Model-based fMRI

Zurich SPM Course
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To model or not to model

GLM

DCM

Classification

Multivariate Bayes
fMRI uses models at different stages

- Hemodynamic response (hrf)
- Activation levels
- Time courses
- Connectivity
- t-tests
Model-based fMRI

- Applying quantitative computational models to generate regressors of interest beyond stimulus inputs and behavioral responses
What is a good model?

The computer engineer, who, when asked to describe how he would write a computer program to recognize a cow, replied, “first, assume a spherical cow.”...
A good model (1)

- A clearly specified object of modeling
  cow
  BOLD response
  expected value of a certain action

- A clearly specified purpose
  recognize cows
  analyze fMRI data, inferences about neural processes
  model ventromedial frontal projections from the midbrain

- Tractable
  computationally efficient
A good model (2)

- Realistic
  hrf better than spherical cow
  e.g., incorporate knowledge about brain anatomy and neuronal responses

- Simple
  spherical cow better than hrf

- BUT Occam’s razor: as simple as possible, as flexible as needed
  hrf better than spherical cow
As simple as possible, as flexible as needed
Model-based fMRI

- Applying quantitative computational models to generate regressors of interest beyond stimulus inputs and behavioral responses

- Goal: uncover hidden variables or processes
Model-based fMRI: questions answered

- How (i.e., by activation of which areas) does the brain implement a particular cognitive process?

![Participant response diagram]

Participant response
Classic designs vs. model-based designs

- Classic event and block related designs
  - Conditions are predefined by the experimental design or given by the participant’s response and are limited to discrete values.

- Parametric designs
  - Continuous spectrum of levels and responses; leaves more degrees of freedom.

- Model-based
  - Access hidden variables and cognitive processes.
Outline

1. Basic recipe for model-based fMRI
2. Using model-based regressors in the GLM
3. Examples
Model-based fMRI

1. Decide on a model
   - This should happen *before* you run the experiment.
   - Start with a research question and choose a model that adequately addresses this question.
   - Design your experiment with this model in mind.
   - E.g., reinforcement learning model, hierarchical bayesian model.
Model-based fMRI

1. Decide on a model
   - Reinforcement learning model

Participant response
Model-based fMRI

2. Pass individual subject trial history to model

\[ \delta = R_{t+1} + \gamma V_{t+1} - V_t \]

\[ V^A_{t+1} = V^A_t + \alpha \delta \]
Model-based fMRI

3. Find best-fitting parameters of the model (e.g., learning rate) to behavioral data

4. Generate

   a. parametric modulators (first level)
   b. model-based time series (first level)
   c. subject-specific parameters (e.g., second level, DCM)
Model-based fMRI

5. Convolve time series with hemodynamic response function

Adapted from O’Doherty et al., (2007)
Model-based fMRI

6. Regress against fMRI data

Hampton et al., (2006)
Model-based fMRI

1. Decide on a model

2. Pass individual subject trial history to model

3. Find best-fitting parameters of model to behavioral data

4. Generate parametric modulators & model-based time series

5. Convolve time series with hemodynamic response function

6. Regress against fMRI data
From classic design to model based fMRI

1. Classic event/block design

2. Adding parametric regressors

3. Model-based design
Model-based fMRI: comparisons

- Classical event/block design
Model-based fMRI: comparisons

- Classical event/block design
Model-based fMRI: comparisons

- Parametric regressors
Model-based fMRI: comparisons

- Parametric regressors
Model-based fMRI: comparisons

- Parametric regressors
How do we construct regressors that correspond to cognitive processes and use them in SPM?

4. Generate

   a. parametric modulators (first level)
   b. model-based time series (first level)
   c. subject-specific parameters (e.g., second level, DCM)
How do we construct regressors from a time series and use them in SPM?

Sample time-series at points of interest (e.g., participant response)

Enter as parametric modulation for condition ‘participant response’
How do we construct regressors from a time series and use them in SPM?

Convolve time series with hemodynamic response function

Adapted from O’Doherty et al., (2007)
How do we construct regressors from a time series and use them in SPM?

Convolve time series with hemodynamic response function

- sample time series at the same rate as the basis functions

- convolve with the basis functions
  \texttt{SPM.xBF.bf}

- sample at TR (i.e., one sample per functional volume)

- add to design matrix as (multiple) regressor(s)
How do we construct regressors from a time series and use them in SPM?

Convolve time series with hemodynamic response function

- add to design matrix as (multiple) regressor(s)
How do we include individual model parameters?

e.g., enter as covariates at the second level
Model-based fMRI recipe

1. Decide on a model (*before* finishing your experimental design)

2. Pass individual subject trial history to model

3. Find best-fitting parameters of model to behavioral data

4. Generate parametric modulators & model-based time series

5. Convolve time series with hemodynamic response function

6. Regress against fMRI data
Design efficiency

- Regressors and design matrix not fully specified before data collection.

