Principles of Dynamic Causal Modelling

SPM course for EEG/MEG, May 2023

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Dynamic causal modelling

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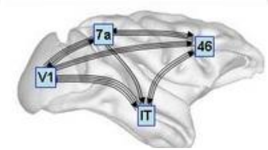
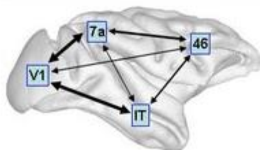
Karl Friston

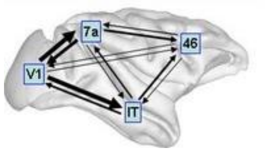
- ***Structural*** **connectivity**
  Presence of axonal connections



- ***Functional*** **connectivity**
  Statistical dependencies between regional time series

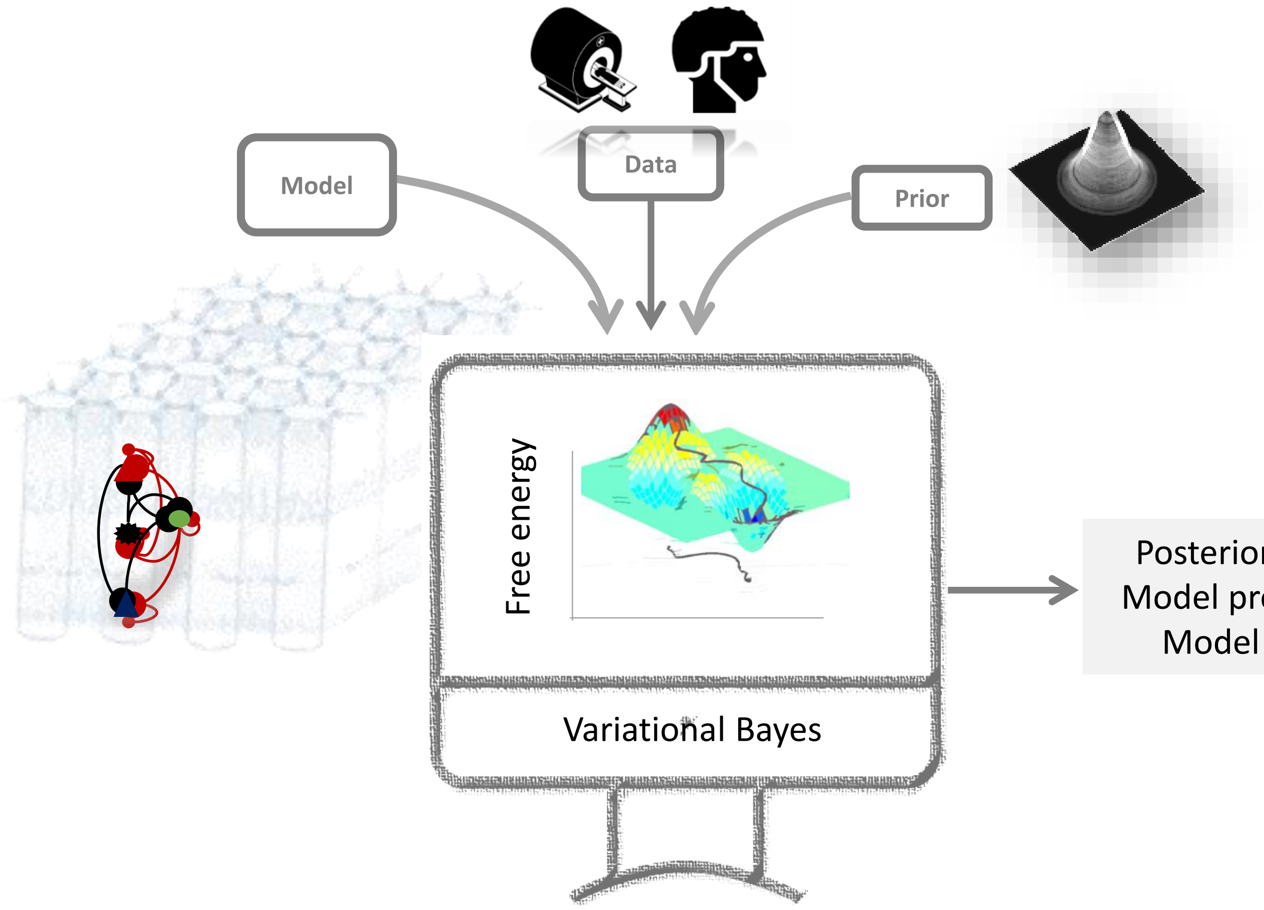


- ***Effective*** **connectivity**
  Causal influences between neuronal populations, and experimental contexts!

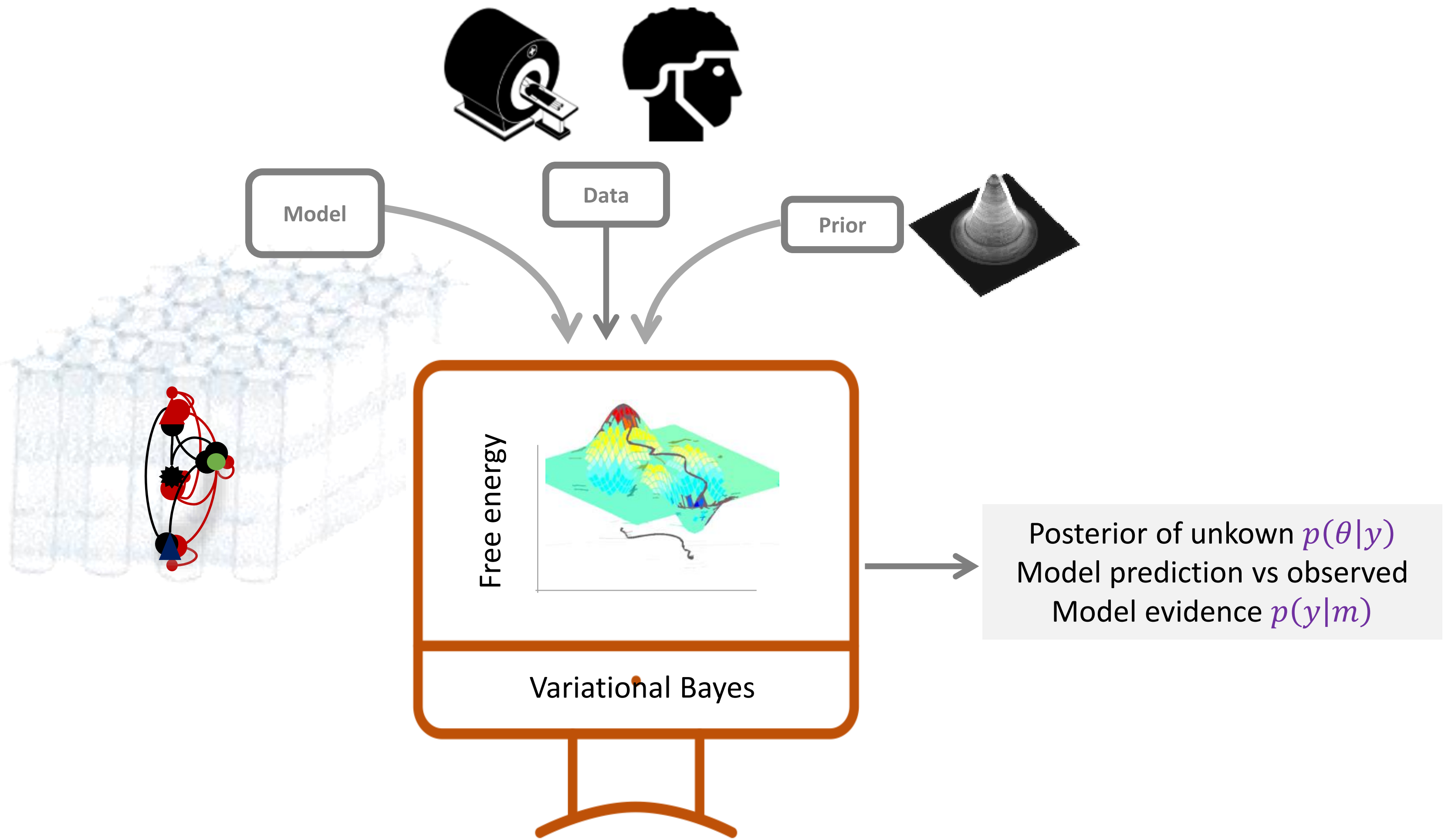


O. Sporns 2007, *Scholarpedia*

Principal of DCM



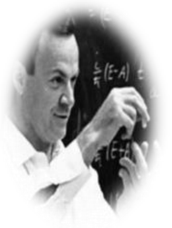
Posterior of unkown $p(\theta|y)$
Model prediction vs observed
Model evidence $p(y|m)$


Model
Data
Prior
Free energy
Variational Bayes
Posterior of unkown $p(\theta|y)$
Model prediction vs observed
Model evidence $p(y|m)$

Model estimation and comparison in DCM

$$F \approx \ln p(y|m) = \text{accuracy(m)} - \text{complexity(m)}$$

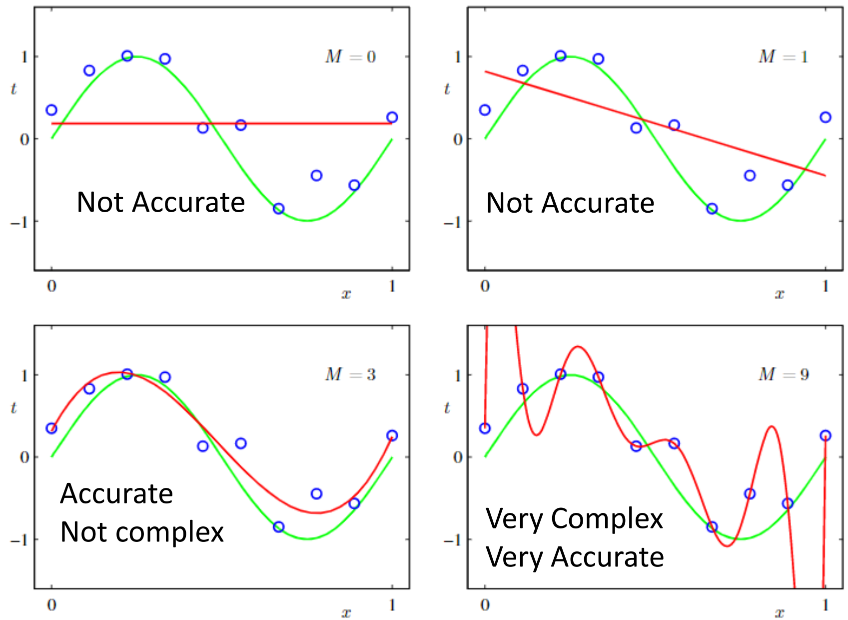
DCM optimises the free energy of a model (wrt parameters) to infer parameters that can accurately replicate the data and are not too complex!



Richard Feynman

Hermann von Helmholtz

Example: curve fitting! $y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M$



Hidden generator of data

Noisy observed data

Fitted curve to data

Model estimation and comparison in DCM

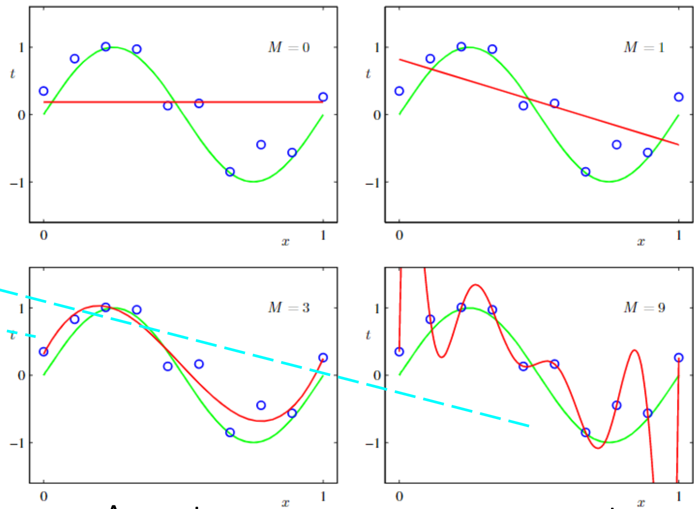
$$F \approx \ln p(y|m) = \text{accuracy(m)} - \text{complexity(m)}$$

Example: curve fitting!

$$y(x, \mathbf{w}) = w_0 + w_1 x + w_2 x^2 + \ldots + w_M x^M$$

Not Accurate

Not Accurate



Accurate
Not complex

Very Complex
Very Accurate

Model (hypothesis) comparison using Bays factor (or log BF)

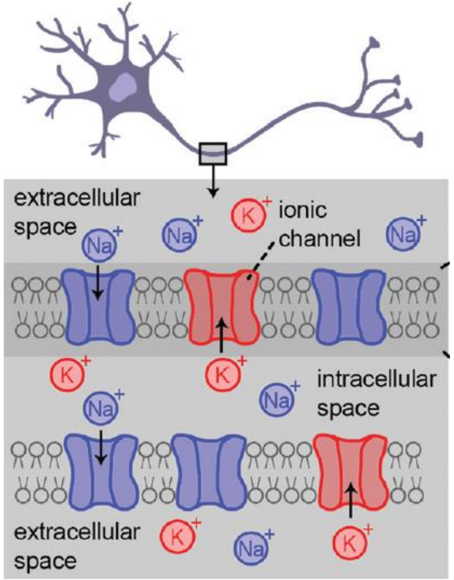
$$\ln BF = F_1 - F_2$$

Categories for levels of evidence from Kass and Raftery (1995)

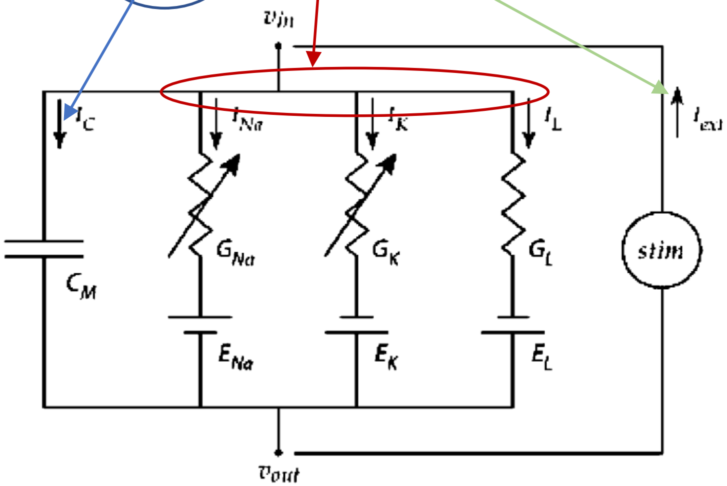
Bayes factor	Log (base e) Bayes factor	Posterior probability	Evidence level
1 to 3	0 to 1	0.5 to 0.73	Not worth more than a bare mention
3 to 20	1 to 3	0.73 to 0.95	Positive
20 to 150	3 to 5	0.95 to 0.99	Strong
> 150	> 5	> 0.99	Very strong

Intuition of Hodgkin & Huxley in balancing complexity and accuracy

$$I_{ion} = G_{Na}(V_m - E_{Na}) + G_K(V_m - E_K) + G_L(V_m - E_L)$$

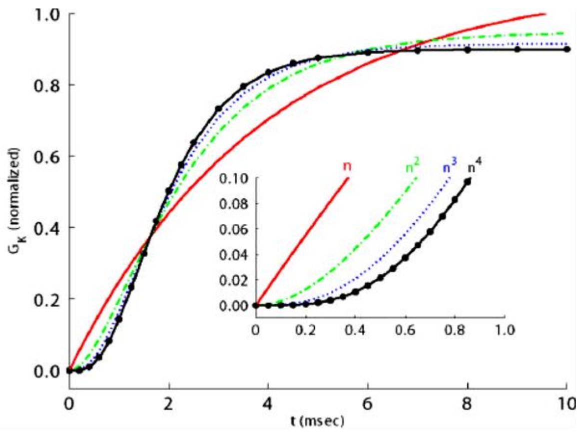


$$C_m \frac{dV_m}{dt} + I_{ion} = I_{ext}$$

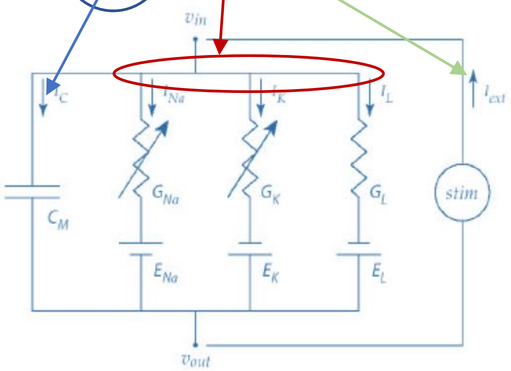


Sir Alan Hodgkin

Sir Andrew Huxley



$$C_m \frac{dV_m}{dt} + I_{ion} = I_{ext}$$



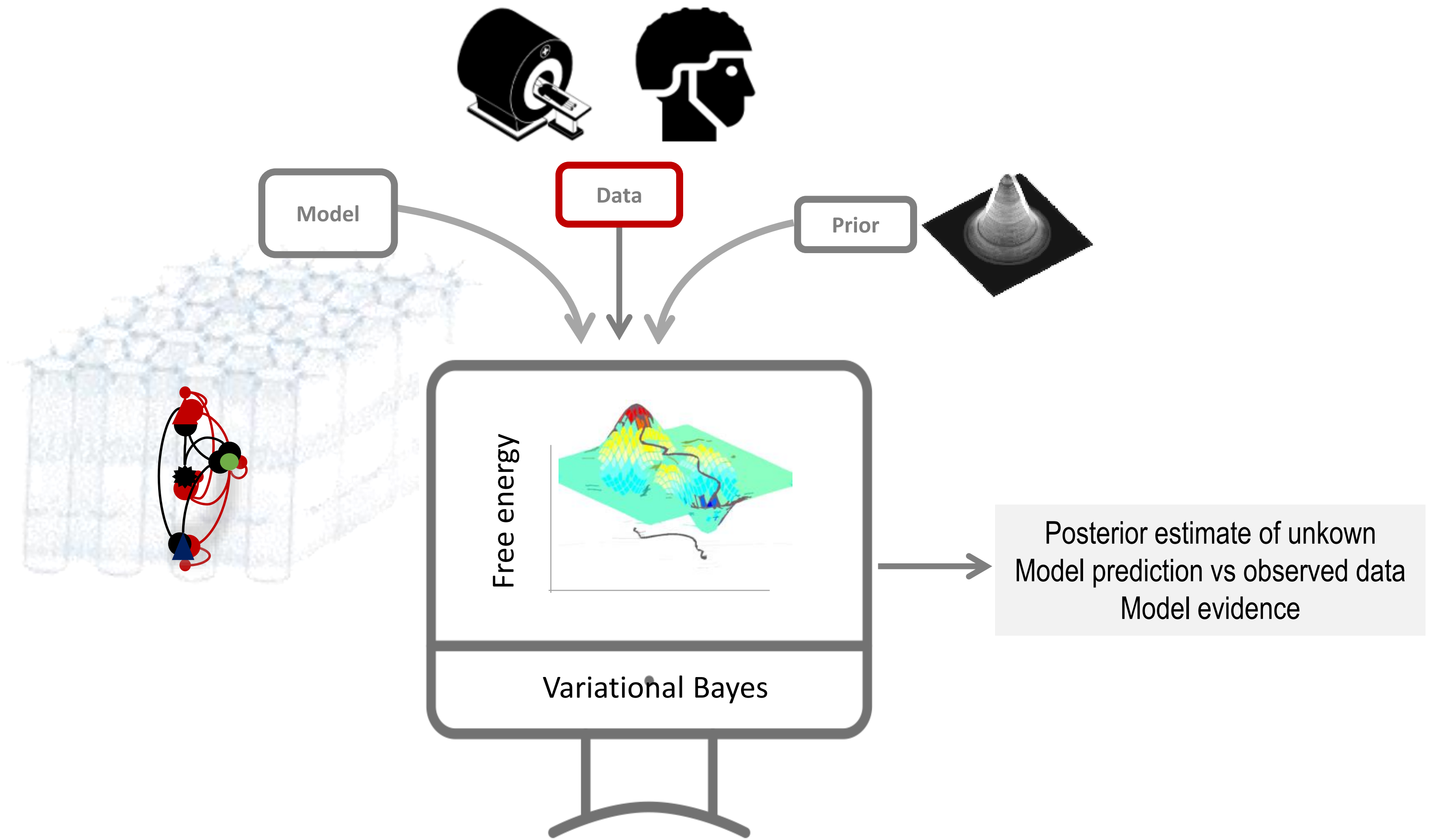
$$I_{ion} = G_{Na}(V_m - E_{Na}) + G_K(V_m - E_K) + G_L(V_m - E_L)$$

Fig. 3 Best fit curves of the form $G_k = \bar{g}_K n^j$ $(j = 1\text{–}4)$ for simulated conductance vs. time data. The inset shows an enlargement of the first millisecond of the response. The initial inflection in the curve cannot be well-fit by a simple exponential (dotted line) which rises linearly from zero. Successively higher powers of j (j=2: dot-dashed; j=3: dashed line) result in a better fit to the initial inflection. In this case, j=4 (solid line) gives the best fit.

Sir Alan Hodgkin

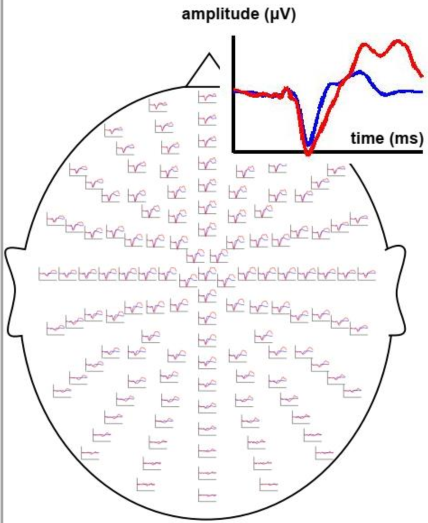
Sir Andrew Huxley

H&H refined the linear model by inclusion of nonlinear terms, having considered balancing complexity with accuracy which is highly relevant to the Free energy concept.

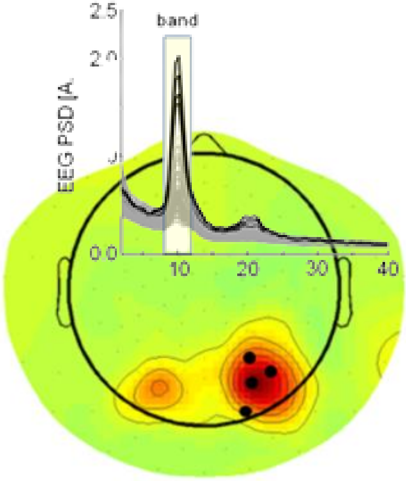

Model
Data
Prior
Free energy
Variational Bayes
Posterior estimate of unkown
Model prediction vs observed data
Model evidence

Data features in DCM:

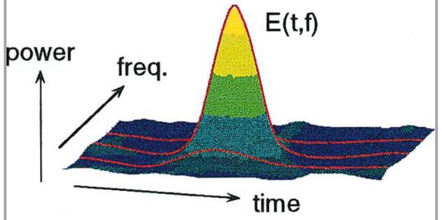
Evoked potentials



Power spectral density



Induced or Time frequency



Mixed data features

Modelling of data features in DCM

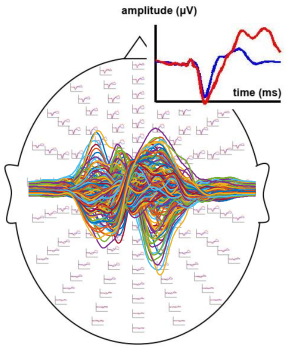
In the absence of any input neuronal dynamics are at stable equilibrium! Different types of inputs induce different data features

Experimental inputs (visual/ auditory stimuli) induce ERPs

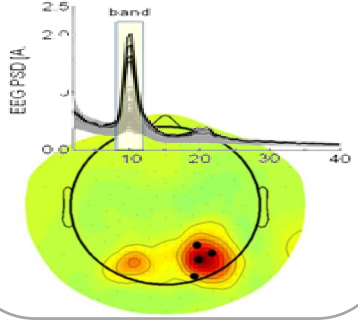
Random neuronal fluctuations induce oscillation around the neuronal equilibrium!

Random neuronal fluctuations about the mean evoked response can generate induce response

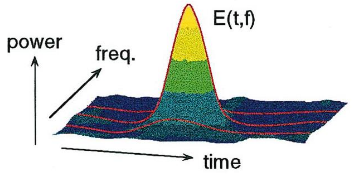
Evoked potentials

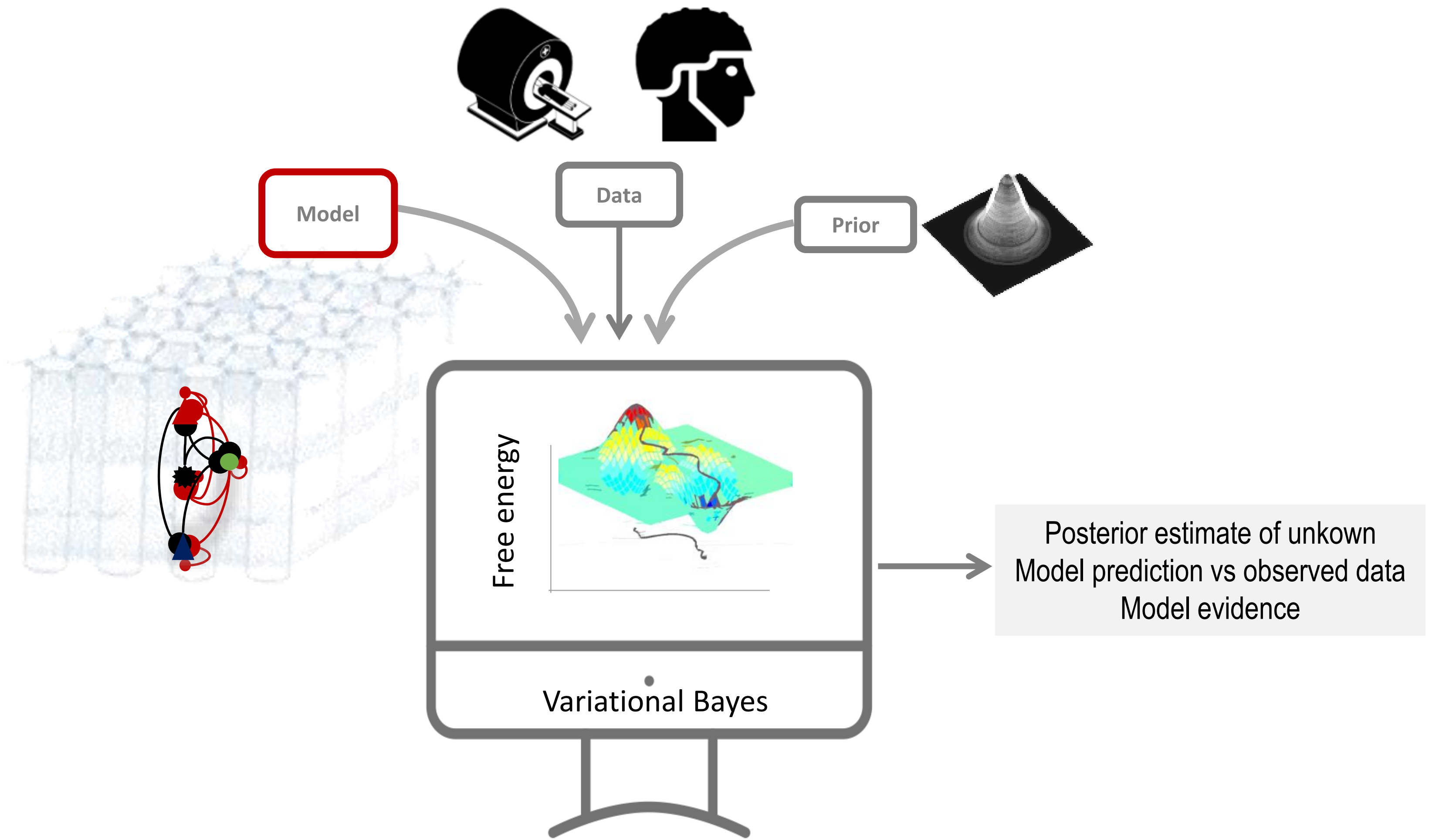


Power spectral density



Induce response




Model
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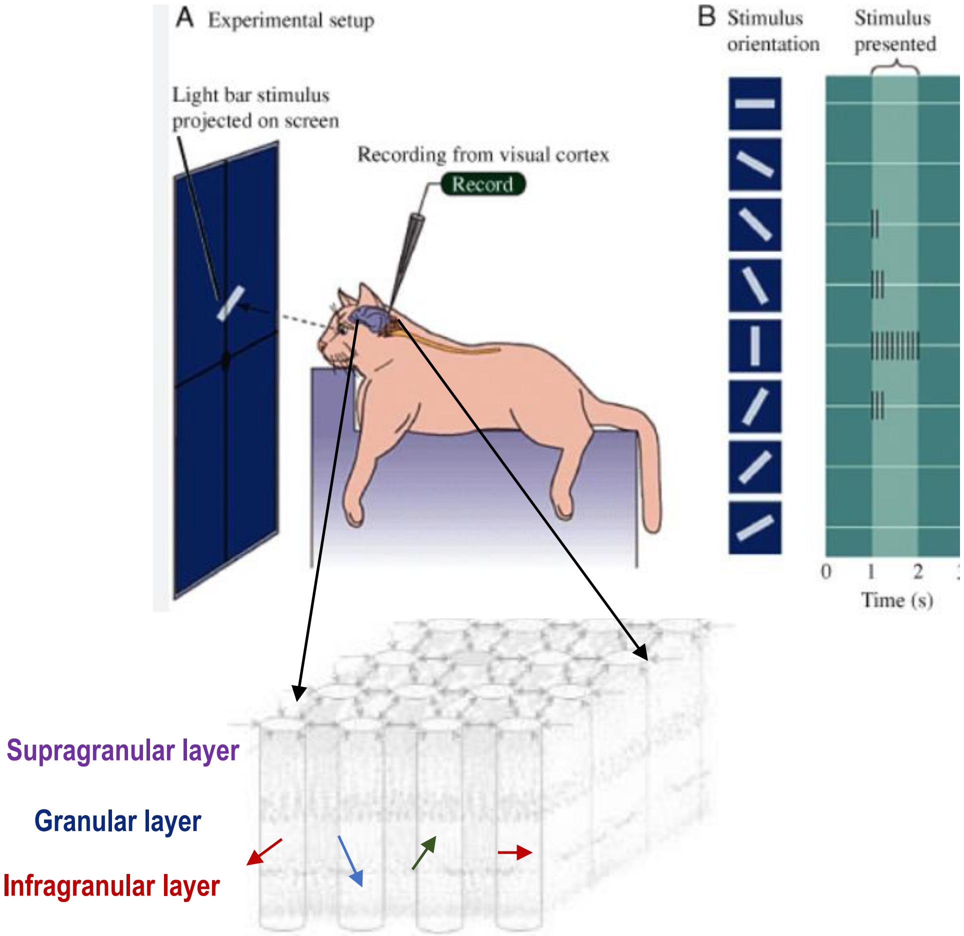
Cortical Column, and brain activity!

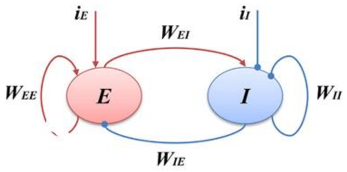
Montcastle

Hubel and Wiesel



Mesoscale models of cortical column’s electrical activity


H Wilson J Cowan



It is hoped that the relative simplicity of the model may serve as a basis for a better understanding of the functional significance of cortical complexity

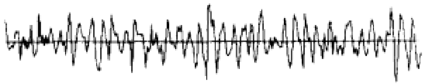
(Hugh Wilson and Jack Cowan, 1973)

F Lopes da Silva

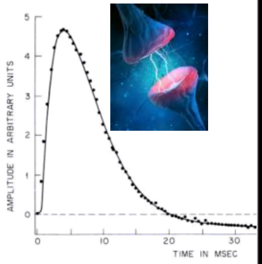
Model of Brain Rhythmic Activity

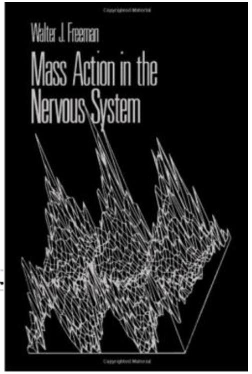
The Alpha-Rhythm of the Thalamus

F. H. Lopes da Silva, A. Hoeks*, H. Smits, and L. H. Zetterberg

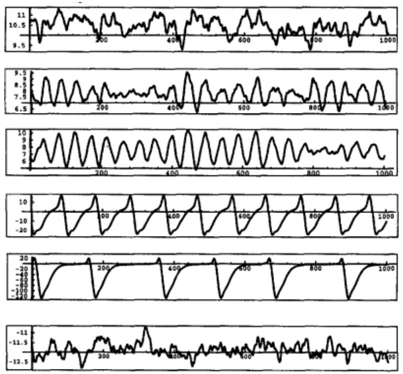


W Freeman



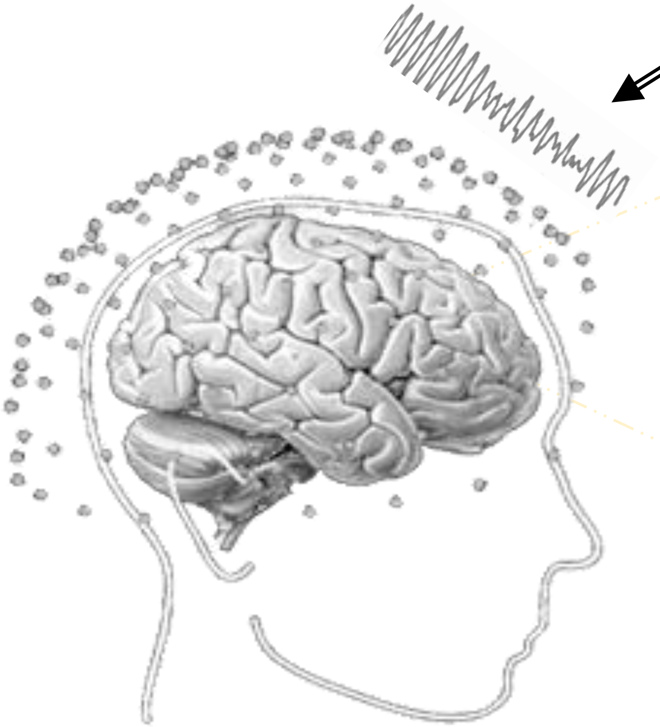


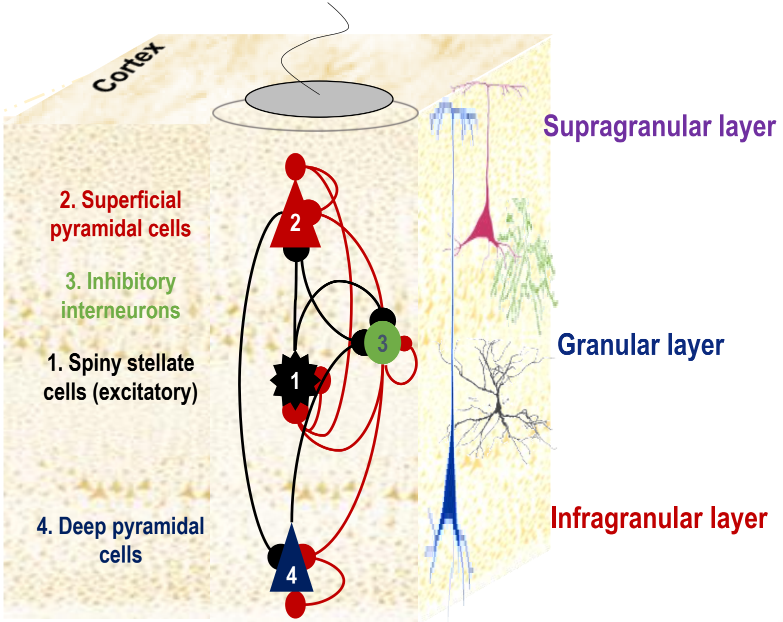
B Jansen



Canonical neural mass models

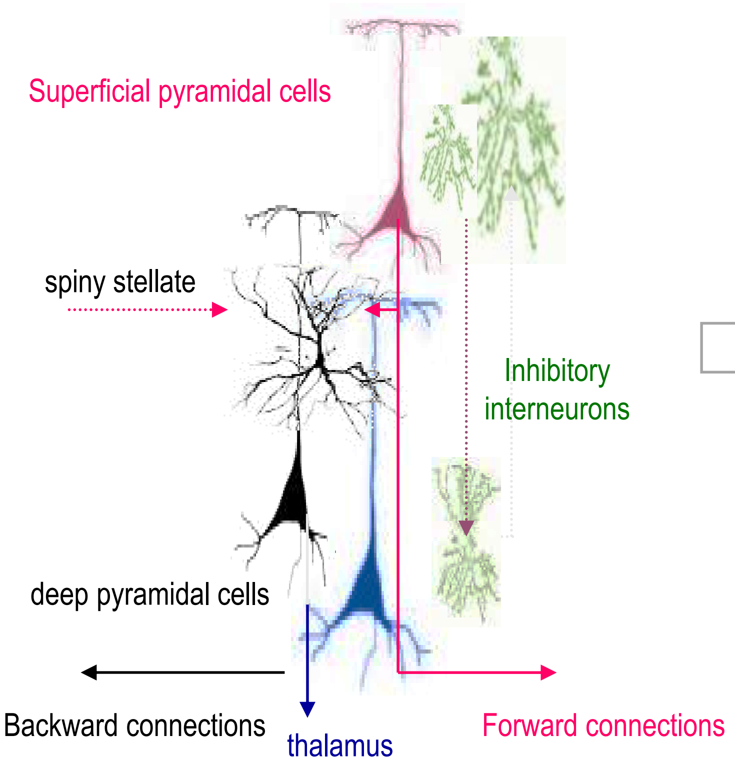
$$y = L\left(V_{sp} + \alpha_1 \times V_{ss} + \alpha_2 \times V_{dp}\right) + e$$

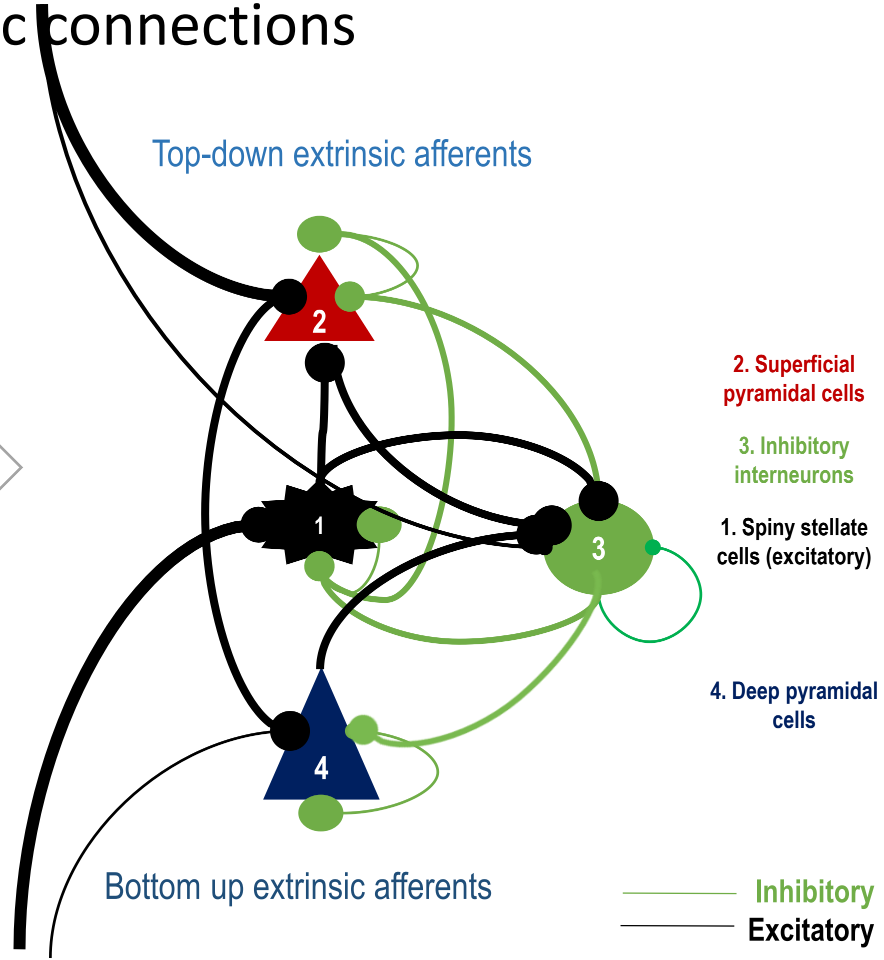




Karl Friston

Canonical neuronal mass models: Intrinsic and extrinsic connections

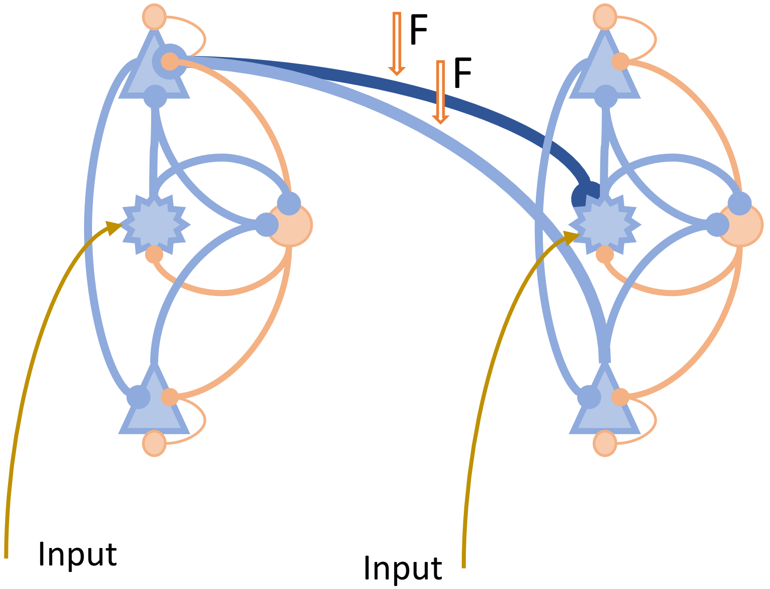


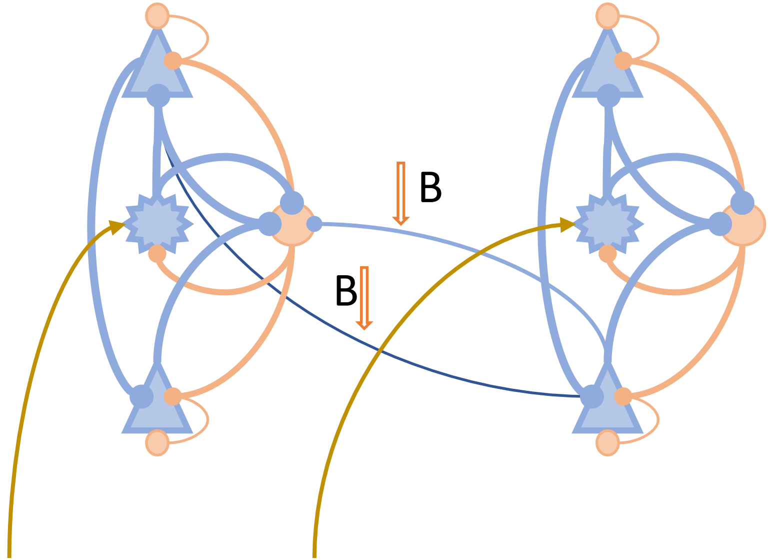


Forward extrinsic connections

Superficial pyramidal
→ spiny stellate

Superficial pyramidal
→ deep pyramidal



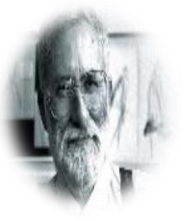


Backward extrinsic connections

Deep pyramidal
→ inhibitory interneurons

Deep pyramidal
→ superficial pyramidal

Canonical neuronal mass models: population model



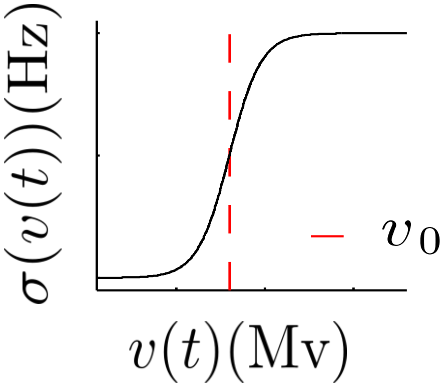
W Freeman

intrinsic → / extrinsic → / experimental → **Firing rate to Potential conversion $h(t)$** →

$$v_i^{(j)} = \Big(\underbrace{\sum\nolimits_k \boldsymbol{G}_{ik}^{(j)}\sigma(v_k^{(j)})}_{\text{intrinsic}} + \underbrace{\sum\nolimits_{kl} \boldsymbol{A}_{jl}^{(ik)}\sigma(v_k^{(l)})}_{\text{extrinsic}} + \underbrace{\sum\nolimits_m C_{jm}^{(i)} u_m}_{\text{experimental}}\Big) \otimes h(t)$$

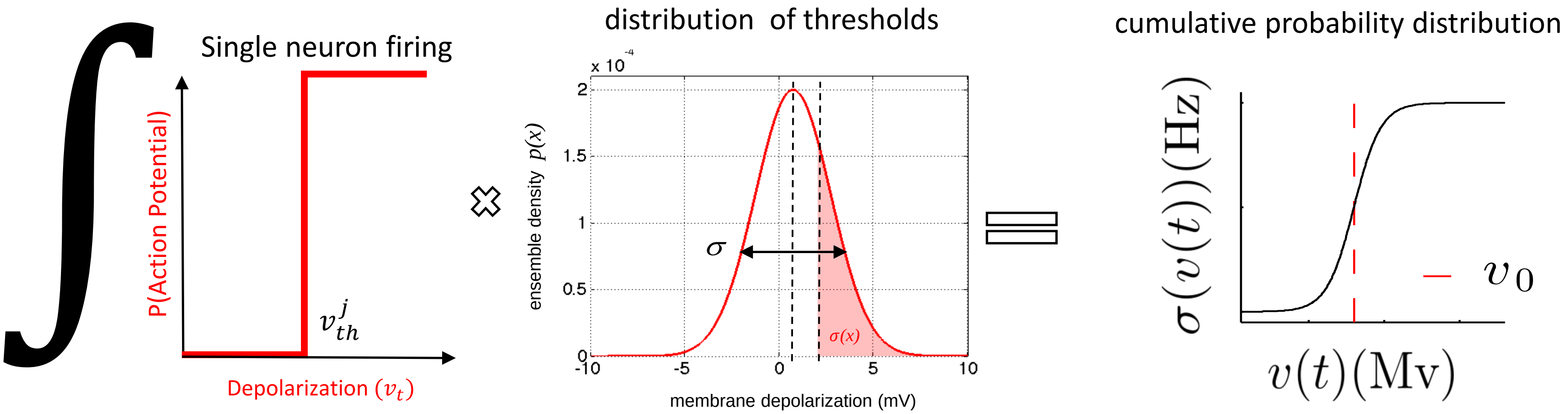
H Wilson J Cowan

v → **Potential to firing rate conversion** → $\sigma(v)$

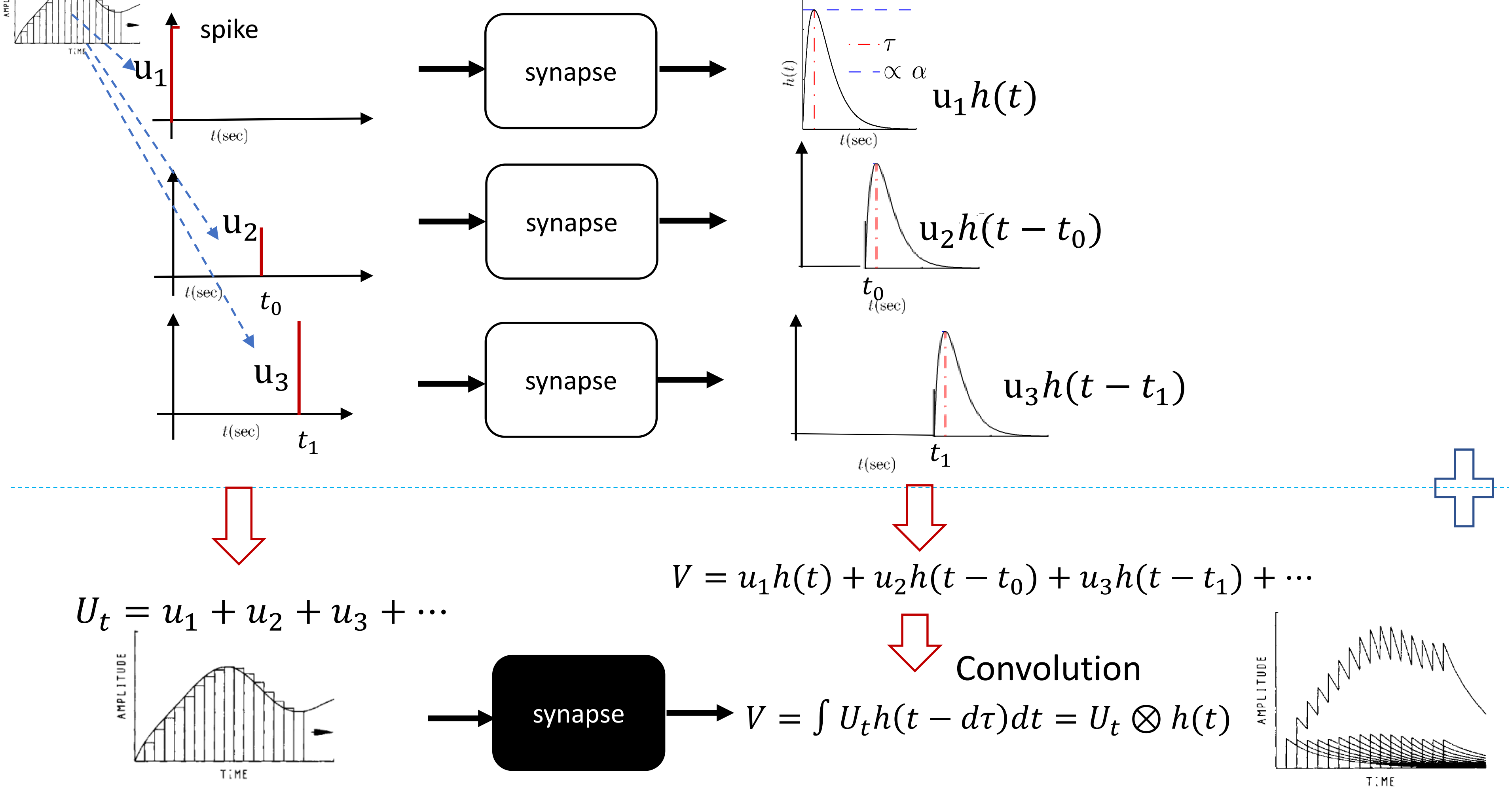


Mean Potential to firing rate conversion

For small membrane potentials, firing rates are small. By increasing the potential firing rate increases but up to a point where it saturate!



Mean firing rate to PSP conversion:



Different NMM models in SPM12:

Neural mass model

ERP NMM based on Jansen & Rit (1995)

SEP ERP with faster dynamics for evoked potentials

CMC, TFM Canonical Microcircuit Model

LFP J&R with spike frequency adaptation mechanisms

NFM Extension of ERP model to a neural field model

In (Convolution) NMM, synaptic mechanisms $h(t)$ has a fixed shape regardless of input firing rates.

Different NMM models in SPM12:

Conductance based models (Morris-Lecar)

NMM based on Morris & Lecar (1981)

MFM dynamics of mean & cov of neuronal population ensemble

CMM, CMM_NMDA canonical micro circuit and mean field model, includes (ligand gated) NMDA receptors

In conductance based models, presynaptic inputs directly influence synaptic mechanisms and they incudes details physiology.

Membrane potential

passive leak current

$$\frac{dV}{dt} = \frac{1}{C}\left[g_L(V_L - V) + g_{AMPA}(V_{AMPA} - V) + g_{GABA}(V_{GABA} - V) + g_{NMDA}\, m(V)(V_{NMDA} - V)\right] + u$$

Mg^{+}

membrane capacitance

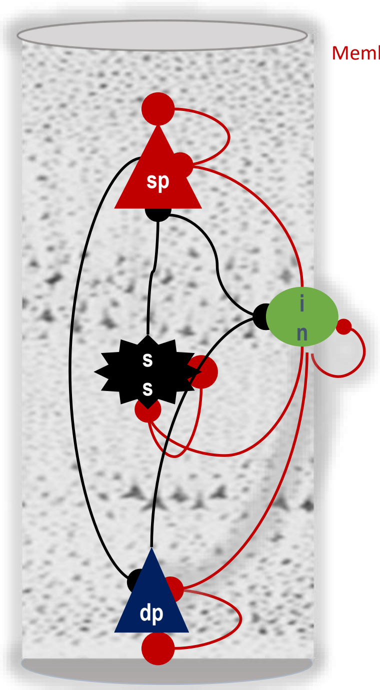
conductance

Firing rates

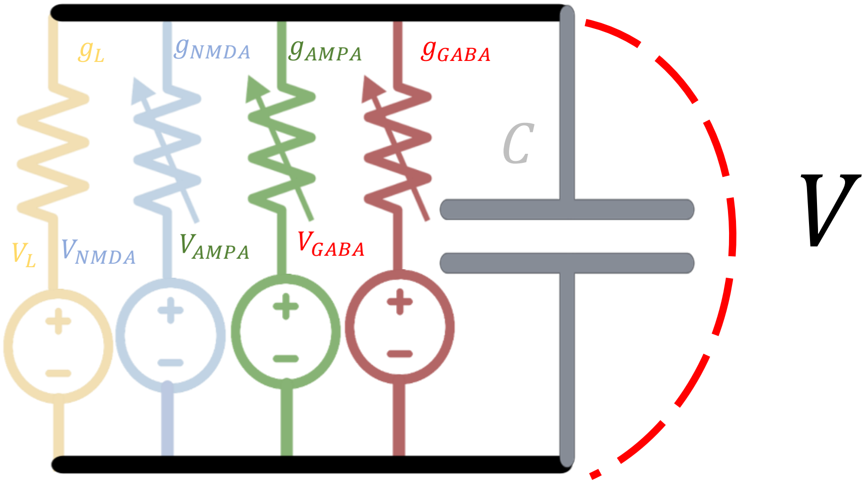
$$\frac{dg_*}{dt} = \frac{1}{\tau_*}\left(\sum_{k=sp,inh,dp,ss} H_k\, \sigma_k - g_*\right) + u_2, \quad * = [AMPA, GABA, NMDA]$$

Time constant

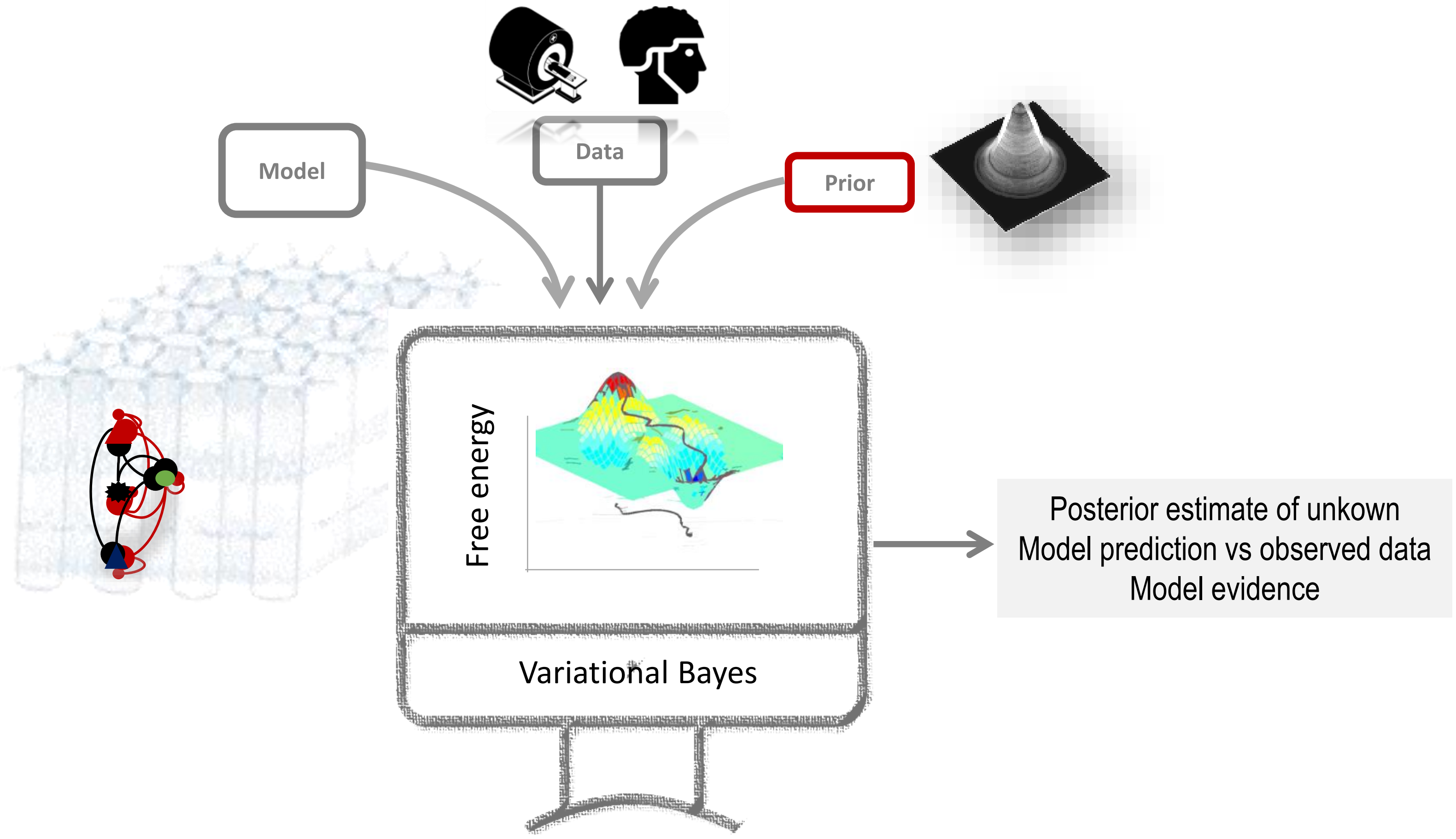
Synaptic gains



ss: Spiny stellate cells
sp: Superficial pyramidal cells
in: Inhibitory interneurons
dp: Deep pyramidal cells



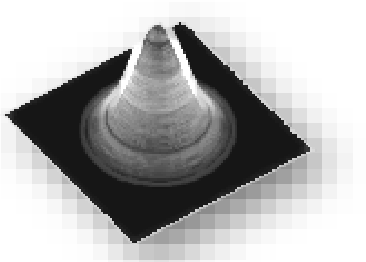
Principal of DCM

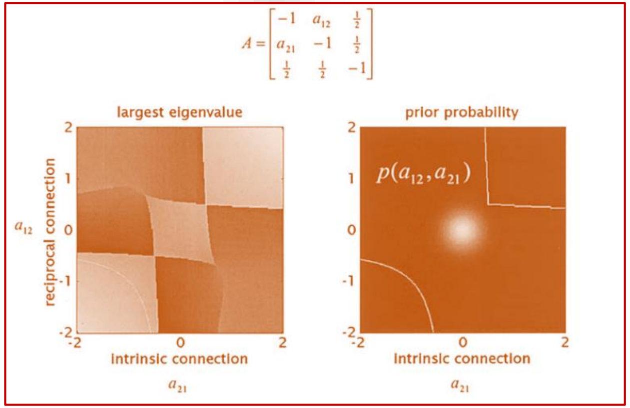


Prior in DCM

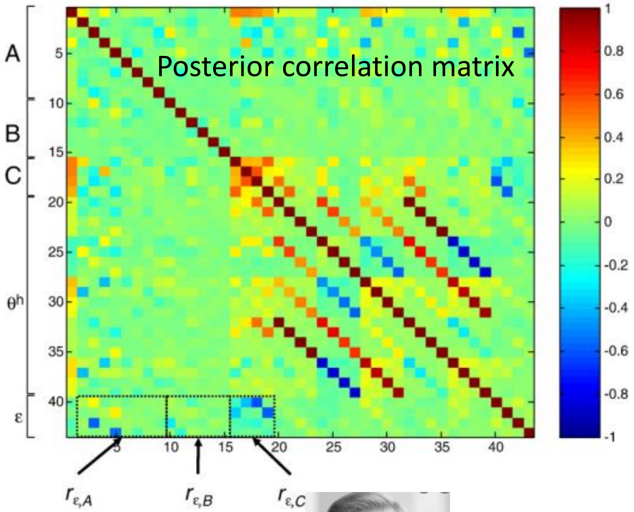
1- Model should be stable in prior space !

$$P(\theta) = N(\mu, \Sigma)$$



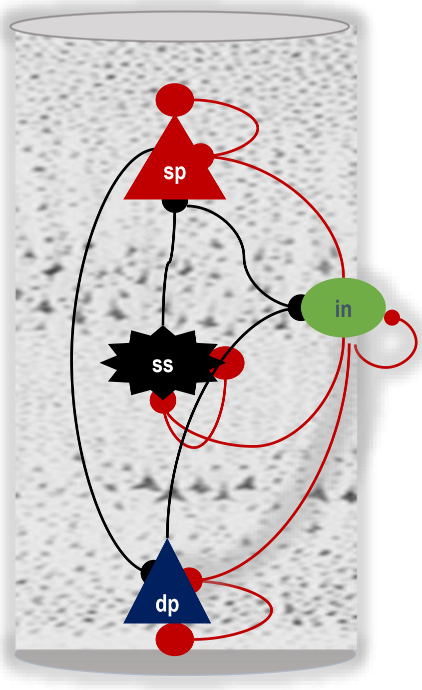


2- Model parametrisation and prior can help indefinability!

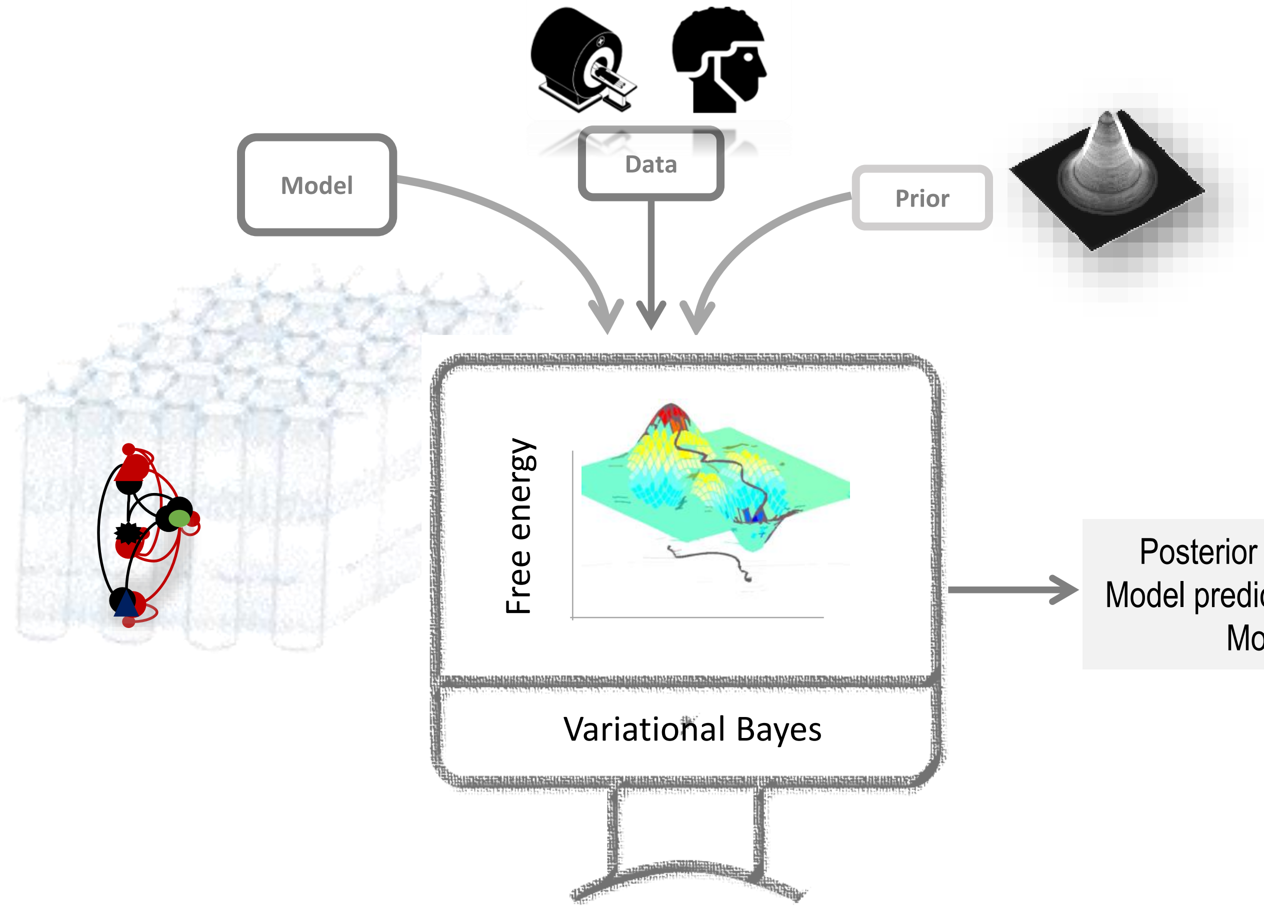


Klass Stephan

3- Prior can add a sense of geometry and/or interpretability to the model!

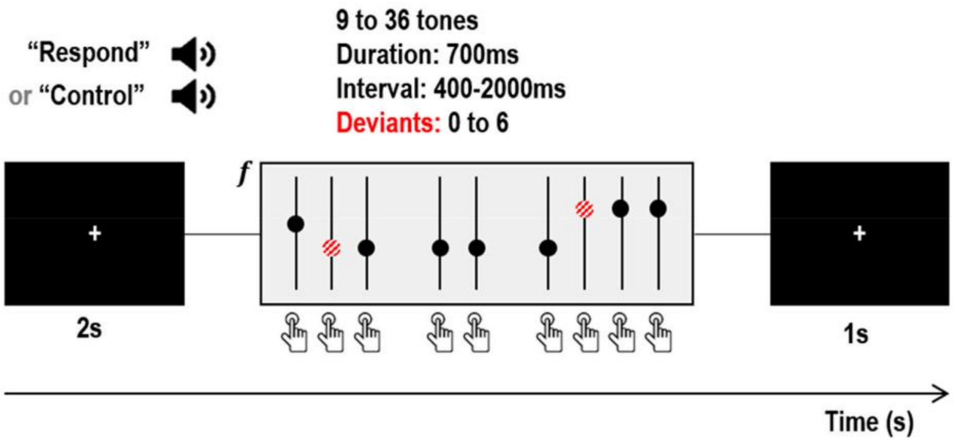


Examples



Posterior estimate of unkown
Model prediction vs observed data
Model evidence

Miss match negativity with agency recorded by MEG, fMRI

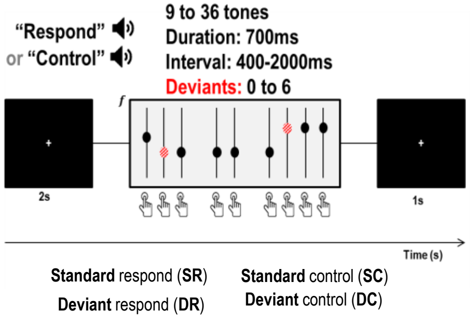


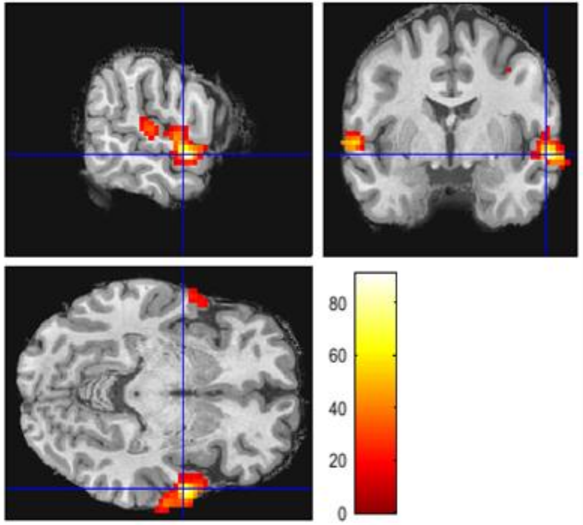
The subject received an auditory cue, instructing them to respond to auditory tones or control the tones (by pressing a button).

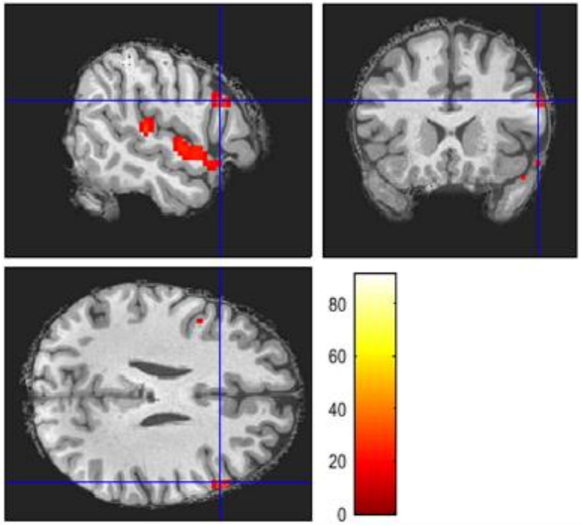
After 2s, a series of tones was presented. Deviant tones (red striped circles) differed in frequency from the preceding tone.

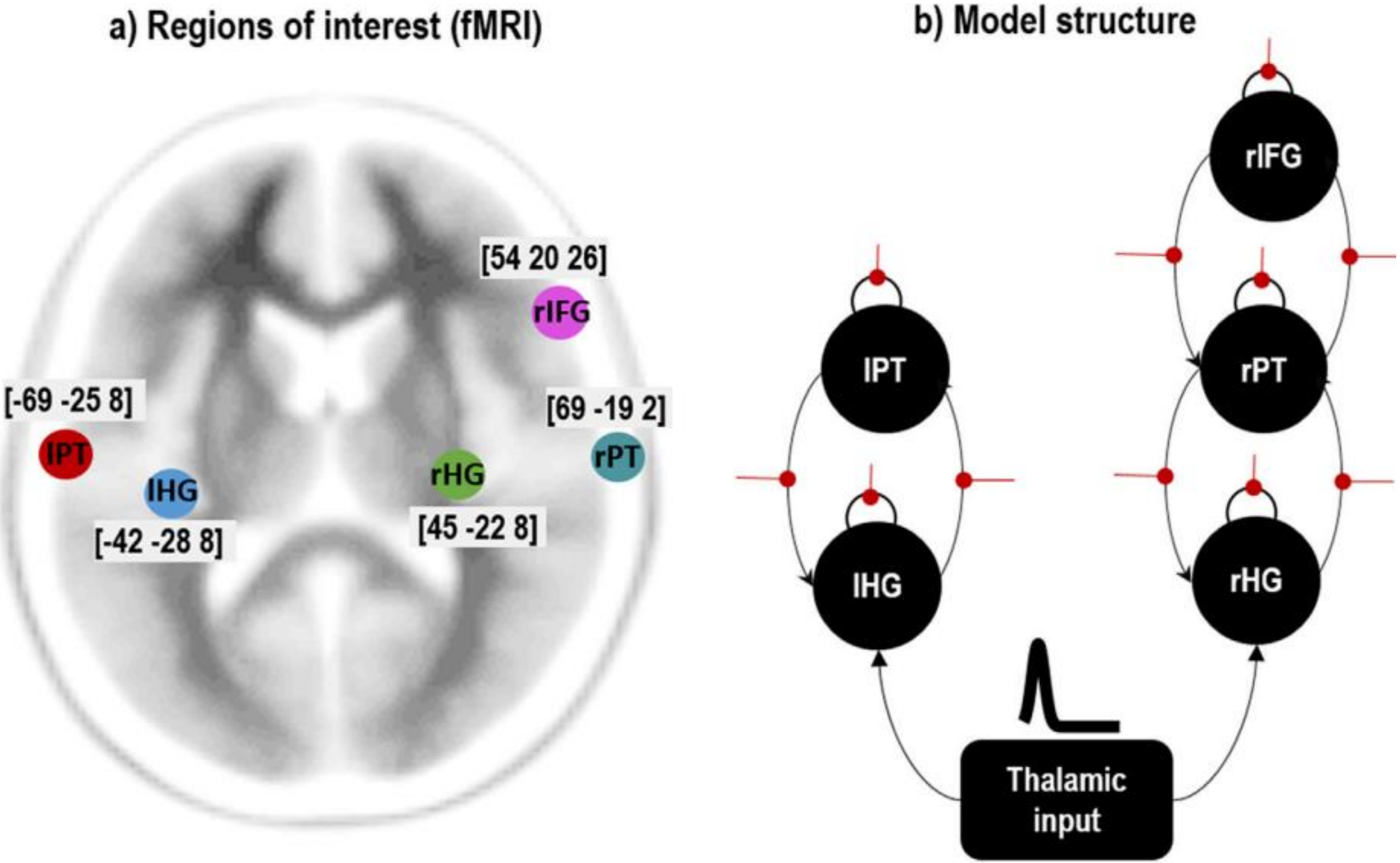
Whether a tone was a standard or deviant was independent of whether the tone was triggered by the computer or the subject.

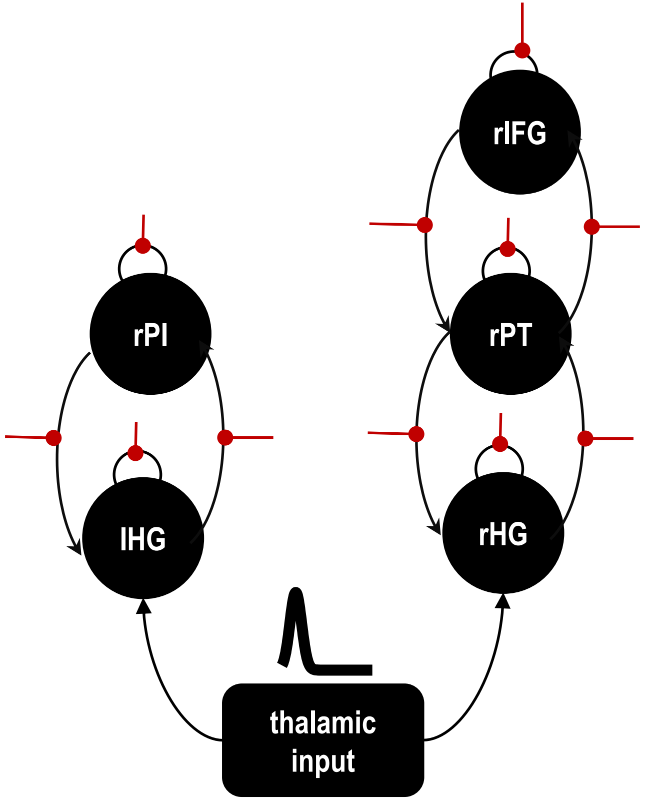
Statistical analysis of fMRI data

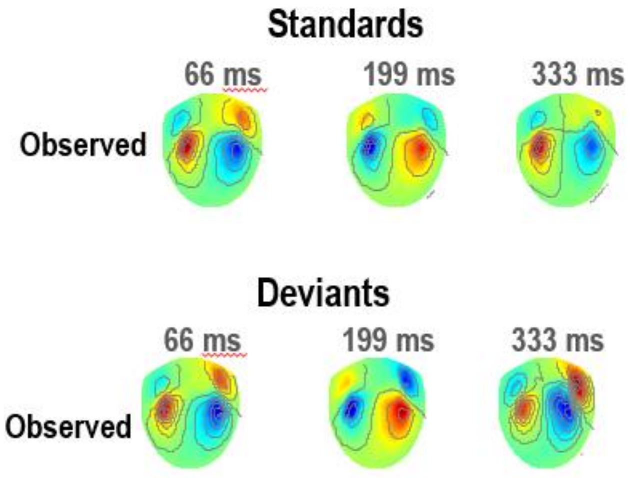






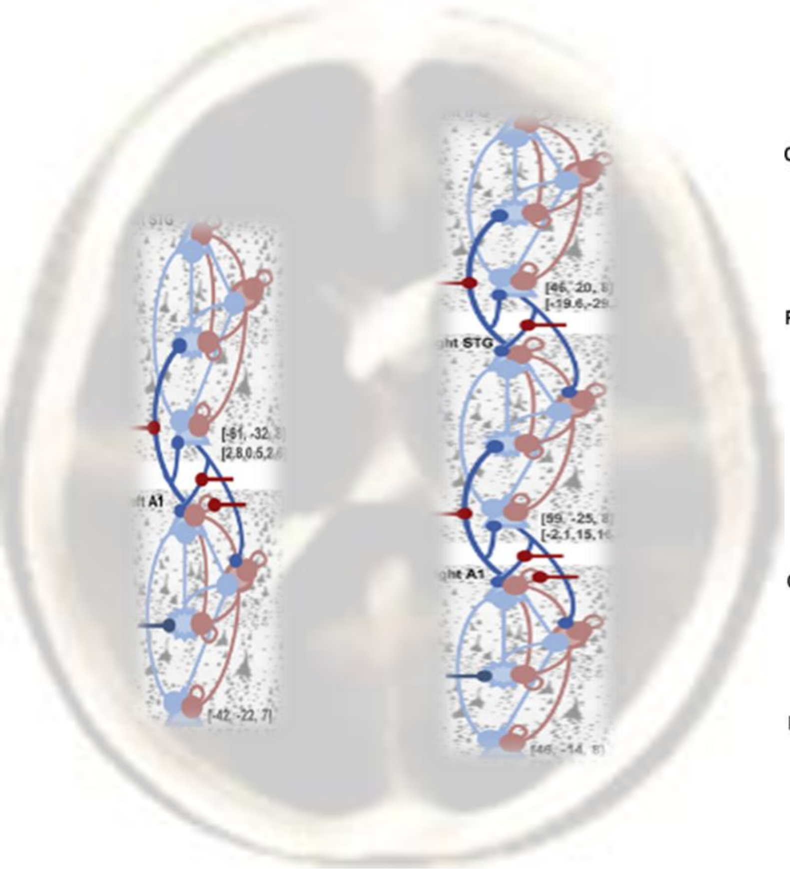

a) Regions of interest (fMRI)
[54 20 26]
rIFG
[-69 -25 8]
lPT
lHG
[-42 -28 8]
rHG
[45 -22 8]
[69 -19 2]
rPT
b) Model structure
rIFG
lPT
rPT
lHG
rHG
Thalamic input

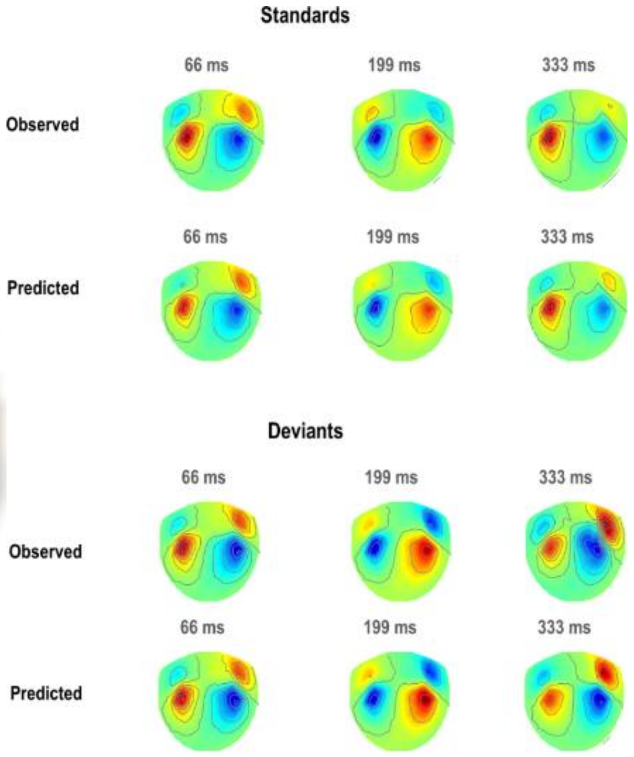




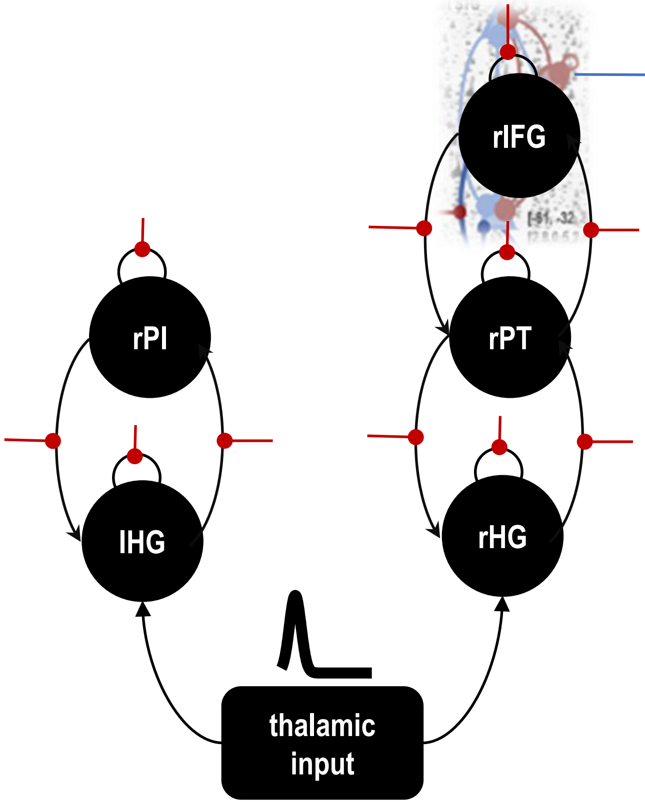
DCM infers neuronal parameters that generates standard response (baseline condition).

Condition-specific parameters is embedded in DCM to model other conditions (deviants) with respect to the baseline condition!

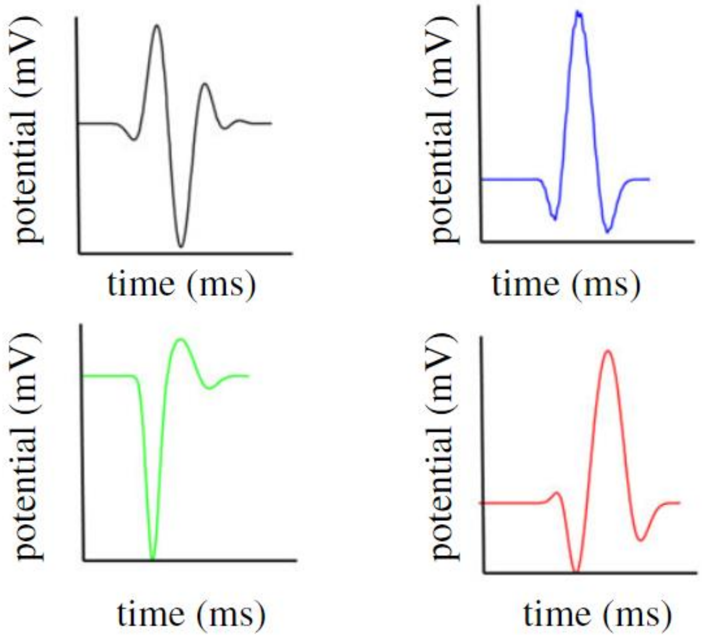

[-42, -22, 7]


Standards
66 ms
199 ms
333 ms
Observed
66 ms
199 ms
333 ms
Predicted
Deviants
66 ms
199 ms
333 ms
Observed
66 ms
199 ms
333 ms
Predicted

Prediction of the model given the estimated parameters



Simulated response of a population to each stimulation



Principles of multimodal dynamic causal Modelling

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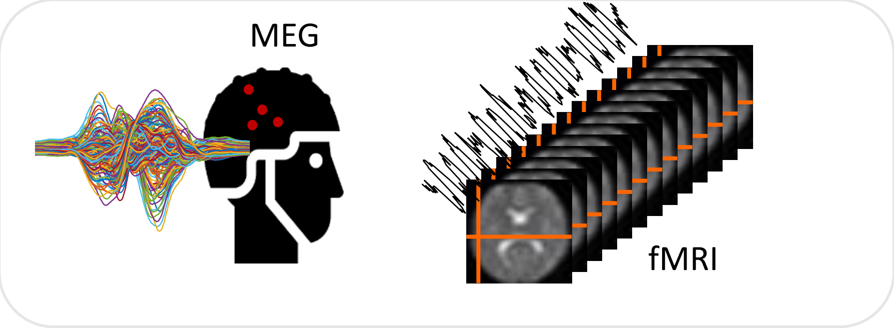
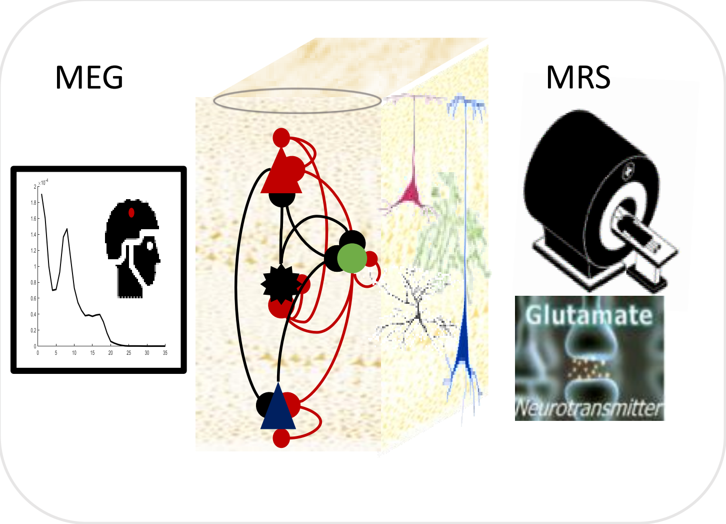
UNIVERSITY OF CAMBRIDGE

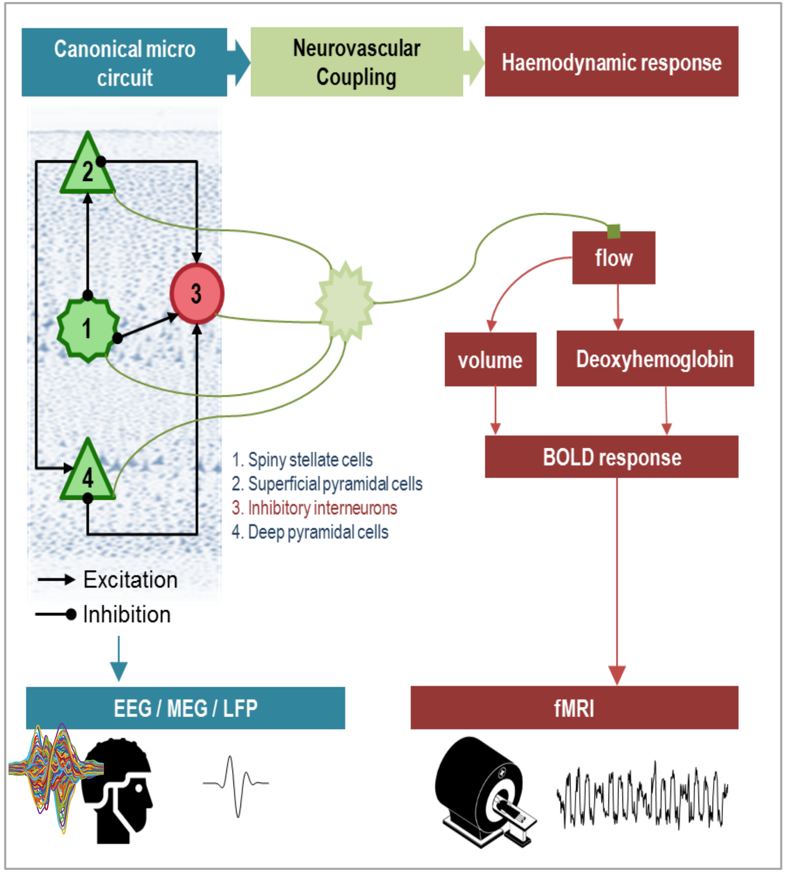
Multimodal DCM: functional (MEG) +functional (fMRI)

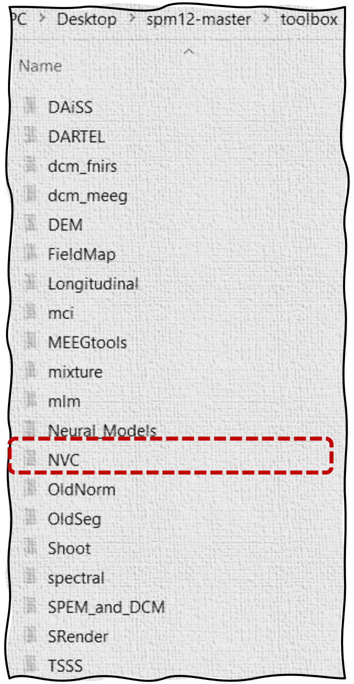


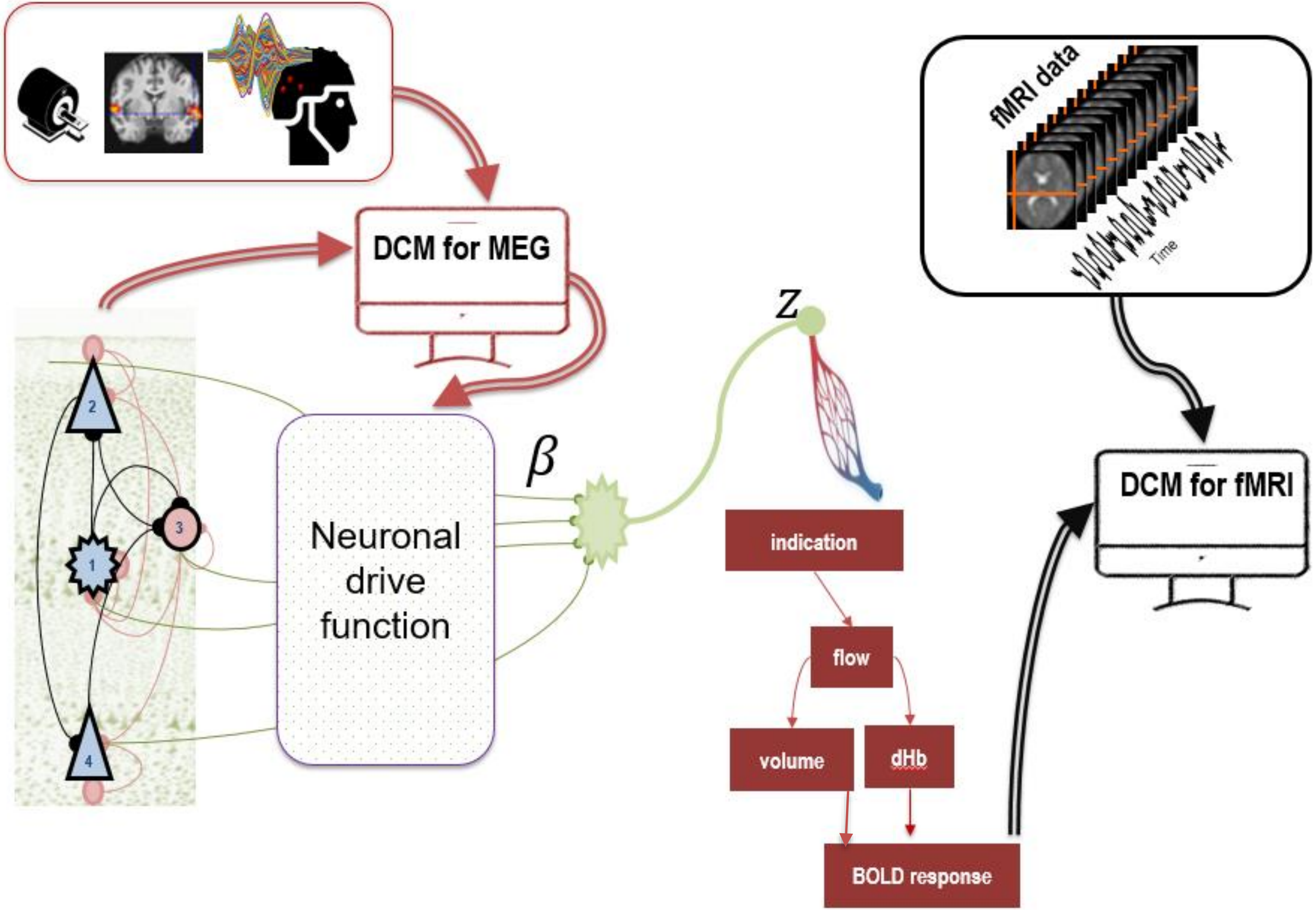
Multimodal DCM: structural (MRS) + functional (MEG)



Multimodal dynamic casual modelling of MEG and fMRI

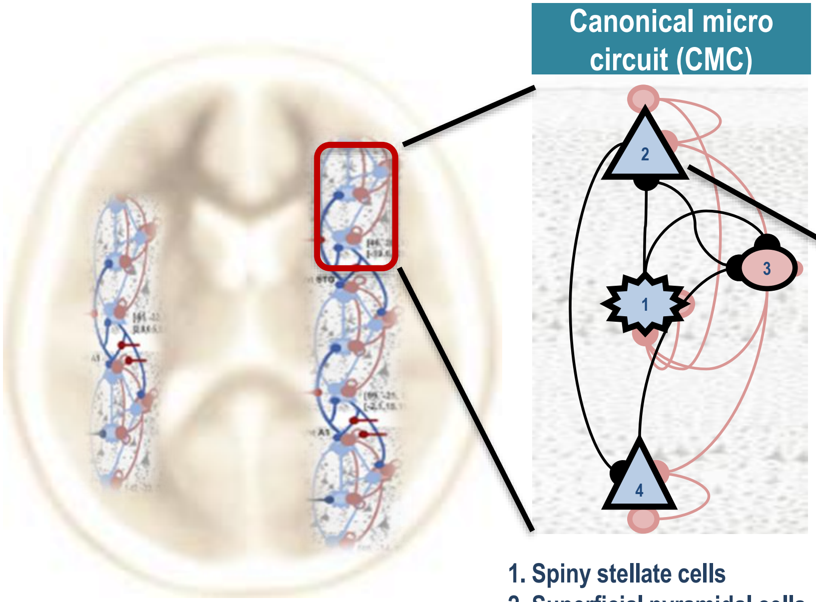




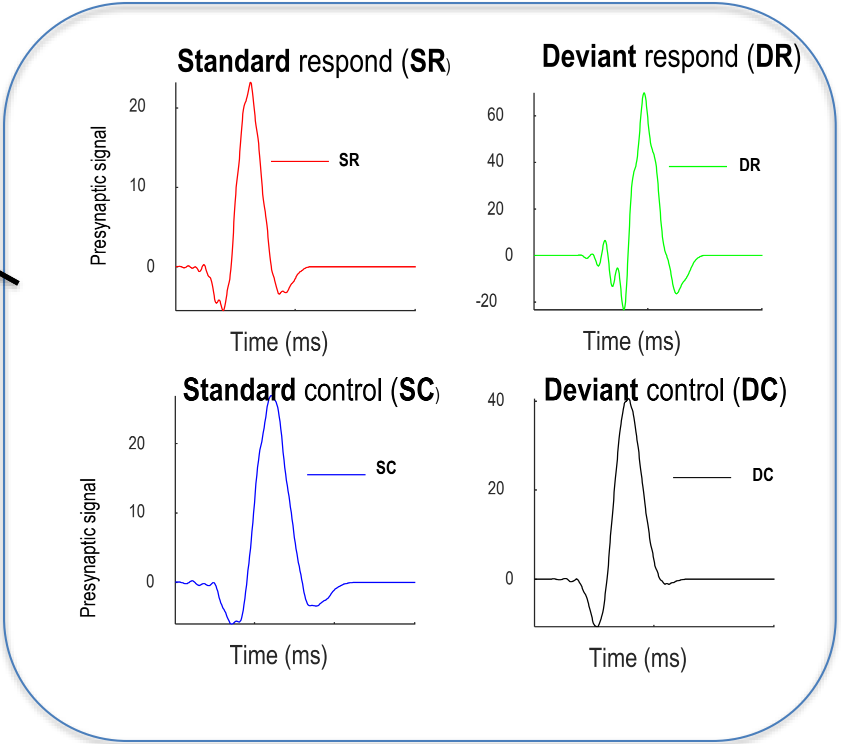

fMRI data
Time
DCM for MEG
DCM for fMRI
z
β
2
3
1
4
Neuronal drive function
indication
flow
volume
dHb
BOLD response

Neuronal drive function

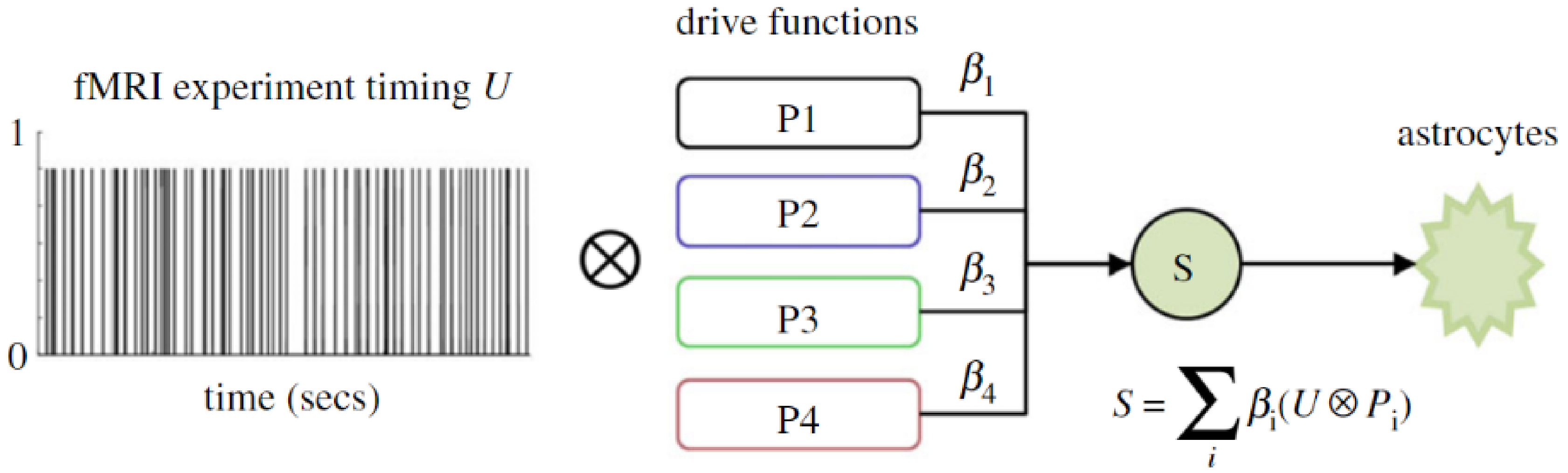
Simulation of pre/post-synaptic responses of a population in a source for each experiential condition (e.g., 4 conditions)

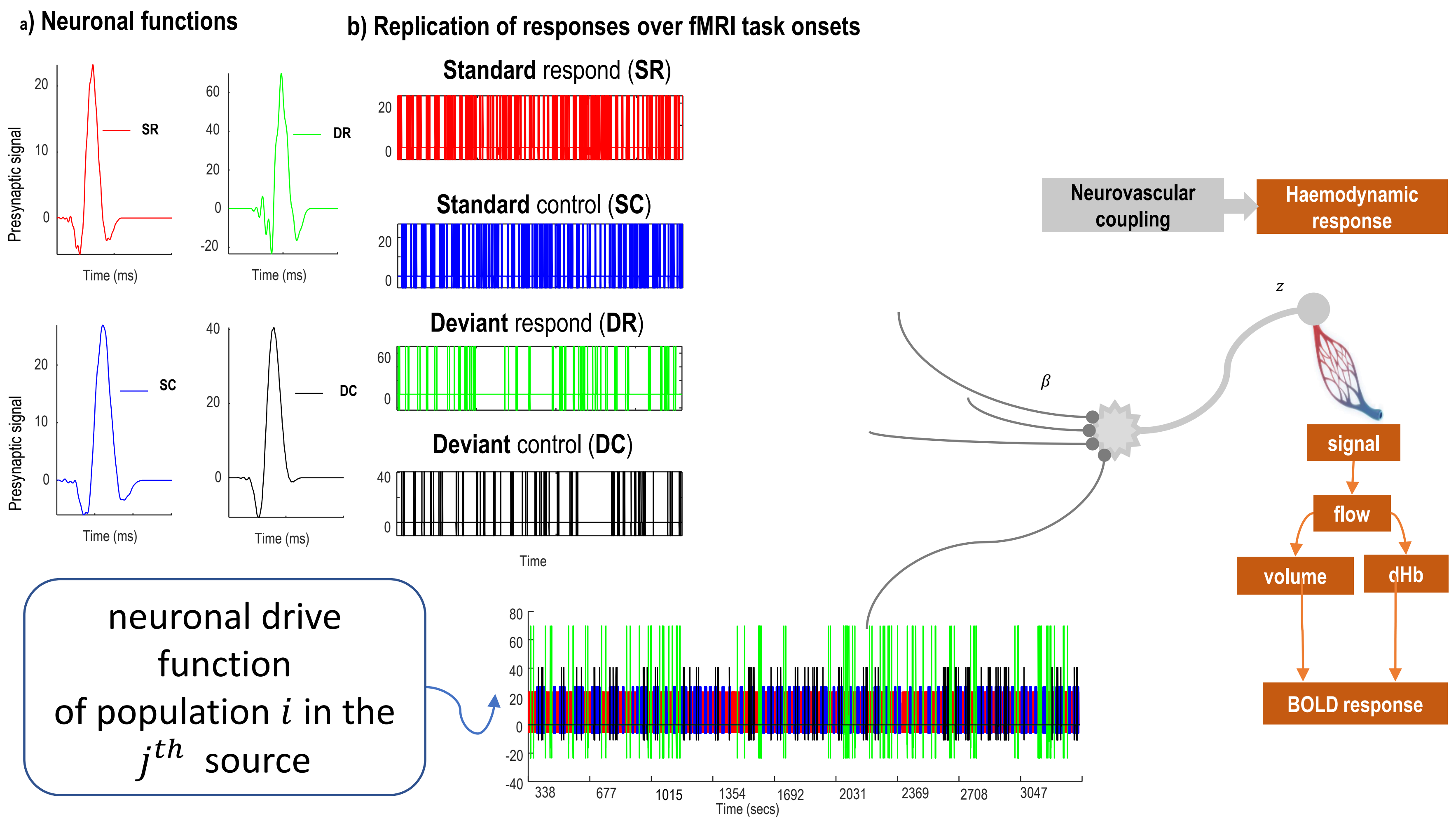


1. Spiny stellate cells
2. Superficial pyramidal cells
3. Inhibitory interneurons
4. Deep pyramidal cells



Simulating neuronal drive function




a) Neuronal functions
b) Replication of responses over fMRI task onsets
Standard respond (SR)
Standard control (SC)
Deviant respond (DR)
Deviant control (DC)
SR
DR
SC
DC
Presynaptic signal
Time (ms)
Time
Time (secs)
Neurovascular coupling
Haemodynamic response
z
β
signal
flow
volume
dHb
BOLD response
neuronal drive function of population i in the j^{th} source

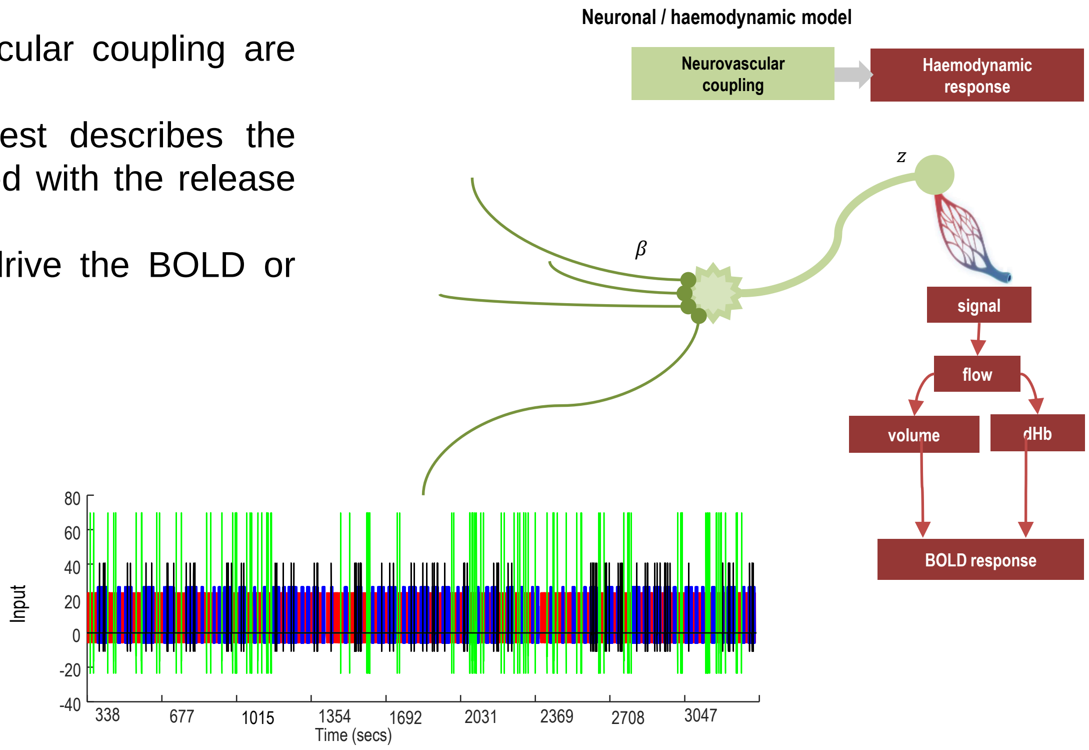
Q1) Whether neurovascular coupling is excited by presynaptic versus postsynaptic neuronal drive;

Q2) Whether distal neuronal sources exert changes on regional BOLD responses;

Q3) Whether the parameters of neurovascular coupling are region specific or equal for all regions

Q4) Whether a static vs linear model best describes the dynamics of astrocyte responses associated with the release of vasoactive agents (e.g., calcium).

Q5) Whether the power of ERP signals drive the BOLD or ERP signals?



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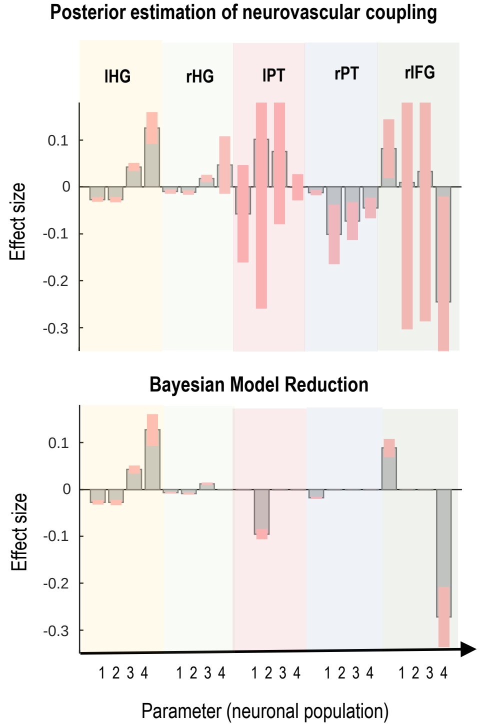
Structure learning: Comparing hypothesis

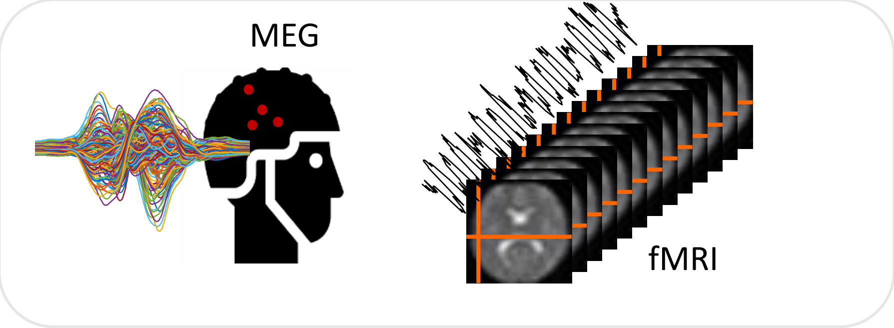
Model space design to investigate function of neurovascular coupling.

