The general linear model and Statistical Parametric Mapping
II: GLM for fMRI

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Overview

• Introduction

• General linear model(s) for fMRI
  – Time series
  – Haemodynamic response
  – Low frequency noise
  – Two GLMs fitted in 2-stage procedure

• Summary
Modelling with SPM

Preprocessed data: single voxel

Design matrix

General linear model

Parameter estimates

Contrasts

SPMs
GLM review

• Design matrix – the model
  – Effects of interest
  – Confounds (aka effects of no interest)
  – Residuals (error measures of the whole model)

• Estimate effects and error for data
  – Specific effects are quantified as contrasts of parameter estimates (aka betas)

• Statistic
  – Compare estimated effects – the contrasts – with appropriate error measures
  – Are the effects surprisingly large?
fMRI analysis

- Data can be filtered to remove low-frequency (1/f) noise
- Effects of interest are convolved with haemodynamic (BOLD) response function (HRF), to capture sluggish nature of response
- Scans must be treated as a timeseries, not as independent observations
  - i.e. typically temporally autocorrelated (for TRs<8s)
fMRI analysis

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Low frequency noise

- **Physical** (scanner drifts)
- **Physiological** (aliased)
  - cardiac (~1 Hz)
  - respiratory (~0.25 Hz)
fMRI example

One session

Passive word listening versus rest

7 cycles of rest and listening

Each epoch 6 scans with 7 sec TR

Question: Is there a change in the BOLD response between listening and rest?

Time series of BOLD responses in one voxel

Stimulus function
Regression model

\[ Y = X_1 \beta_1 + \hat{\beta}_2 + \hat{\epsilon}_1 \]
Add high pass filter

This means ‘taking out’ fluctuations below the specified frequency.

SPM implements by fitting low frequency fluctuations as effects of no interest.

Single subject

Frequency domain
128 second High-pass filter

relative spectral density

0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45

0 0.02 0.04 0.06

Frequency (Hz)
Fitted & adjusted data

Raw fMRI timeseries
Fitted & adjusted data

Raw fMRI timeseries

highpass filtered (and scaled)

fitted high-pass filter
Fitted & adjusted data

Raw fMRI timeseries

Adjusted data

highpass filtered (and scaled)

fitted high-pass filter

fitted box-car
Fitted & adjusted data

Raw fMRI timeseries

Adjusted data

highpass filtered (and scaled)

Residuals
Regression model

\[ Y = X_1 \ast \hat{\beta}_1 + \hat{\beta}_2 + \hat{\epsilon}_1 \]

(High-pass filter not visible)
Regression model

\[ Y = X_1 \beta \hat{\beta}_1 + \hat{\beta}_2 + \hat{\epsilon}_1 \]

What's wrong with this model?

1. Stimulus function is not expected BOLD response
2. Data is serially correlated
fMRI analysis

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Convolution with HRF

Boxcar function \( \ast \) hæmodynamic response = convolved with HRF
Convolution with HRF

- **Unconvolved fit**
- **Residuals**
- **Convolved fit**
- **Residuals (less structure)**

**Boxcar function convolved with HRF**

**Unconvolved fit**

**Convolved fit**

**Residuals**

**Residuals (less structure)**

- **Boxcar function**
- **Hæmodynamic response**
- **Convolved with HRF**
fMRI analysis

• Data can be filtered to remove low-frequency (1/f) noise

• Effects of interest are convolved with haemodynamic (BOLD) response function (HRF), to capture sluggish nature of response

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Temporal autocorrelation

• Because scans are not independent measures, the number of degrees of freedom is less than the number of scans
• This means that under the null hypothesis the data are less free to vary than might be assumed
• A given statistic, e.g. T value, is therefore less surprising and so less significant than we think…

...the next talk
2-stage GLM

Each has an independently acquired set of data
These are modelled separately
Models account for **within subjects variability**
Parameter estimates apply to individual subjects

**Single subject**

Single subject **contrasts of parameter estimates** taken forward to 2\textsuperscript{nd} level as (spm\_con*.img) **‘con images’**

‘Summary statistic’
random effects method

1\textsuperscript{st} level
2-stage GLM

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Models account for **within subjects variability**
Parameter estimates apply to individual subjects

**Single subject**
- **contrasts of parameter estimates** taken forward to 2nd level as (spm_con*.img) ‘con images’

**Group/s of subjects**
- **b**etav**ar**iability
- **b**etas measure each subject’s effects
- **b**etas measure group effect/s

**1st level**

To make an inference that generalises to the population, must also model the **between subjects variability**

**2nd level**

Statistics compare **contrasts of 2nd level parameter estimates** to 2nd level error

‘Summary statistic’ random effects method
Single subject design matrix

\[ Y = X_1 \hat{\beta}_1 + \hat{\epsilon}_1 \]
Group level design matrix

\[
\hat{\beta}_1 = X_2 \hat{\beta}_2 + \hat{\epsilon}_2
\]

Number of effects in model

Group analysis
Summary

• For fMRI studies the GLM specifically needs to take account of
  – Low frequency noise
  – The sluggish haemodynamic response
  – The temporally autocorrelated nature of the timeseries of scans

• A computationally efficient 2-stage GLM is used
  – Continued in next talk