- To estimate design efficiency:
  - Simulate behavioral data, conduct behavioral pilot study
  - Obtain simulated/pilot time course from the model
  - Optimize design efficiency
Model-based fMRI: design efficiency

- Model based fMRI
Model-based fMRI: comparisons

- Model based fMRI
Model-based fMRI recipe

1. Decide on a model
2. Pass individual subject trial history to model
3. Find best-fitting parameters of model to behavioral data
4. Generate model-based time series
5. Convolve time series with hemodynamic response function
6. Regress against fMRI data
Model-based fMRI – an example

- The task
- The decision model
- The learning model
- Combined inversion
- fMRI results
The task – single trial
The task – probabilistic structure

Probability of Face, given high tone = Probability of House, given low tone
The decision model

- Softmax decision rule
- Curve shape is determined by the parameter $\zeta$
- Translates beliefs into decision probabilities

Probability of decision “1”, (i.e., of betting on “1”)

Prediction (“certainty”) that next stimulus is “1”
The learning model

<table>
<thead>
<tr>
<th>State of the world</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-volatility $x_3$ of tendency</td>
<td>Gaussian random walk with constant step size $\vartheta$</td>
</tr>
<tr>
<td>$p(x_3^{(k)}) \sim N(x_3^{(k-1)}, \vartheta)$</td>
<td></td>
</tr>
<tr>
<td>Tendency $x_2$ towards category “1”</td>
<td>Gaussian random walk with step size $\exp(\kappa x_3 + \omega)$</td>
</tr>
<tr>
<td>$p(x_2^{(k)}) \sim N(x_2^{(k-1)}, \exp(\kappa x_3 + \omega))$</td>
<td></td>
</tr>
<tr>
<td>Stimulus category $x_1$ (“0” or “1”)</td>
<td>Sigmoid transformation of $x_2$</td>
</tr>
<tr>
<td>$p(x_1 = 1) = s(x_2)$, $p(x_1 = 0) = 1 - s(x_2)$</td>
<td></td>
</tr>
</tbody>
</table>

Mathys et al. (2011)
Learning and decision models combined

\[ x_3 \leq \theta \leq x_2 \leq \kappa, \omega \]

\[ p(y = 1) = s(\mu_2^{(k-1)}) \]
Notation

\[ \zeta \rightarrow y \quad \text{def} = \quad y(k) \rightarrow y(k-1) \rightarrow y(k+1) \]

\[ k = 1, \ldots, n \]

\[ \zeta \rightarrow y \quad \text{def} = \quad y(k) \rightarrow y(k) \]

\[ k = 1, \ldots, n \]

\[ x \rightarrow u \quad \text{def} = \quad x(k) \rightarrow x(k-1) \rightarrow x(k+1) \]

\[ k = 1, \ldots, n \]
Model inversion
Regressor: Uncertainty
The update equations

<table>
<thead>
<tr>
<th>Level 3</th>
<th>$\Delta \mu_3 = \sigma_3 \cdot \frac{\kappa}{2} \cdot w_2 \cdot \delta_2$ with $\Delta \mu_3 = \mu_3^{(k)} - \mu_3^{(k-1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_3 = \sigma_3^{(k)}$</td>
</tr>
<tr>
<td></td>
<td>$w_2 = \frac{e^{\kappa \mu_3^{(k-1)} + \omega}}{\sigma_2^{(k-1)} + e^{\kappa \mu_3^{(k-1)} + \omega}}$</td>
</tr>
<tr>
<td></td>
<td>$\delta_2 = \frac{\sigma_2^{(k)} + (\mu_2^{(k)} - \mu_2^{(k-1)})^2}{\sigma_2^{(k-1)} + e^{\kappa \mu_3^{(k-1)} + \omega}} - 1$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2</th>
<th>$\Delta \mu_2 = \sigma_2 \cdot \delta_1$ with $\Delta \mu_2 = \mu_2^{(k)} - \mu_2^{(k-1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_2 = \sigma_2^{(k)}$</td>
</tr>
<tr>
<td></td>
<td>$\delta_1 = \mu_1^{(k)} - s(\mu_2^{(k-1)})$</td>
</tr>
</tbody>
</table>

Expectation update
(Unweighted) learning rate
Weighting factor
Prediction error
Regressor: Uncertainty-weighted prediction error
Summary

- Model-based fMRI:
  - Application of quantitative computational models to generate regressors of interest beyond stimulus inputs and behavioral responses.
  - Serves to uncover hidden variables and cognitive processes
- A model may be realistic but it is never correct.
- In most cases, hrf beats