Dynamic Causal Modelling for evoked responses

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Overview

1 DCM: introduction

2 Neural ensembles dynamics

3 Bayesian inference

4 Conclusion
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DCM: introduction

*structural, functional and effective* connectivity

- **Structural connectivity**
  = presence of axonal connections

- **Functional connectivity**
  = statistical dependencies between regional time series

- **Effective connectivity**
  = causal (directed) influences between neuronal populations
DCM: introduction
connections are recruited in a *context-dependent* fashion

- meta-analysis on single-word reading (Turkeltaub, 2002)
Introduction

DCM for evoked responses: auditory mismatch negativity

S-D: reorganisation of the connectivity structure

sequence of auditory stimuli

standard condition (S)

deviant condition (D)

t ~ 200 ms
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Neural ensembles dynamics
systems of neural populations

macro-scale

meso-scale

micro-scale

Golgi
Nissl

external granular layer
external pyramidal layer
internal granular layer
internal pyramidal layer

mean-field firing rate
synaptic dynamics

mean-field firing rate

membrane potential (mV)

time (s)

firing rate (Hz)

membrane potential (mV)
Neural ensembles dynamics
from micro- to meso-scale: mean-field treatment

\[ x_j : \text{post-synaptic potential of } j^{th} \text{ neuron within its ensemble} \]

\[ \frac{1}{N-1} \sum_{j \neq j'} H(x_j, -\theta) \xrightarrow[N \to \infty]{} \int H(x - \theta) p(x) dx \]

\[ = \int_{\theta}^{\infty} p(x) dx \approx S(\mu) \]

mean firing rate

\( \sigma \)

\( \mu \)

\( \theta \)

ensemble density \( p(x) \)

mean membrane depolarization (mV)

\( \sigma \)

\( \mu \)

\( \theta \)

\( S(x) \)

mean firing rate (Hz)

membrane depolarization (mV)
Neural ensembles dynamics

synaptic dynamics

\[ \mu(t) = S(u(t)) \otimes \text{kernel}_{PSP}(t) \]

\[ \begin{aligned} \dot{\mu}_1 &= \mu_2 \\ \dot{\mu}_2 &= \kappa_{i/e}^2 S(u) - 2\kappa_{i/e} \mu_2 - \kappa_{i/e}^2 \mu_1 \end{aligned} \]
Neural ensembles dynamics

intrinsc connections within the cortical column

\[ \dot{\mu}_1 = \mu_8 \]
\[ \dot{\mu}_8 = \gamma_3 \kappa \mu (\mu_0) - 2\kappa \mu_8 - \kappa \mu_1 \]

\[ \gamma_4 \]
\[ \dot{\mu}_4 = \gamma_4 \kappa \mu (\mu_0) - 2\kappa \mu_4 - \kappa \mu_1 \]

\[ \gamma_2 \]
\[ \dot{\mu}_0 = \mu_5 - \mu_6 \]
\[ \dot{\mu}_2 = \mu_5 \]
\[ \dot{\mu}_5 = \gamma_2 \kappa \mu (\mu_0) - 2\kappa \mu_5 - \kappa \mu_2 \]
\[ \dot{\mu}_3 = x_6 \]
\[ \dot{\mu}_6 = \gamma_4 \kappa \mu (\mu_0) - 2\kappa \mu_6 - \kappa \mu_3 \]
Neural ensembles dynamics
from meso- to macro-scale: neural fields

local wave propagation equation:

\[
\left( \frac{\partial^2}{\partial t^2} + 2\kappa \frac{\partial}{\partial t} + \kappa^2 - \frac{3}{2} c^2 \nabla^2 \right) \mu^{(i)}(\mathbf{r}, t) \approx c\kappa \zeta^{(i)}(\mathbf{r}, t)
\]

\[
\zeta^{(i)} = \sum_{i'} \gamma_{ii'} S\left( \mu^{(i')} \right)
\]
Neural ensembles dynamics
extrinsic connections between brain regions

\[
\dot{\mu}_7 = \mu_8 \\
\mu_8 = \kappa_7^2 ((\gamma_B + \gamma_L + \gamma_3 I)S(\mu_0)) - 2\kappa_7 \mu_8 - \kappa_7^2 \mu_7
\]

\[
\gamma_4 \
\mu_1 = \mu_4 \\
\mu_4 = \kappa_4^2 ((\gamma_L + \gamma_I)S(\mu_0) + \gamma_d \mu) - 2\kappa_4 \mu_4 - \kappa_4^2 \mu_1
\]

\[
\gamma_2 \
\mu_0 = \mu_5 - \mu_6 \\
\dot{\mu}_2 = \mu_5 \\
\mu_5 = \kappa_5^2 ((\gamma_B + \gamma_L)S(\mu_0) + \gamma_2 S(\mu_1)) - 2\kappa_5 \mu_5 - \kappa_5^2 \mu_2 \\
\dot{\mu}_3 = \mu_6 \\
\dot{\mu}_6 = \kappa_6^2 \gamma_4 S(\mu_7) - 2\kappa_6 \mu_6 - \kappa_6^2 \mu_3
\]

