolution to differential equation



$$v_{out}(t) = u(t) * h(t)$$
$$h(t) = a\frac{-t}{\kappa}e^{-\frac{t}{\kappa}}$$

Laplace transform

$$x(s) = \int x(t)e^{-st}\mathrm{d}t$$

$$v_{out}(s) = u(s)h(s) = u(s)\frac{a\kappa}{s^2 + 2\kappa s + \kappa^2}$$

$$s^2 v_{out}(s) + 2\kappa s v_{out}(s) + \kappa^2 v_{out}(s) = a\kappa u(s)$$

Inverse Laplace transform

$$x(t) = \int x(s)e^{st}\mathrm{d}s$$

$$v''_{out}(t) + 2\kappa v'_{out}(t) + \kappa^2 v_{out}(t) = a\kappa u(t)$$

$$v'_{out}(t) = I(t)$$
$$I'(t) = -2\kappa I(t) - \kappa^2 v_{out}(t) - a\kappa u(t)$$