Model	F1: Parameterisation	F2: Distal inputs?	F3: Region-specific?	F4: Direct vs Delay
1	Pre	Yes	Yes	Direct
2	Pre	No	Yes	Direct
3	Pre	Yes	No	Direct
4	Pre	No	No	Direct
5	Post	N/A	Yes	Direct
6	Post	N/A	No	Direct
7	Pre (Friston et al., 2017)	Yes	No	Direct
8	Pre (Friston et al., 2017)	No	No	Direct
9	Pre	Yes	Yes	Delay
10	Pre	No	Yes	Delay
11	Pre	Yes	No	Delay
12	Pre	No	No	Delay
13	Post	N/A	Yes	Delay
14	Post	N/A	No	Delay
15	Pre (Friston et al., 2017)	Yes	No	Delay
16	Pre (Friston et al., 2017)	No	No	Delay

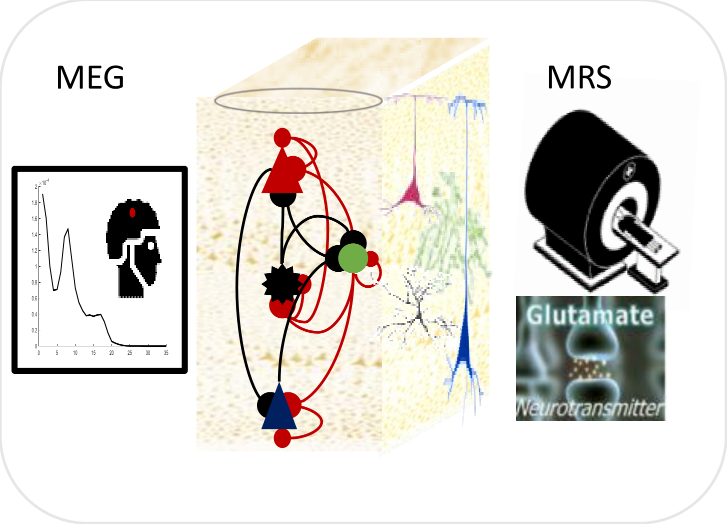


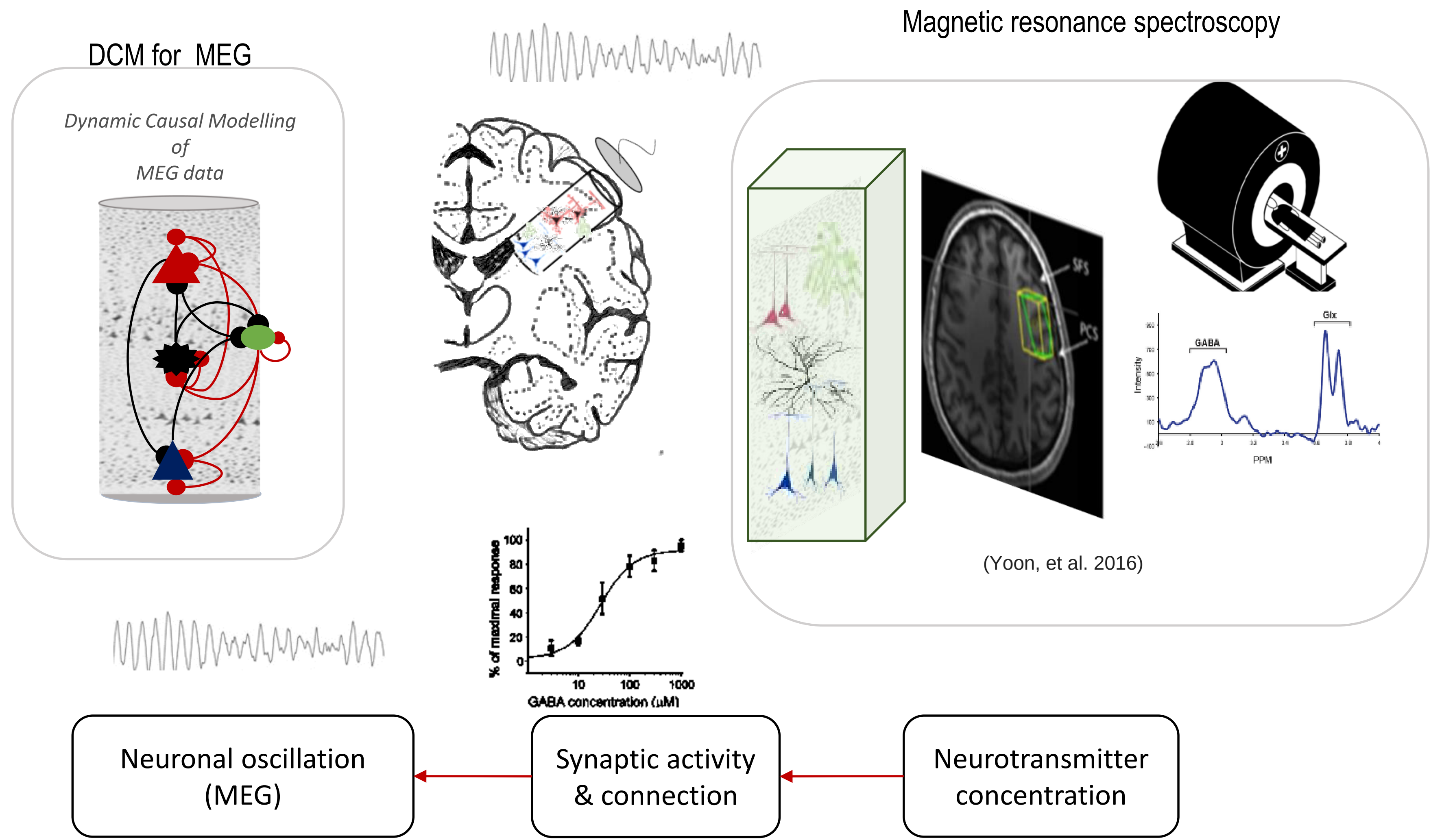
The most likely neurovascular coupling mechanisms that induce BOLD responses receive instantaneous local presynaptic neuronal activity, with region-specific parameterization!

Multimodal DCM: functional (MEG) +functional (fMRI)

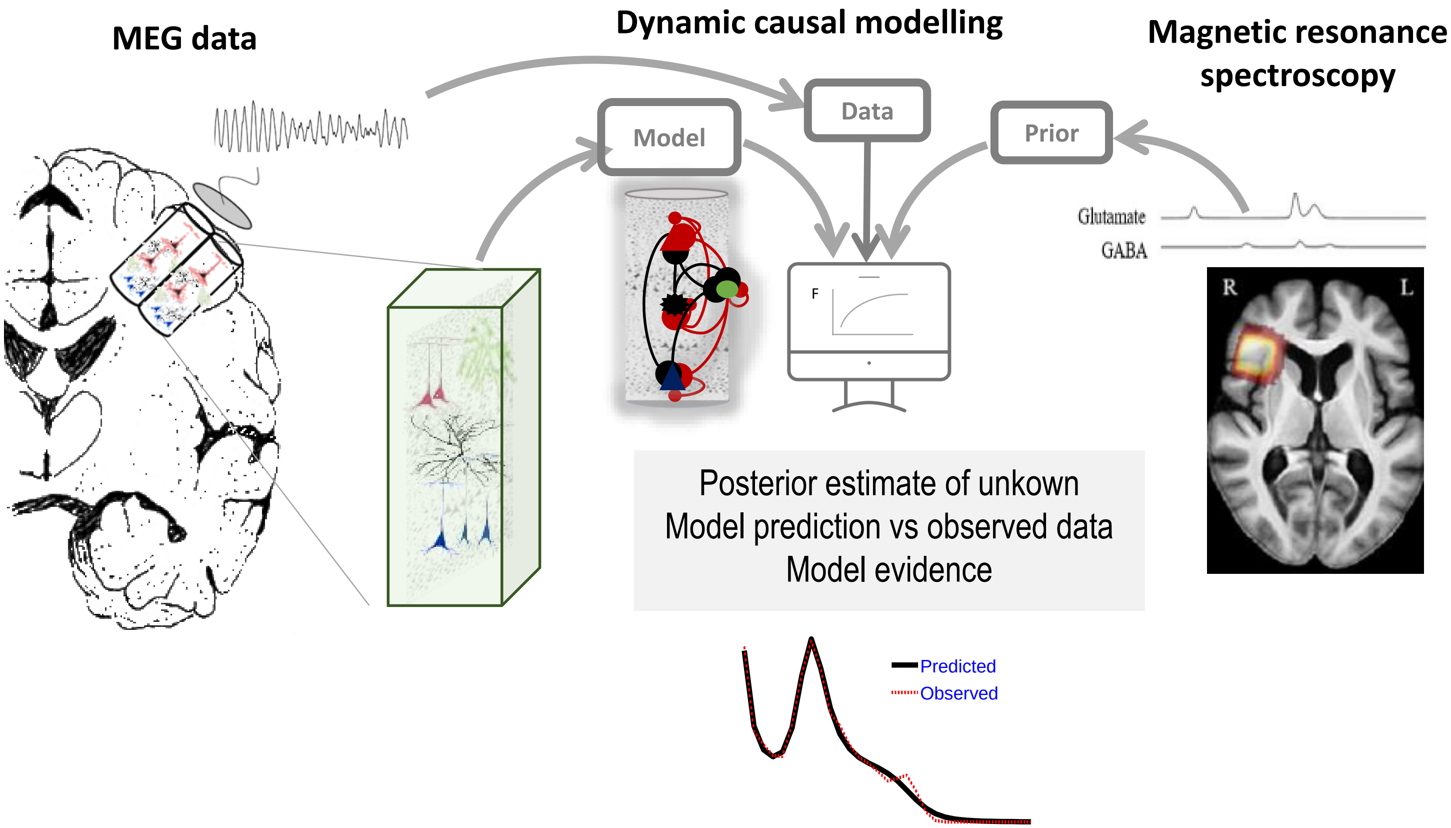


Multimodal DCM: structural (MRS) + functional (MEG)

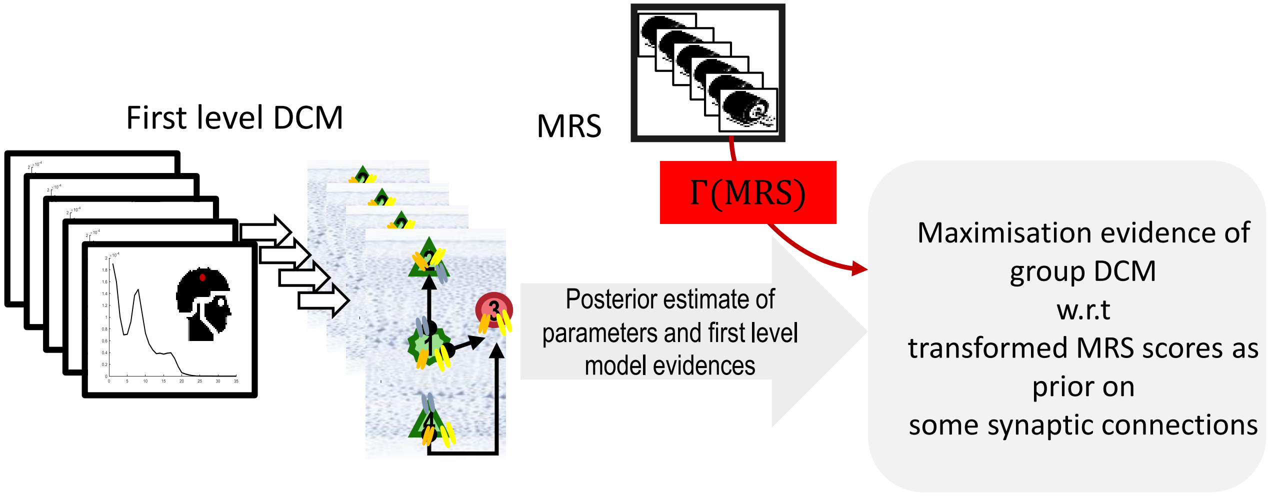



DCM for MEG
Dynamic Causal Modelling
of
MEG data
Magnetic resonance spectroscopy
SFS
PCS
GABA
Glx
Intensity
PPM
(Yoon, et al. 2016)
% of maximal response
0
20
40
60
80
100
10
100
1000
GABA concentration (μM)
Neuronal oscillation
(MEG)
Synaptic activity
& connection
Neurotransmitter
concentration

Inclusion of empirical prior -MRS data- into DCM pipeline



Which specific synaptic connections should be influenced by transformed MRS data and how?

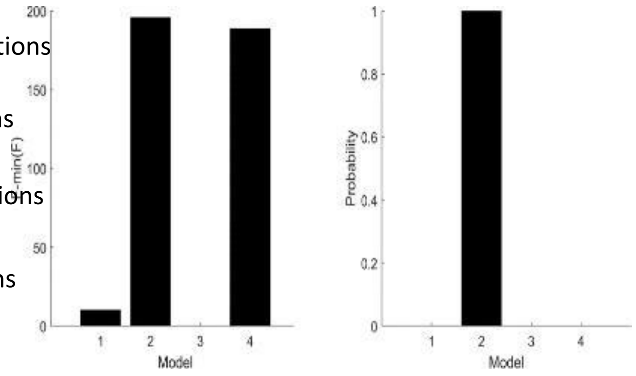


Model 1: Polynomial transformations of MRS GABA & self-inh connections

Model 2: Sigmoid transformations of MRS GABA & self-inh connections

Model 3: Polynomial transformations of MRS GABA & intrinsic connections

Model 4: Sigmoid transformations of MRS GABA & intrinsic connections



Appendix:

Input-output relation: Convolution of input and synaptic Impulse response

$$v_i(t) = u(t) * h(t)$$

$$h(t) = \alpha \frac{t}{k} e^{\frac{-t}{k}}$$

Laplace transform

$$x(s) = \int x(t) e^{-st} \mathrm{dt}$$

After Laplace transform: convolution becomes product

Laplace transform of input

Laplace transform of h(t)

$$v_i(s) = u(s)h(s) = u(s) \frac{\alpha\kappa}{s^2 + 2\kappa s + \kappa^2}$$

We multiply the denominator In right hand side with $v_i(s)$

$$s^2 v_i(s) + 2\kappa s v_i(s) + \kappa^2 v_i(s) = \alpha\kappa u(s)$$

$s^n v(s)$ in Laplace domain is equal to n^{th} derivative of v(t) in time domain

Inverse Laplace transform

$$x(t) = \int x(s) e^{st} \mathrm{ds}$$

$$v_i''(t) + 2\kappa v_i'(t) + \kappa^2 v_i(t) = \alpha\kappa u(t)$$

$$\left(\frac{d^2}{dt^2} + 2\kappa \frac{d}{dt} + \kappa^2\right) v_i(t) = \alpha\kappa u(t)$$

$$\Gamma v_i(t) = \alpha\kappa u(t)$$

Using Γ notation to simplify writing of the left hand side!

$$v_i'(t) = I(t)$$

$$I'(t) = -2\kappa I(t) - \kappa^2 v_i(t) - \alpha\kappa u(t)$$

First order differential equations equivalent of the convolution integral.

Thanks for your attention

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