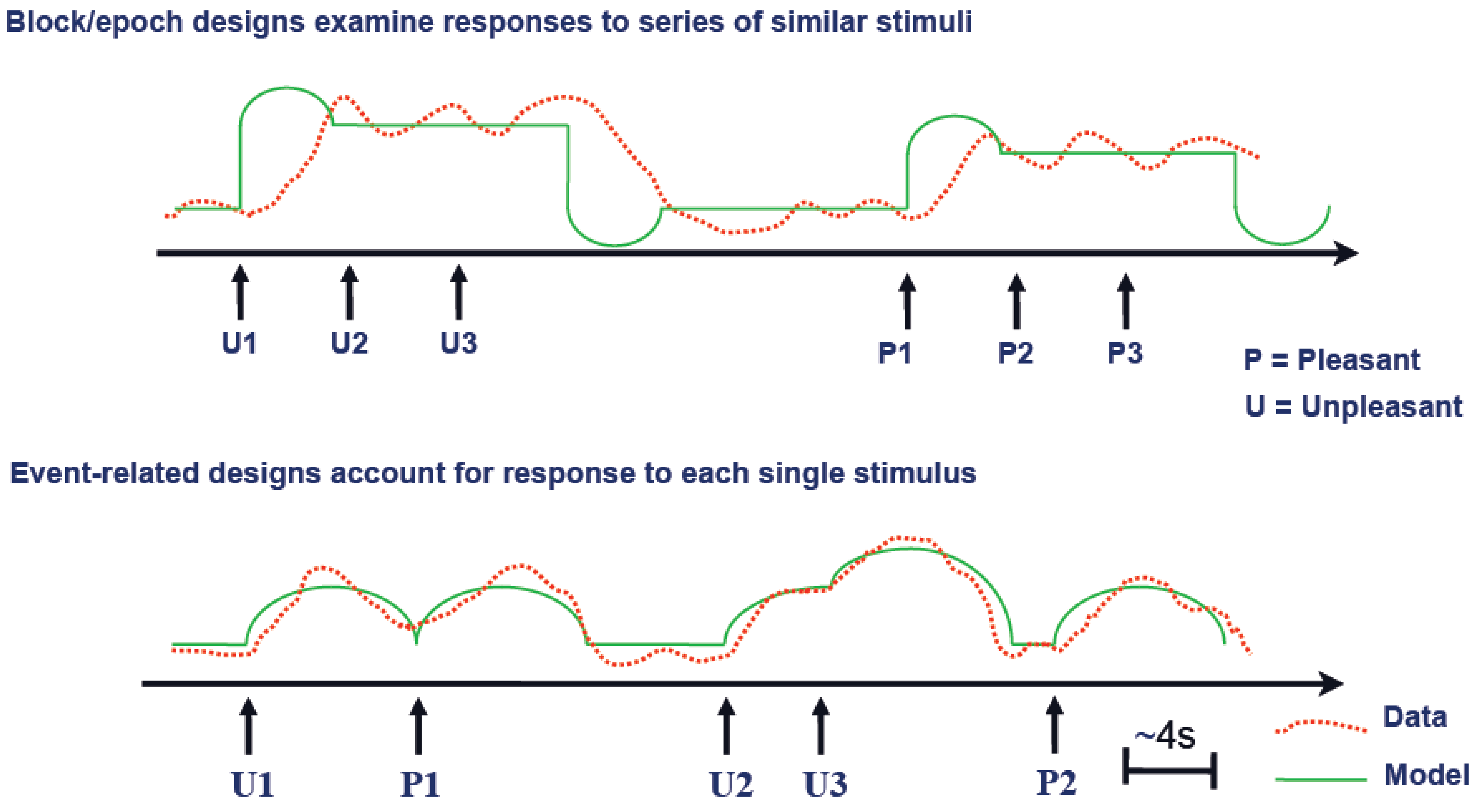
1st level analysis

Basis functions, parametric modulation and correlated regressors

First Level Analysis

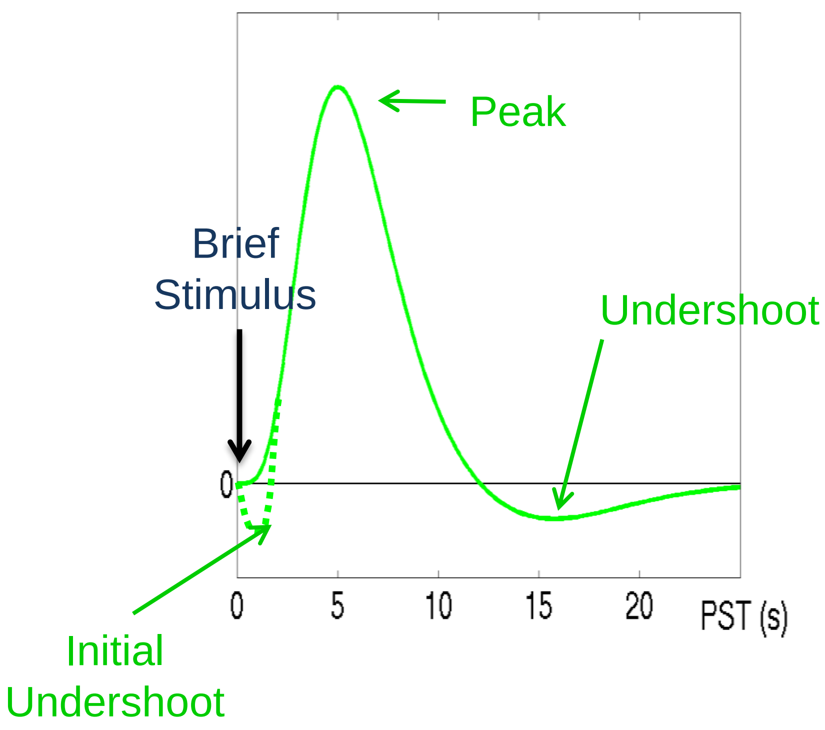
- Bold impulse response
- Temporal Basis Functions
- Parametric modulation
- Correlated regressors

Blocked design vs. event-related design



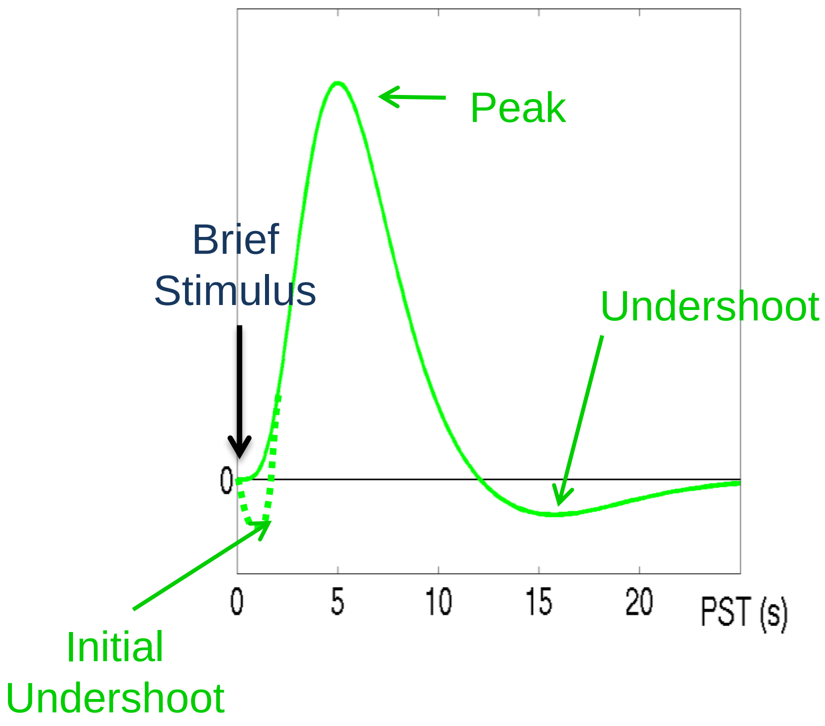
Hemodynamic Response Function (HRF)