extrinsic forward connections
extrinsic lateral connections
extrinsic backward connections
Neural ensembles dynamics

systems of neural populations

macro-scale

meso-scale

Golgi

Nissl

external granular layer

eexternal pyramidal layer

internal granular layer

internal pyramidal layer

micro-scale

main DCM evolution parameters:
- action potential firing threshold + ensemble PSP spread
- synaptic time constants + axonal propagation delays
- effective coupling strengths + modulatory effects
Neural ensembles dynamics

the observation mapping

main DCM observation parameters:
- sources location/orientation (ECD) or spatial profile (distributed responses)
- relative contribution of cortical layers to measured signal
Neural ensembles dynamics

*a note on causality*

\[
\dot{x} = f(x, u, \theta)
\]
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Bayesian inference

**forward and inverse problems**

forward problem

\[ p(y|\mathcal{G},m) \]

likelihood

posterior distribution

\[ p(\mathcal{G}|y,m) \]

inverse problem
Bayesian inference
deriving the likelihood function

- Model of data with unknown parameters:

\[ y = \tilde{g}(\theta) \quad \text{e.g., GLM:} \quad \tilde{g}(\theta) = X\theta \]

- But data is noisy:

\[ y = \tilde{g}(\theta) + \varepsilon \]

- Assume noise/residuals is ‘small’:

\[ p(\varepsilon) \propto \exp\left(-\frac{1}{2\sigma^2}\varepsilon^2\right) \]

\[ P(|\varepsilon| > 4\sigma) \approx 0.05 \]

→ Distribution of data, given fixed parameters:

\[ p(y|\theta) \propto \exp\left(-\frac{1}{2\sigma^2} (y - \tilde{g}(\theta))^2\right) \]
Bayesian inference

likelihood and priors

\[ p(y|\mathcal{G}, m) \]
\[ p(\mathcal{G}|m) \]

posterior

\[ p(\mathcal{G}|y, m) = \frac{p(y|\mathcal{G}, m) p(\mathcal{G}|m)}{p(y|m)} \]
Bayesian inference

zooming in the VB algorithm

measured data → specify generative forward model (with prior distributions of parameters)
 Variational Bayesian (VB) algorithm

iterative procedure:
1. compute model response using current set of parameters
2. compare model response with data
3. improve parameters, if possible

1. posterior distributions of parameters
2. model evidence (free energy)
Frequentist versus Bayesian inference

*testing point hypotheses*

- define the null and the alternative hypothesis *in terms of priors*, e.g.:

\[
H_0 : p(\theta|H_0) = \begin{cases} 1 & \text{if } \theta = 0 \\ 0 & \text{otherwise} \end{cases}
\]

\[
H_1 : p(\theta|H_1) = N(0, \Sigma)
\]

- apply decision rule, i.e.: if \[ \frac{P(H_0|y)}{P(H_1|y)} \leq 1 \] then reject H0

- **Savage-Dickey ratios** (nested models, i.i.d. priors):

\[
p(y|H_0) = p(y|H_1) \frac{p(\theta = 0|y, H_1)}{p(\theta = 0|H_1)}
\]
Bayesian inference
model comparison for group studies

\[
\ln p(y|m_1) - \ln p(y|m_2)
\]

- **fixed effect**: assume all subjects correspond to the same model
- **random effect**: assume different subjects might correspond to different models
Bayesian inference
key DCM parameters

\( (\theta_{21}, \theta_{32}, \theta_{13}) \)

state-state coupling

\( \theta_{13}^u \)

input-state coupling

\( \theta_{13}^u \)

input-dependent modulatory effect
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Conclusion

back to the auditory mismatch negativity

sequence of auditory stimuli

... S S S S D S S S S D S S ...

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S-D: reorganisation of the connectivity structure
Conclusion

**DCM for EEG/MEG: variants**

- second-order mean-field DCM

- DCM for steady-state responses

- DCM for induced responses

- DCM for phase coupling
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Bayesian inference
the variational Bayesian approach

\[ \ln p( y|m) = \left\langle \ln p(\mathcal{G}, y|m) \right\rangle_q + S(q) + D_{KL}(q(\mathcal{G}); p(\mathcal{G}|y,m)) \]

free energy : functional of \( q \)

approximate (marginal) posterior distributions: \( \{q(\mathcal{G}_1), q(\mathcal{G}_2)\} \)
Bayesian inference

*model comparison*

**Principle of parsimony**: « plurality should not be assumed without necessity »

Model evidence:

$$p(y|m) = \int p(y|\mathcal{G},m) p(\mathcal{G}|m) d\mathcal{G}$$

“Occam’s razor”: