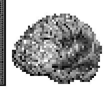
Spatial Preprocessing

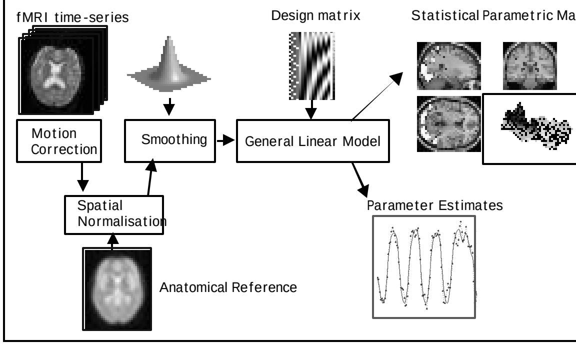
John Ashburner
john@fil.ion.ucl.ac.uk

- ⌘ Smoothing
- ⌘ Rigid registration
- ⌘ Spatial normalisation

With slides by Chloe Hutton and Jesper Andersson



Overview of SPM Analysis

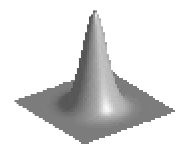


Contents

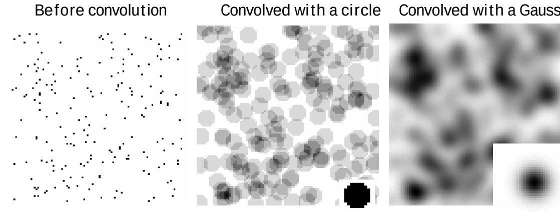
- ⌘Smoothing
- ⌘Rigid registration
- ⌘Spatial normalisation

Smoothing

Each voxel after smoothing effectively becomes the result of applying a weighted region of interest (ROI).

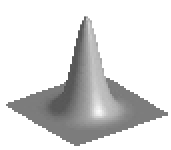


Before convolution — Convolved with a circle — Convolved with a Gaussian

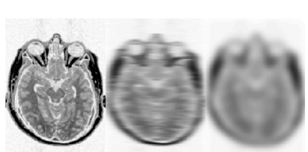


Smoothing

- ⌘Why smooth?
  - Potentially increase sensitivity
  - Inter-subject averaging
  - Increase validity of SPM
- ⌘Smoothing is a convolution with a Gaussian kernel



Gaussian convolution is separable



Contents

- ⌘Smoothing
- ⌘Rigid registration
  - Rigid-body transforms
  - Optimisation & objective functions
  - Interpolation
- ⌘Spatial normalisation

Within-subject Registration

- Assumes there is no shape change, and motion is rigid-body
- Used by [realign] and [coregister] functions
- The steps are:
  - **Registration** - i.e. Optimising the parameters that describe a rigid body transformation between the source and reference images
  - **Transformation** - i.e. Re-sampling according to the determined transformation

Affine Transforms

- Rigid-body transformations are a subset
- Parallel lines remain parallel
- Operations can be represented by:

$x_1 = m_{11}x_0 + m_{12}y_0 + m_{13}z_0 + m_{14}$

$y_1 = m_{21}x_0 + m_{22}y_0 + m_{23}z_0 + m_{24}$

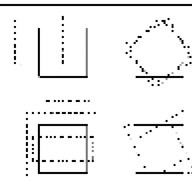
$z_1 = m_{31}x_0 + m_{32}y_0 + m_{33}z_0 + m_{34}$

- Or as matrices:

Y=Mx

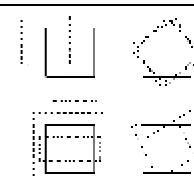
$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$$

2D Affine Transforms



- Translations by t_x and t_y
  - $x_1 = x_0 + t_x$
  - $y_1 = y_0 + t_y$
- Rotation around the origin by Θ radians
  - $x_1 = \cos(\Theta)\, x_0 + \sin(\Theta)\, y_0$
  - $y_1 = -\sin(\Theta)\, x_0 + \cos(\Theta)\, y_0$
- Zooms by s_x and s_y
  - $x_1 = s_x\, x_0$
  - $y_1 = s_y\, y_0$
- Shear
  - $x_1 = x_0 + h\, y_0$
  - $y_1 = y_0$

2D Affine Transforms



- Translations by t_x and t_y
  - $x_1 = 1\, x_0 + 0\, y_0 + t_x$
  - $y_1 = 0\, x_0 + 1\, y_0 + t_y$
- Rotation around the origin by Θ radians
  - $x_1 = \cos(\Theta)\, x_0 + \sin(\Theta)\, y_0 + 0$
  - $y_1 = -\sin(\Theta)\, x_0 + \cos(\Theta)\, y_0 + 0$
- Zooms by s_x and s_y:
  - $x_1 = s_x\, x_0 + 0\, y_0 + 0$
  - $y_1 = 0\, x_0 + s_y\, y_0 + 0$
- Shear
  - $x_1 = 1\, x_0 + h\, y_0 + 0$
  - $y_1 = 0\, x_0 + 1\, y_0 + 0$

3D Rigid-body Transformations

- A 3D rigid body transform is defined by:
  - 3 translations - in X, Y & Z directions
  - 3 rotations - about X, Y & Z axes
- The order of the operations matters

$$\begin{pmatrix} 1 & 0 & 0 & X_{trans} \\ 0 & 1 & 0 & Y_{trans} \\ 0 & 0 & 1 & Z_{trans} \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos F & \sin F & 0 \\ 0 & -\sin F & \cos F & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos T & 0 & \sin T & 0 \\ 0 & 1 & 0 & 0 \\ -\sin T & 0 & \cos T & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos O & \sin O & 0 & 0 \\ -\sin O & \cos O & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

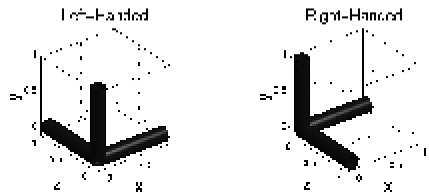
Translations — Pitch about x axis — Roll about y axis — Yaw about z axis

Voxel-to-world Transforms

- Affine transform associated with each image
  - Maps from voxels ($x=1..n_x$, $y=1..n_y$, $z=1..n_z$) to some world co-ordinate system. e.g.,
    - Scanner co-ordinates - images from DICOM toolbox
    - T&T/MNI coordinates - spatially normalised
- Registering image B (source) to image A (target) will update B's vox-to-world mapping
  - Mapping from voxels in A to voxels in B is by
    - A-to-world using M_A, then world-to-B using M_B^{-1}
    - $M_B^{-1} M_A$

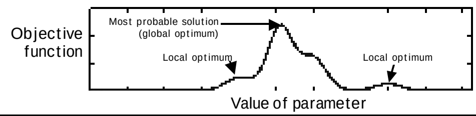
Left- and Right-handed Coordinate Systems

- Analyze™ files are stored in a left-handed system
- Talairach & Tournoux uses a right-handed system
- Mapping between them requires a flip
  - Affine transform with a negative determinant



Optimisation

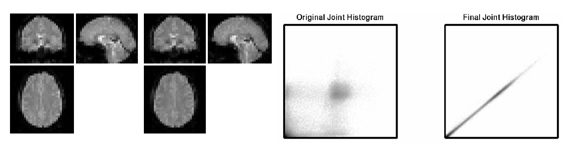
- Optimisation involves finding some "best" parameters according to an "objective function", which is either minimised or maximised
- The "objective function" is often related to a probability based on some model



Objective Functions for Image Registration

- Intra-modal
  - Mean squared difference (minimise)
  - Normalised cross correlation (maximise)
  - Entropy of difference (minimise)
- Inter-modal (or intra-modal)
  - Mutual information (maximise)
  - Normalised mutual information (maximise)
  - Entropy correlation coefficient (maximise)
  - AIR cost function (minimise)

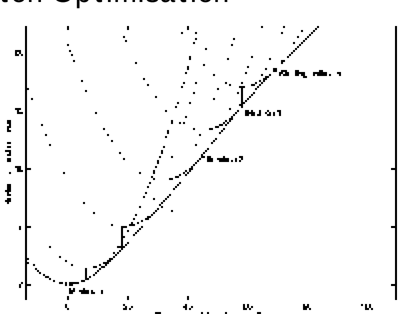
Mean-squared Difference



- Minimising mean-squared difference works for intra-modal registration (realignment)
- Simple relationship between intensities in one image, versus those in the other
  - Assumes normally distributed differences

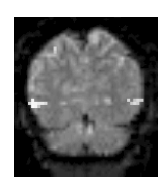
Gauss-newton Optimisation

- Works best for least-squares
- Minimum is estimated by fitting a quadratic at each iteration

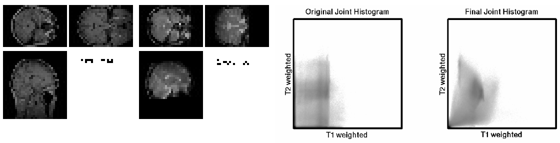


Inter-modal registration

- Match images from same subject but different modalities:
  - anatomical localisation of single subject activations
  - achieve more precise spatial normalisation of functional image using anatomical image.



Mutual Information



- Used for between-modality registration
- Derived from joint histograms
- $MI = \int_{ab} P(a,b) \log_2 [P(a,b)/(P(a) P(b))]$
  - Related to entropy: $MI = -H(a,b) + H(a) + H(b)$
    - Where $H(a) = -\int_a P(a) \log_2 P(a)$ and $H(a,b) = -\int_a P(a,b) \log_2 P(a,b)$

Image Transformations

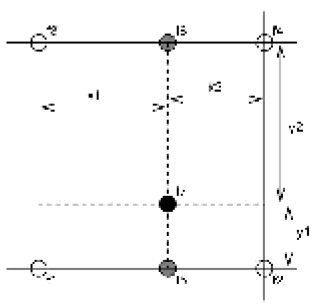
- Images are re-sampled. An example in 2D:

for y_0=1..n_{y0} % loop over rows
  for x_0=1..n_{x0} % loop over pixels in row
    $x_1 = t_x(x_0,y_0,$**q**$)$ % transform according to **q**
    $y_1 = t_y(x_0,y_0,$**q**$)$
    if 1£x_1£ n_{x1} & 1£y_1£n_{y1} then % voxel in range
      $f_1(x_0,y_0) = f_0(x_1,y_1)$ % assign re-sampled value
    end % voxel in range
  end % loop over pixels in row
end % loop over rows

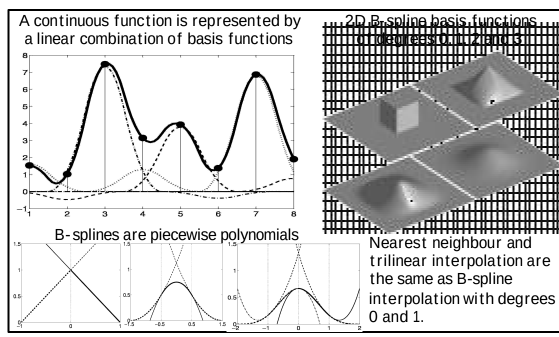
- What happens if x_1 and y_1 are not integers?

Simple Interpolation

- Nearest neighbour
  - Take the value of the closest voxel
- Tri-linear
  - Just a weighted average of the neighbouring voxels
  - $f_5 = f_1 x_2 + f_2 x_1$
  - $f_6 = f_3 x_2 + f_4 x_1$
  - $f_7 = f_5 y_2 + f_6 y_1$



B-spline Interpolation

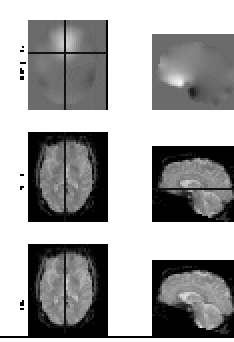


Residual Errors from aligned fMRI

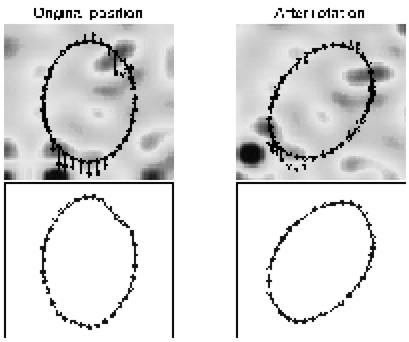
- Re-sampling can introduce interpolation errors
  - especially tri-linear interpolation
- Gaps between slices can cause aliasing artefacts
- Slices are not acquired simultaneously
  - rapid movements not accounted for by rigid body model
- Image artefacts may not move according to a rigid body model
  - image distortion
  - image dropout
  - Nyquist ghost
- Functions of the estimated motion parameters can be modelled as confounds in subsequent analyses

Movement by Distortion Interaction of fMRI

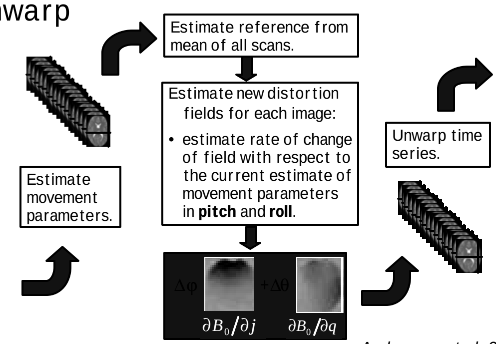
- Subject disrupts B_0 field, rendering it inhomogeneous
  - => distortions in phase-encode direction
- Subject moves during EPI time series
- Distortions vary with subject orientation
  - => shape varies



Movement by distortion interaction



Correcting for distortion changes using Unwarp



Andersson et al, 2001

Contents

- Smoothing
- Rigid registration
- Spatial normalisation
  - Affine registration
  - Nonlinear registration
  - Regularisation

Spatial Normalisation - Reasons

- Inter-subject averaging
  - Increase sensitivity with more subjects
    - Fixed-effects analysis
  - Extrapolate findings to the population as a whole
    - Mixed-effects analysis
- Standard coordinate system
  - e.g., Talairach & Tournoux space

Spatial Normalisation - Objective

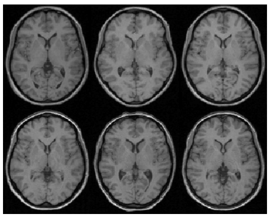
- Warp the images such that functionally homologous regions from different subjects are as close together as possible
  - Problems:
    - No exact match between structure and function
    - Different brains are organised differently
    - Computational problems (local minima, not enough information in the images, computationally expensive)
- Compromise by correcting gross differences followed by smoothing of normalised images

Very hard to define a one-to-one mapping of cortical folding

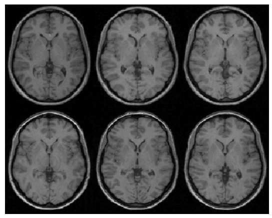
Use only approximate registration.

Spatial Normalisation - Procedure

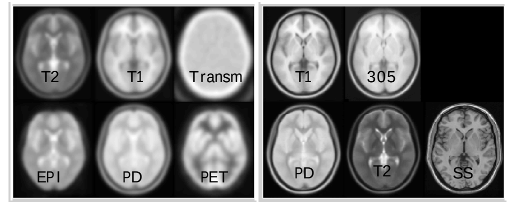
- Minimise mean squared difference from template image(s)



Affine registration

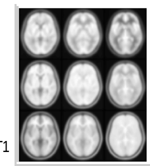


Non-linear registration

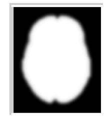


Template Images

"Canonical" images



A wider range of contrasts can be registered to a linear combination of template images.



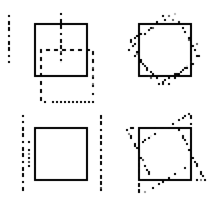
Spatial normalisation can be weighted so that non-brain voxels do not influence the result.

Similar weighting masks can be used for normalising lesioned brains.

Spatial Normalisation - Templates

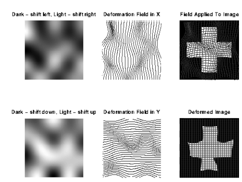
Spatial Normalisation - Affine

- The first part is a 12 parameter affine transform
  - 3 translations
  - 3 rotations
  - 3 zooms
  - 3 shears
- Fits overall shape and size



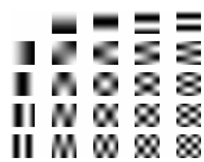
- Algorithm simultaneously minimises
  - Mean-squared difference between template and source image
  - Squared distance between parameters and their expected values (regularisation)

Spatial Normalisation - Non-linear



Deformations consist of a linear combination of smooth basis functions

These are the lowest frequencies of a 3D discrete cosine transform (DCT)

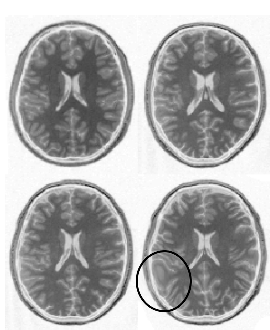


Algorithm simultaneously minimises
- Mean squared difference between template and source image
- Squared distance between parameters and their known expectation

---

Spatial Normalisation - Overfitting

Without regularisation, the non-linear spatial normalisation can introduce unnecessary warps.



References

Friston et al (1995): *Spatial registration and normalisation of images.* Human Brain Mapping 3:165-189

Collignon et al (1995): *Automated multi-modality image registration based on information theory.* IPMI'95 pp 263-274

Andersson et al (2001): *Modeling geometric deformations in EPI time series.* Neuroimage 13:903-919

Thévenaz et al (2000): *Interpolation revisited.* IEEE Trans. Med. Imaging 19:739-758.

Ashburner et al (1997): *Incorporating prior knowledge into image registration.* NeuroImage 6:344-352

Ashburner et al (1999): *Nonlinear spatial normalisation using basis functions.* Human Brain Mapping 7:254-266