- Function of blood oxygenation, flow, volume
- Peak (max. oxygenation) 4-6s poststimulus; baseline after 20-30s
- Initial undershoot can be observed
- Similar across V1, A1, S1… but possible differences across:
  - other regions
  - individuals



Hemodynamic Response Function (HRF)

- Long SOA → BOLD response returns to baseline, no overlap
- Overlap can be accommodated if the BOLD response is explicitly modelled (linear superposition)
- Short SOAs are more sensitive



General Linear (convolution) model

GLM for a single voxel:

$$y(t) = u(t) \otimes h(\tau) + \varepsilon(t)$$

$u(t)$ = neural causes (stimulus train)

$$u(t) = \sum \delta(t - nT)$$

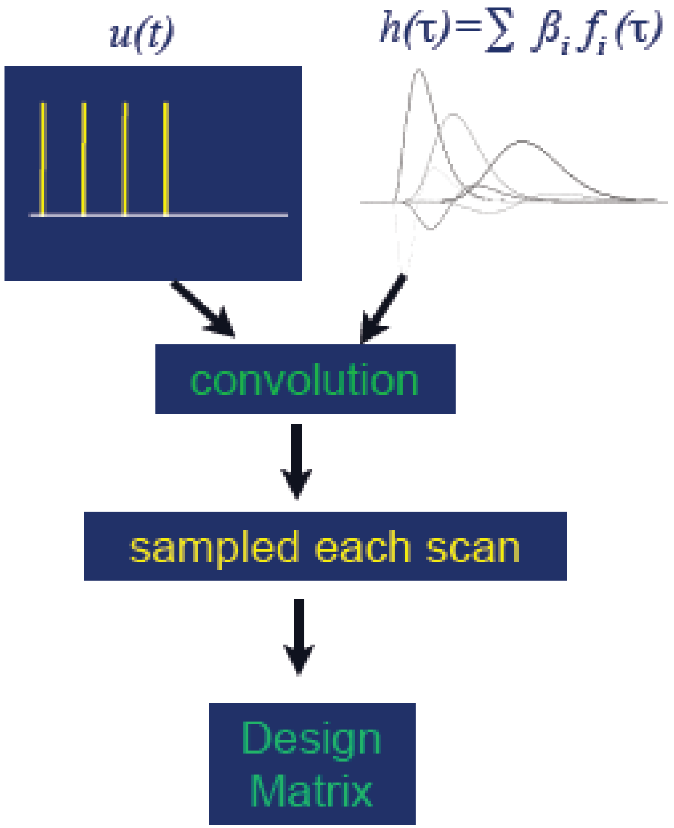
$h(\tau)$ = hemodynamic (BOLD) response

$$h(\tau) = \sum \beta_i f_i(\tau)$$

$f_i(\tau)$ = temporal basis functions

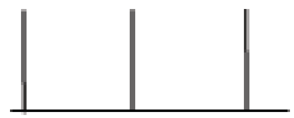
$$y(t) = \sum \sum \beta_i f_i(t - nT) + \varepsilon(t)$$

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

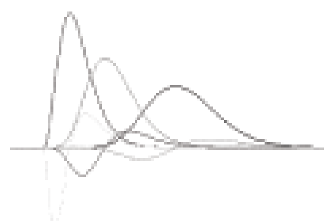


General linear model

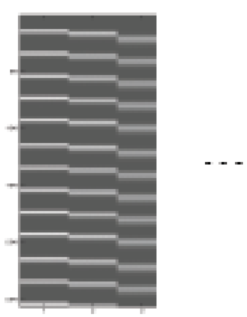
Stimulus every 20s

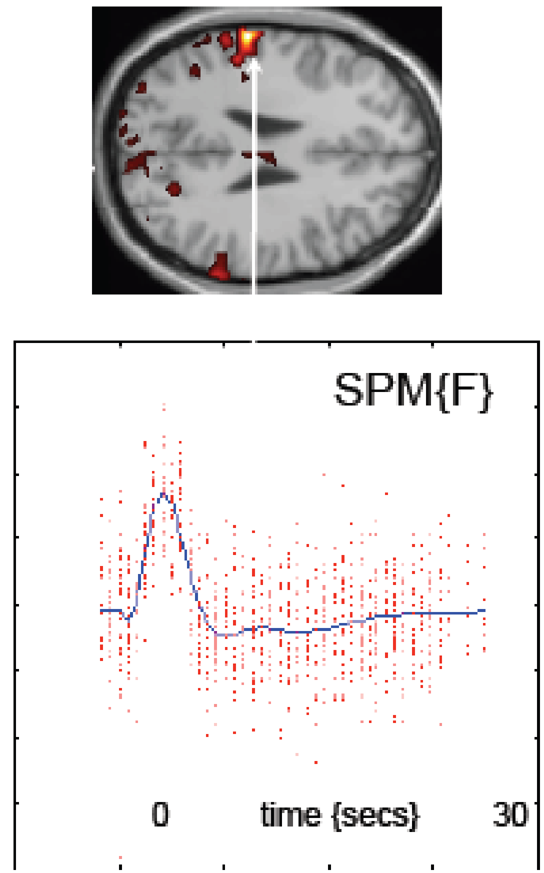


Gamma functions $f_i(\tau)$ of peristimulus time τ (Orthogonalised)

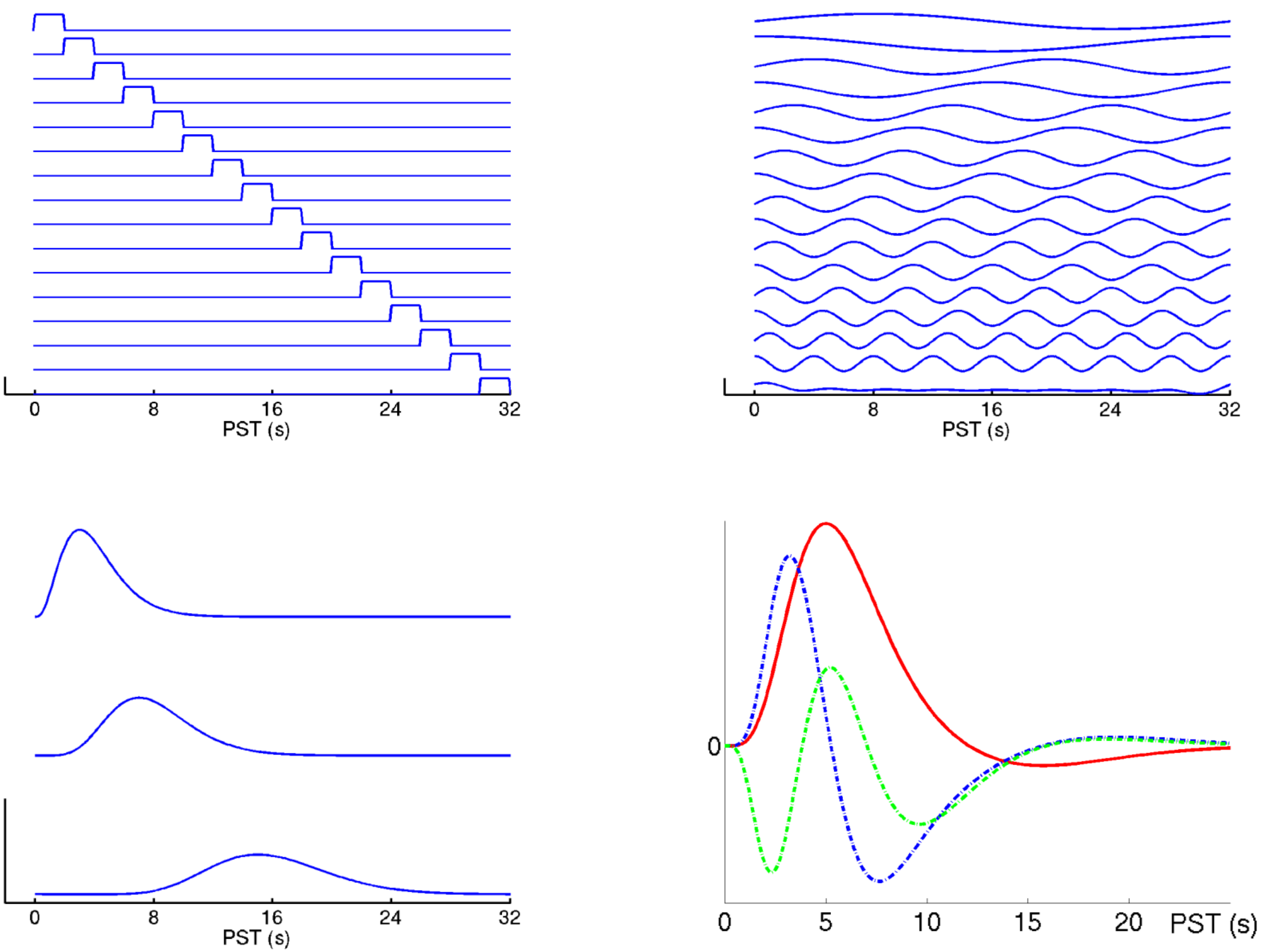


Sampled every TR = 1.7s
Design matrix, $\mathbf{X}$
$[x(t) \otimes f_1(\tau) \mid x(t) \otimes f_2(\tau) \mid \ldots]$





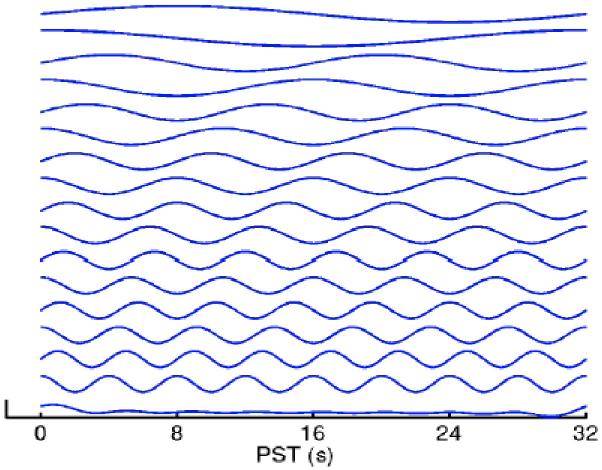
Temporal basis functions



Temporal basis functions

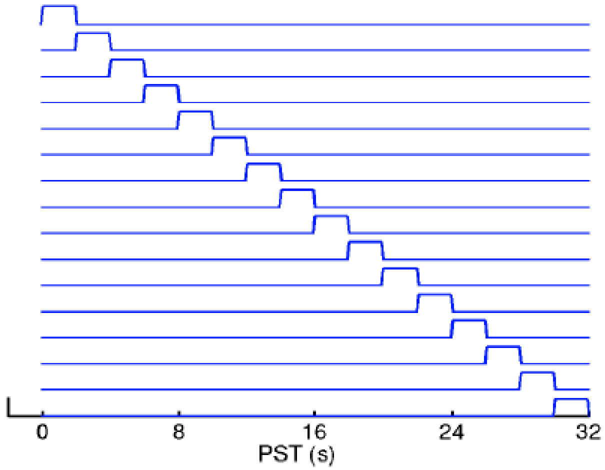
- Fourier Set

  Windowed sines & cosines
  Any shape (up to frequency limit)
  Inference via F-test



- Finite Impulse Response

  Mini “timebins” (selective averaging)
  Any shape (up to frequency limit)
  Inference via F-test



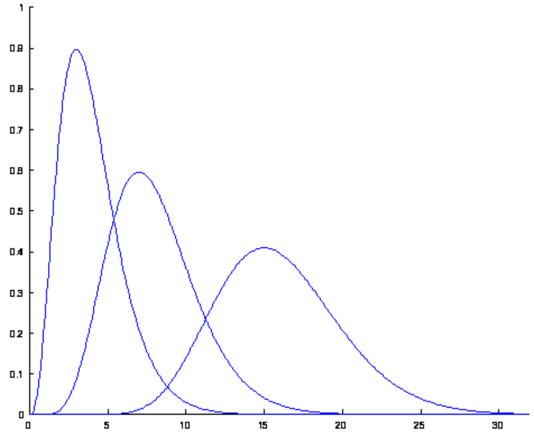
Temporal basis functions

- Gamma Functions

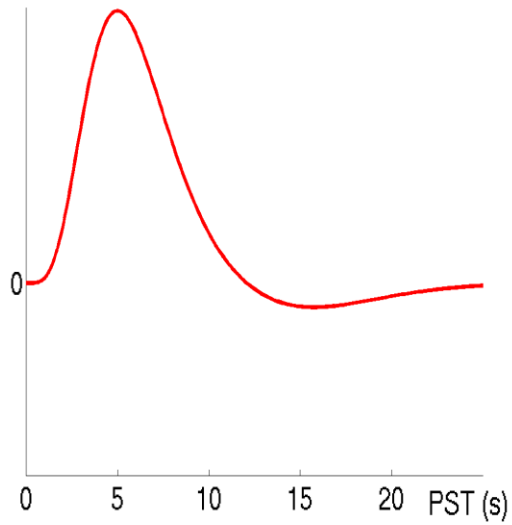
  Bounded, asymmetrical (like BOLD)

  Set of different lags

  Inference via F-test



Two Gamma functions added



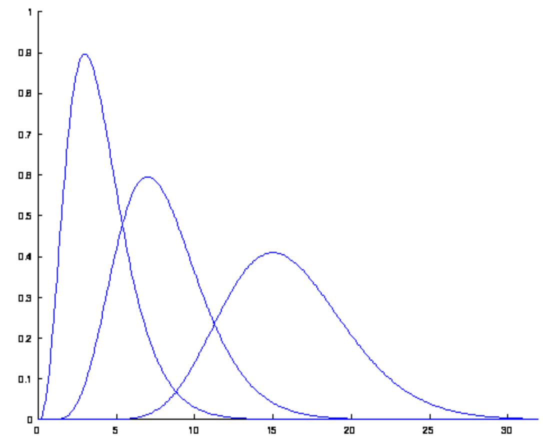
Temporal basis functions

- Gamma Functions

  Bounded, asymmetrical (like BOLD)

  Set of different lags

  Inference via F-test

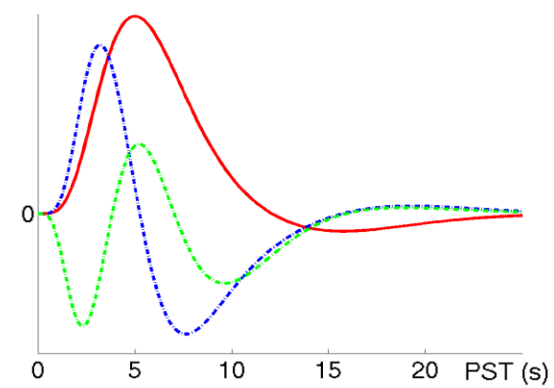


- “Informed” Basis Set

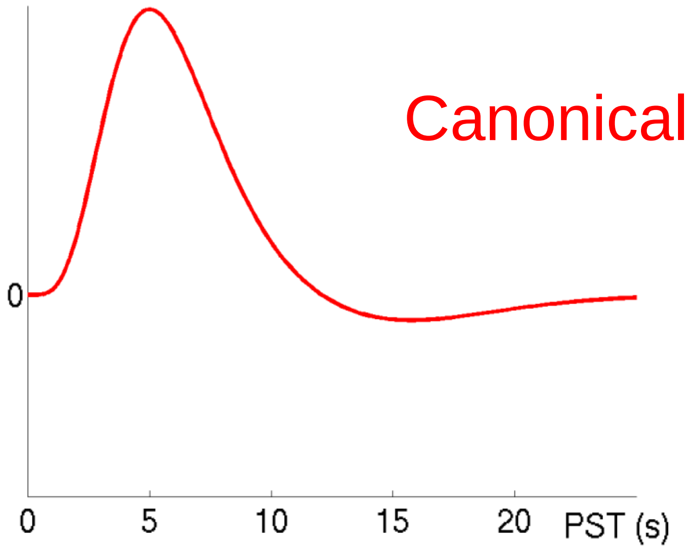
  Best guess of canonical BOLD response

  Variability captured by Taylor expansion

  “Magnitude” inferences via t-test…?



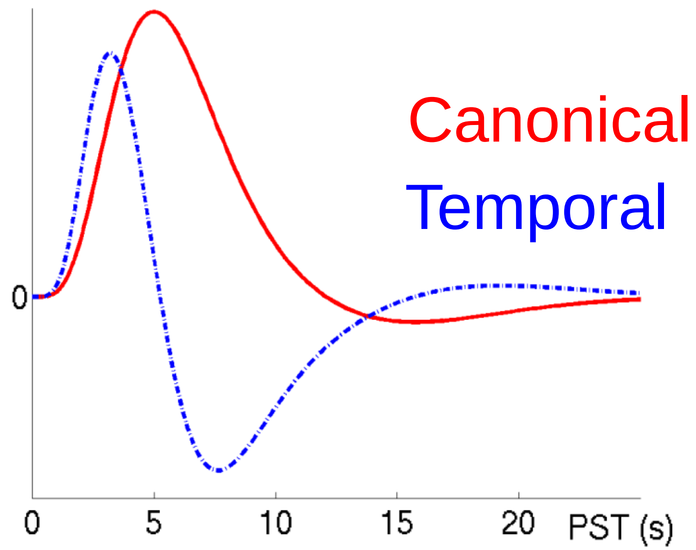
Temporal basis functions



"Informed" Basis Set
(Friston et al. 1998)

Canonical HRF (2 gamma functions)

Temporal basis functions



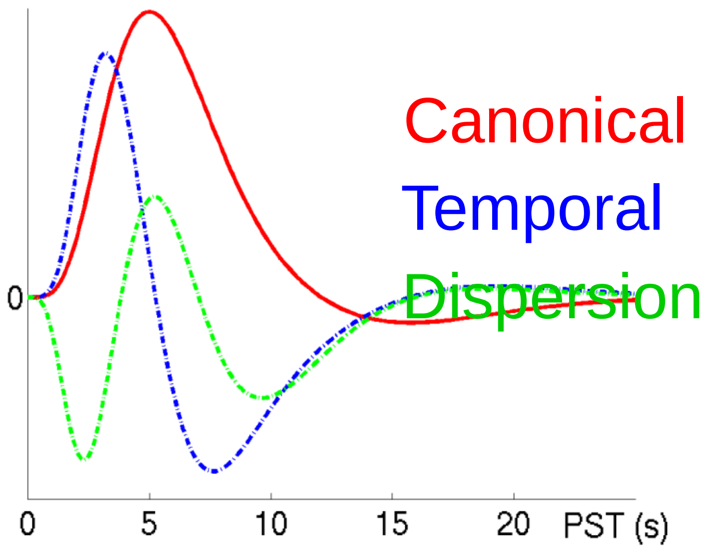
"Informed" Basis Set
(Friston et al. 1998)

Canonical HRF (2 gamma functions)

plus Multivariate Taylor expansion in:

- time (*Temporal Derivative*)

Temporal basis functions



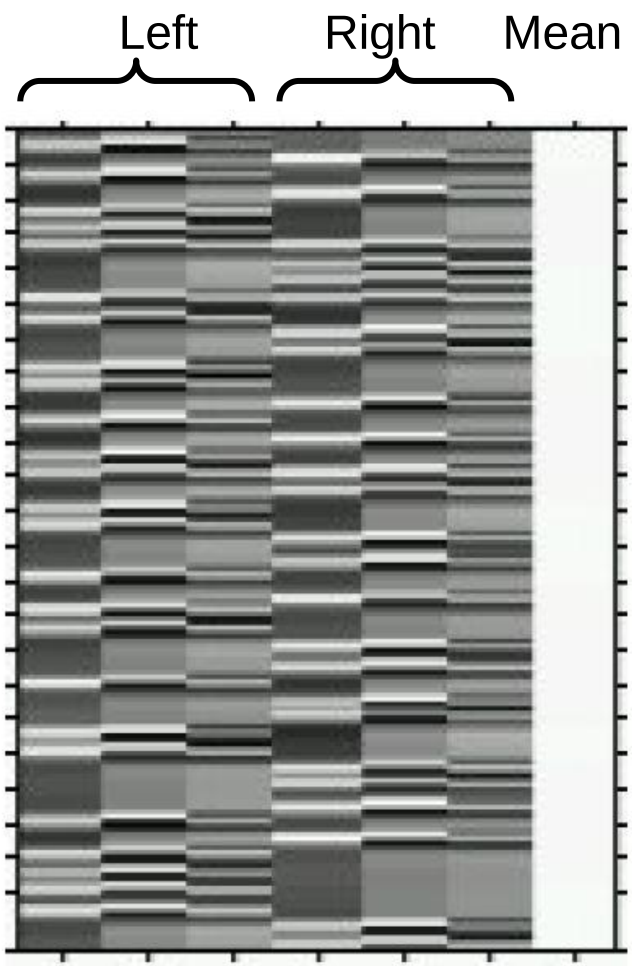
"Informed" Basis Set
(Friston et al. 1998)

Canonical HRF (2 gamma functions)

plus Multivariate Taylor expansion in:

- time (*Temporal Derivative*)
- width (*Dispersion Derivative*)

Design Matrix



3 regressors used to model each condition

The three basis functions are:

1. Canonical HRF

2. Derivatives with respect to time

3. Derivatives with respect to dispersion

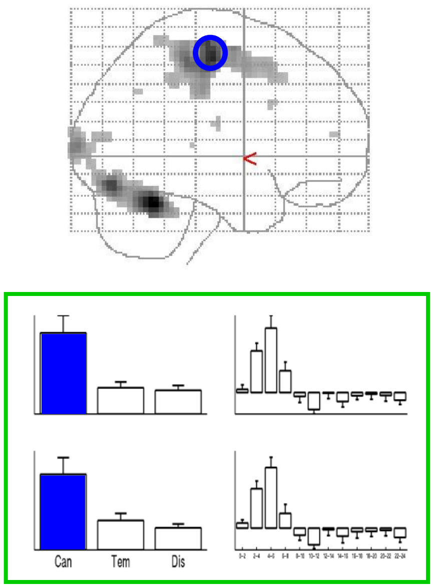
Temporal basis functions

- “Informed” Basis Set
  - “Magnitude” inferences via t-test on canonical parameters (providing canonical is a reasonable fit)
  - “Latency” inferences via tests on ratio of derivative : canonical parameters

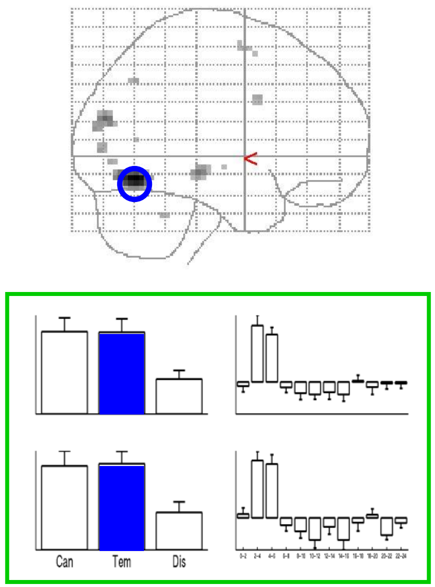
Which temporal basis set?

Example: rapid motor response to faces, *Henson et al, 2001*

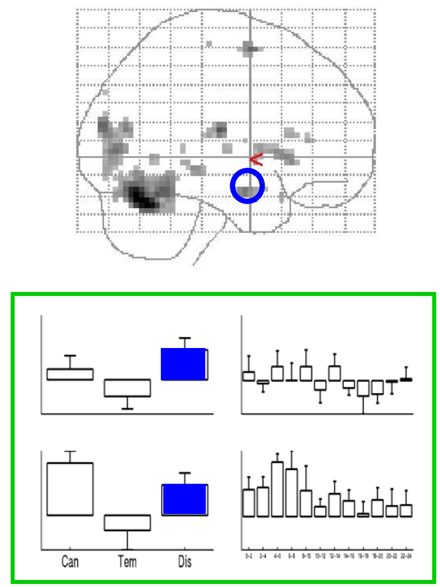
Canonical



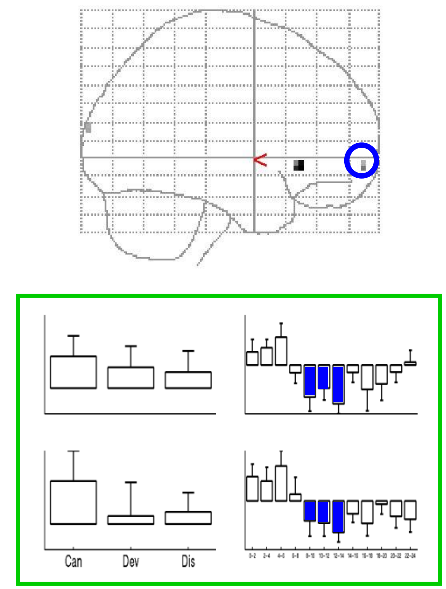
+ Temporal



+ Dispersion



+ FIR



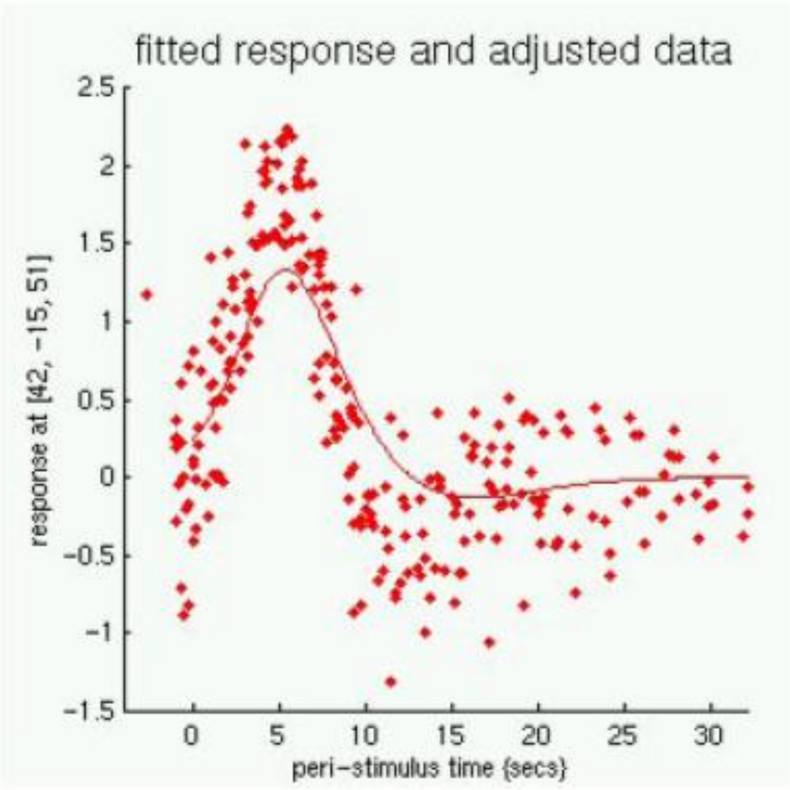
…canonical + temporal + dispersion derivatives appear sufficient

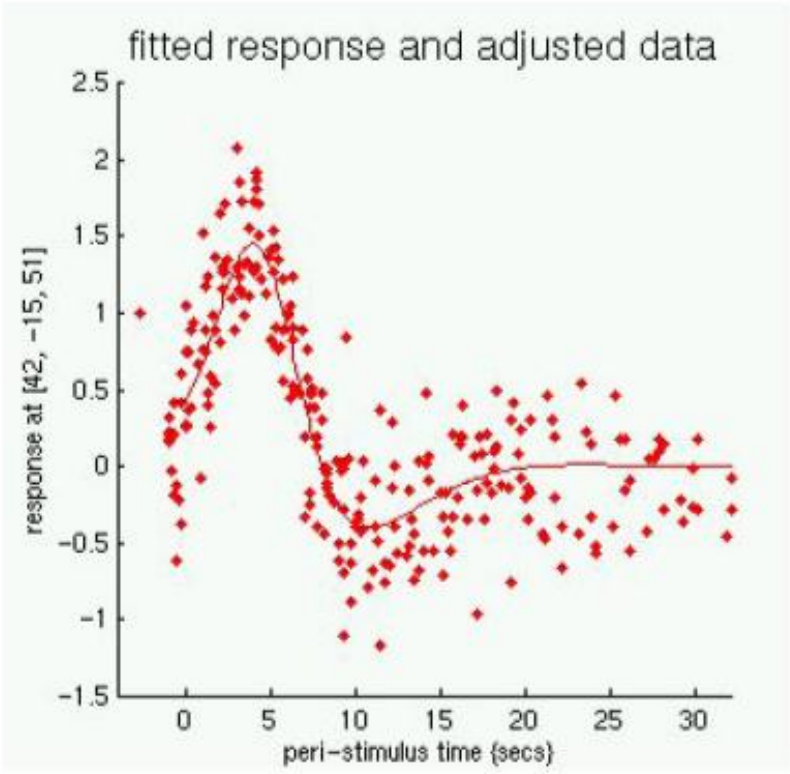
…may not be for more complex trials (eg stimulus-delay-response)

…but then such trials better modelled with separate neural components (ie activity no longer delta function) + constrained HRF (Zarahn, 1999)

Comparison of the fitted response

Haemodynamic response in a single voxel.





Left: Estimation using the simple model

Right: More flexible model with basis functions

Summary

SPM uses basis functions to model the hemodynamic response using a single basis function or a set of functions.

The most common choice is the `Canonical HRF' (Default in SPM)

By adding the time and dispersion derivatives one can account for variability in the signal change over voxels

Part II:
Correlated regressors parametric/non-parametric design

Multicollinearity

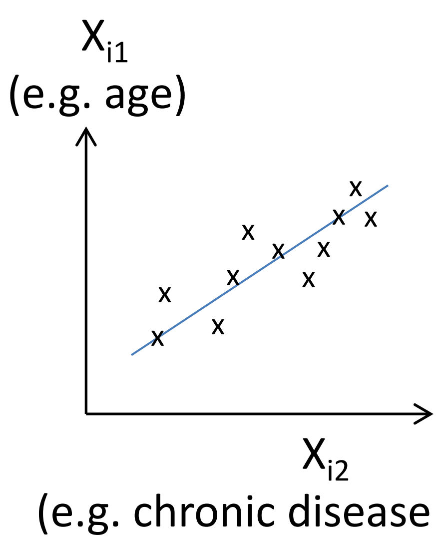
$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_N x_{iN} + \varepsilon$$

Coefficients reflect
an estimated change in y
with every unit change in x_i
while controlling for all other regressors

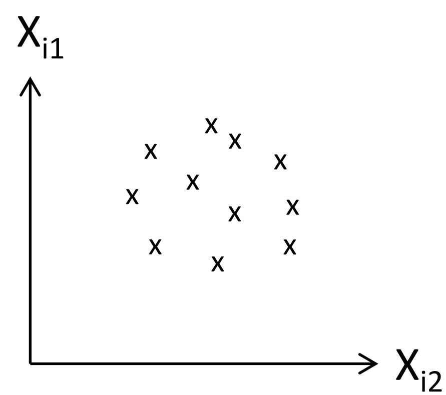
Multicollinearity

$$y_i = \beta_0 + \underbrace{\beta_1 x_{i1} + \beta_2 x_{i2}}_{x_{i1} = \lambda_0 + \lambda x_{i2} + v} + \ldots + \beta_N x_{iN} + \varepsilon$$

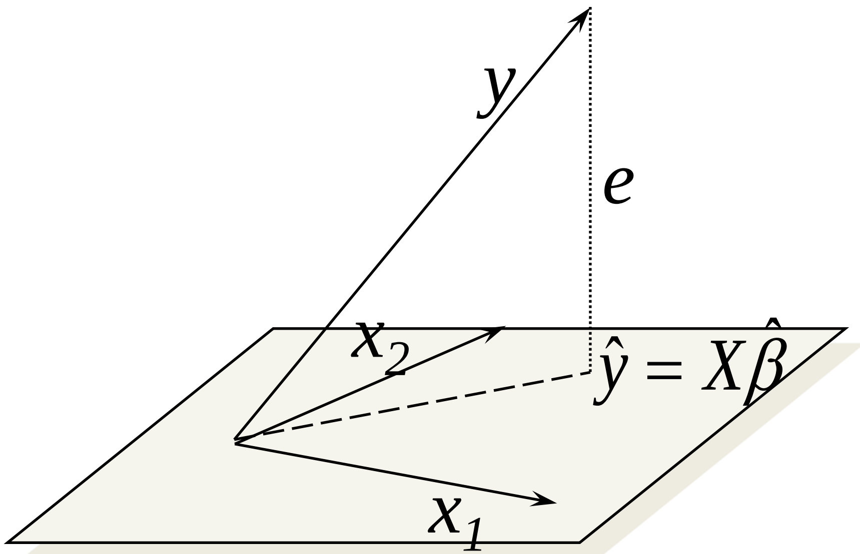
low variance of *v*



high variance of *v*



Multicollinearity and estimability



(SPM course Oct. 2010, Guillaume Flandin)

OLS minimizes *e* by

$$\boldsymbol{Xe = 0}$$

with

$$\boldsymbol{e = Y - (X\beta_{estim})^{-1}}$$

which gives

$$\boldsymbol{\beta_{estim} = (X^T X)^{-1} X^T Y}$$

cf
covariance matrix

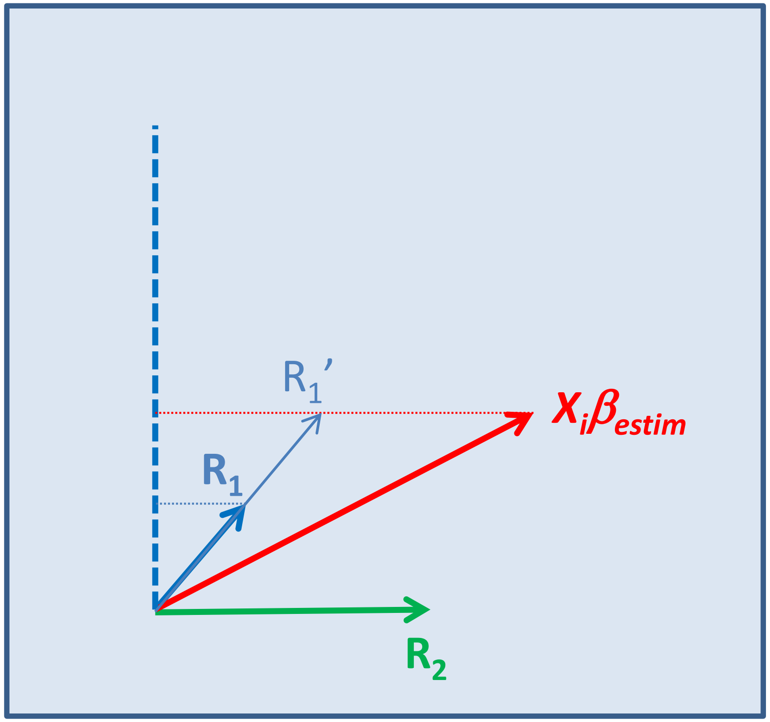
<u>high multicollinearity</u>
(i.e. variance of *v* small)

$\Rightarrow$ inaccuracy of individual β_{estim}, high standard error

<u>perfect multicollinearity</u>
(i.e. variance of *v* = 0)

$\Rightarrow$ *det(X) = 0*
$\Rightarrow$ $(X^T X)$ not invertible
$\Rightarrow$ β_{estim} not unique

Multicollinearity



(t- and [unidimensional] F-) testing of a single regressor (e.g. R_1) ≙ testing for the component that is <u>not explained by</u> (is orthogonal to) the other/the reduced model (e.g. R_2)

$\Rightarrow$ multicollinearity is contrast specific

$\Rightarrow$ “conflating” correlated regressors by means of (multidimensional) F-contrasts permits assessing common contribution to variance

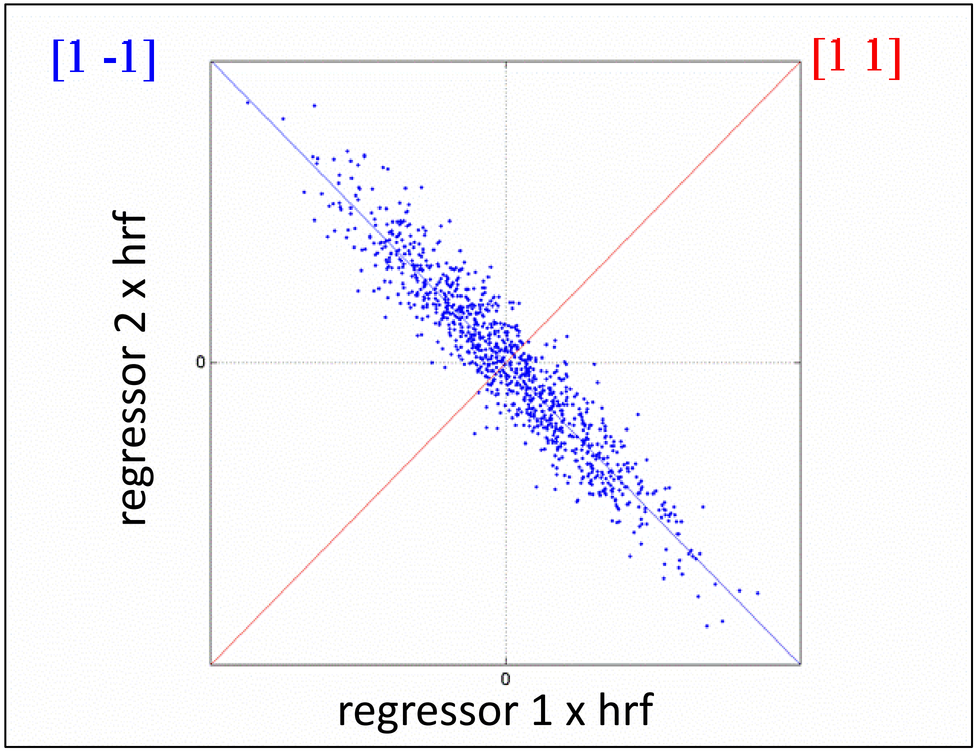
($X_i\beta_{estim}$ = projection of Y_i onto X space)

Multicollinearity

(relatively) little spread after projection onto

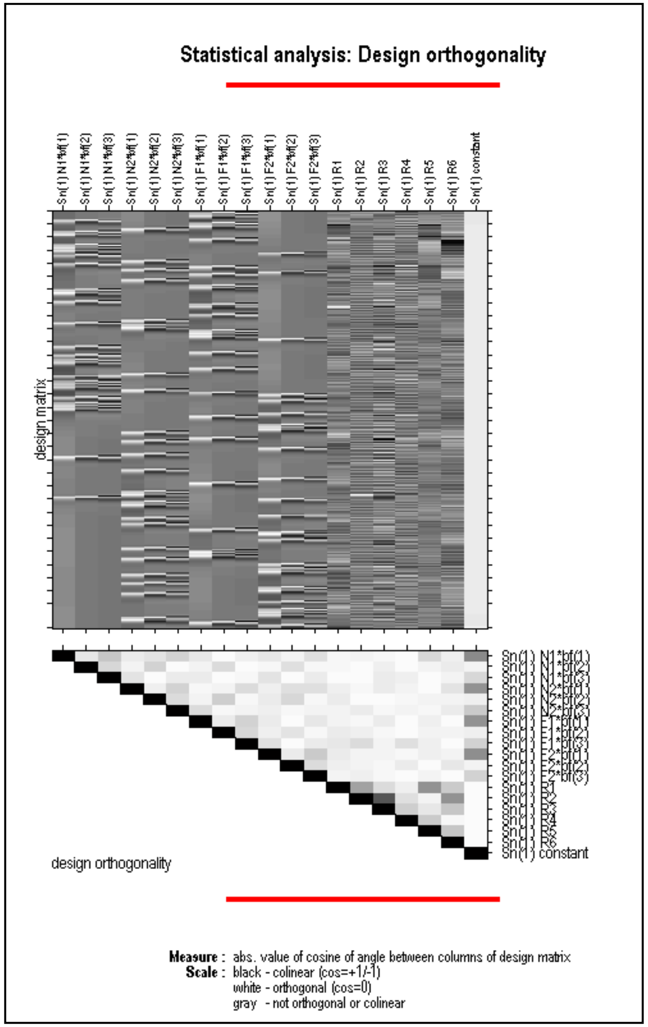
x-axis,
y-axis or
f(x) = x

reflecting reduced efficiency for detecting dependencies of the observed data on the respective (combination of) regressors



(MRC CBU Cambridge,
http://imaging.mrc-cbu.cam.ac.uk/imaging/DesignEfficiency)

Orthogonality matrix



reflects the cosine of the angles between respective pairs of columns

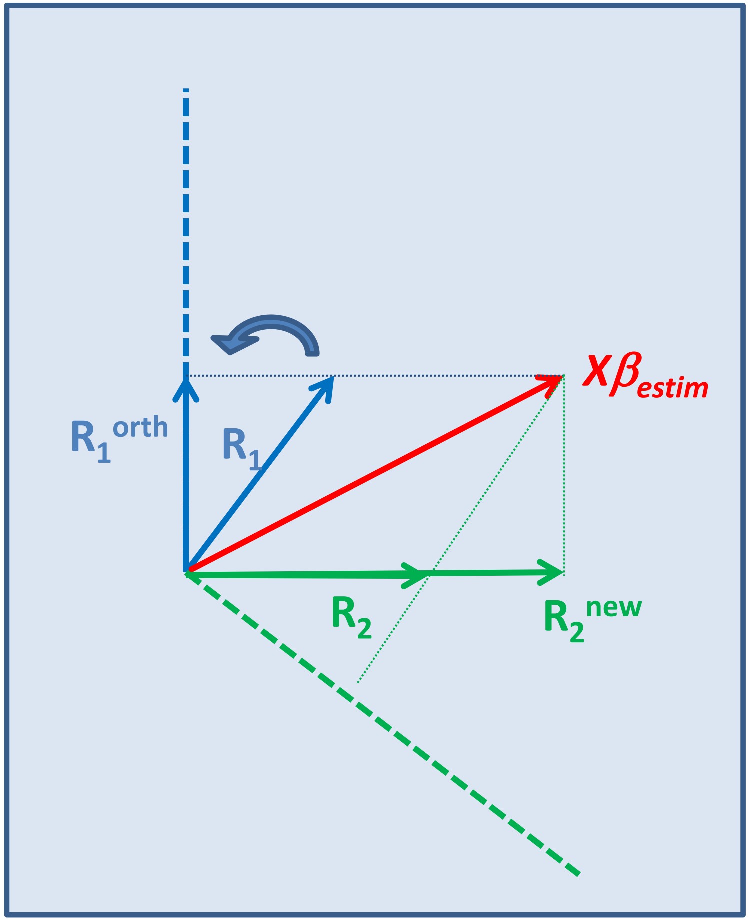
(SPM course Oct. 2010, Guillaume Flandin)

Orthogonalizing

leaves the parameter estimate of R_1 unchanged but alters the estimate of the R_2 parameter

assumes unambiguous causality between the orthogonalized predictor and the dependent variable by attributing the common variance to this one predictor **only**

hence rarely justified



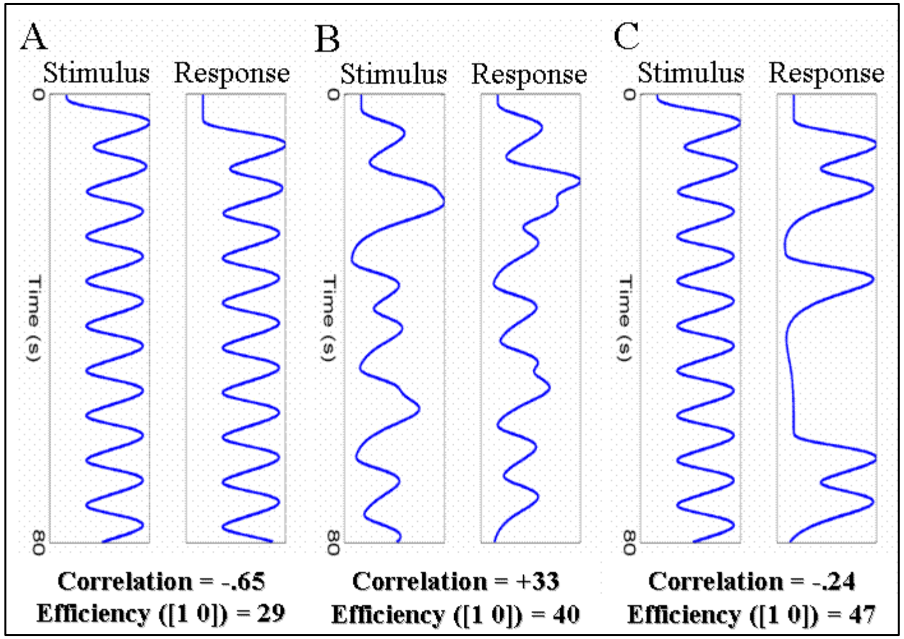
Dealing with multicollinearity

- Avoid.
 (avoid dummy variables; when sequential scheme of predictors (stimulus – response) is inevitable: inject jittered delay (see B) or use a probabilistic R_1-R_2 sequence (see C))

- Obtain more data to decrease standard error of parameter estimates

- Use F-contrasts to assess common contribution to data variance

- Orthogonalizing might lead to self-fulfilling prophecies

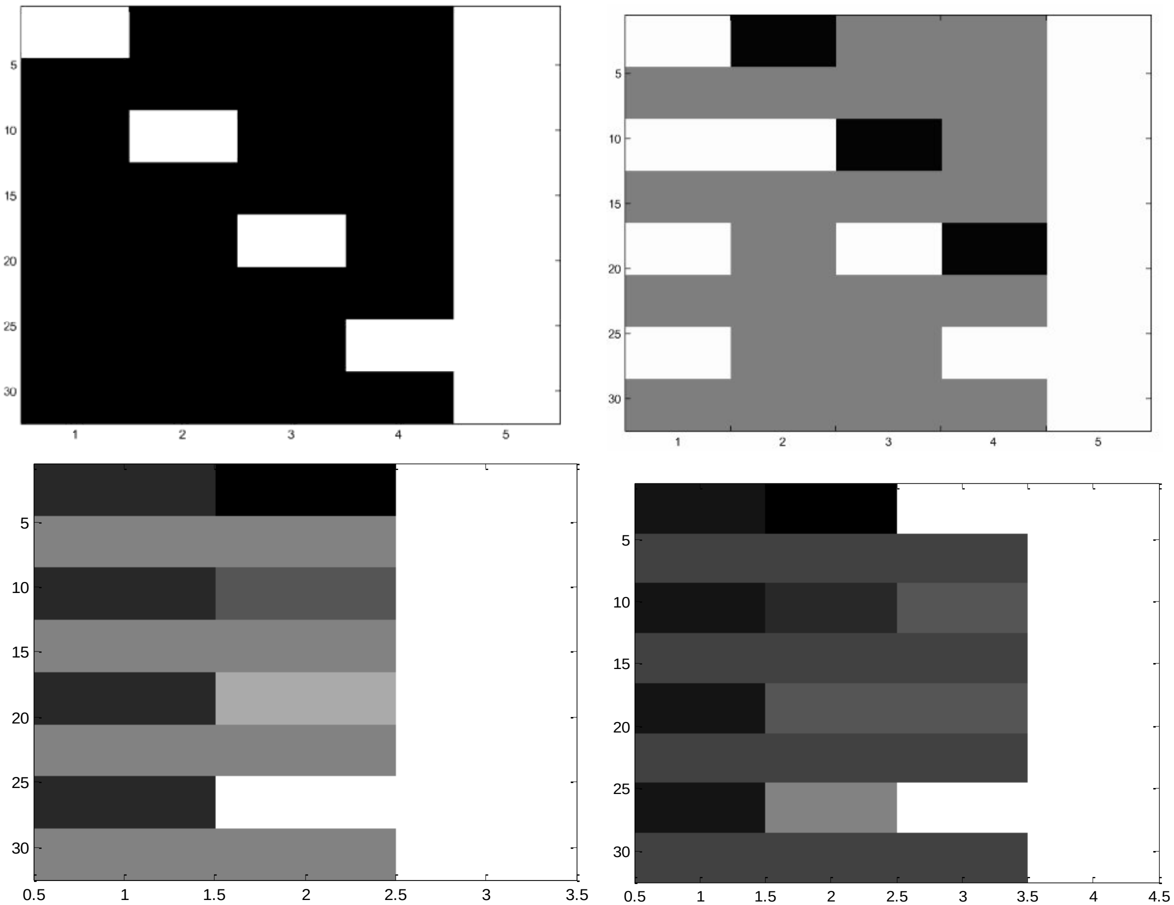


(MRC CBU Cambridge, http://imaging.mrc-cbu.cam.ac.uk/imaging/DesignEfficiency)

Parametric vs. factorial design

factorial

parametric



Widely-used example
(Statistical Parametric Mapping, Friston et al. 2007)

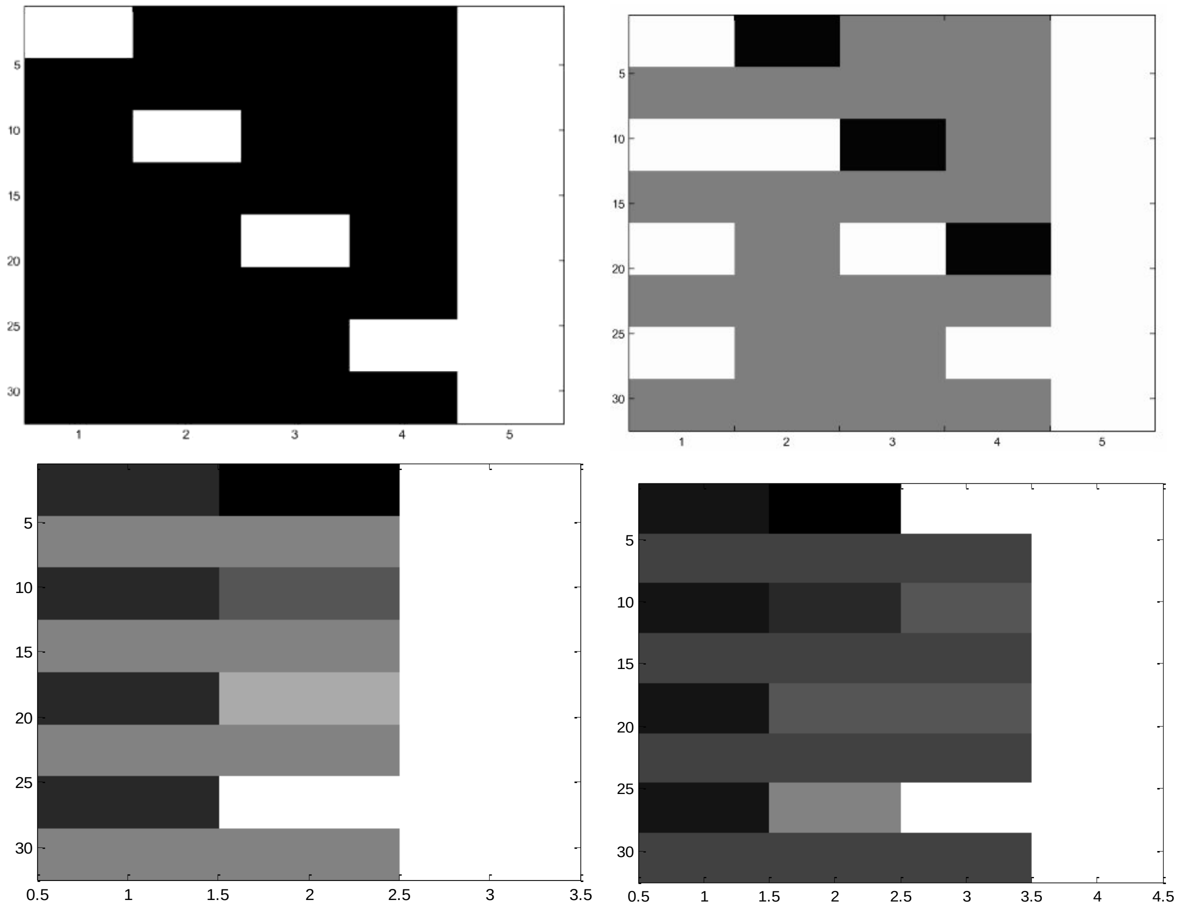
Four button press forces

Parametric vs. factorial design

Which – when?

factorial

parametric



Limited prior knowledge, flexibility in contrasting beneficial (“screening”):

Large number of levels/continuous range:

Resources

- Slides from Methods for Dummies 2011
- Rik Henson Short SPM Course slides
- SPM 2012 Course
- SPM Manual and Data Set

Special thanks to Guillaume Flandin