Principles of Dynamic Causal Modelling for EEG/MEG

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Overview

1 DCM: introduction

2 Differential equations

3 Neural states dynamics

4 Bayesian inference

5 Conclusion
Introduction

structural, functional and effective connectivity

- **structural connectivity**
  = presence of axonal connections

- **functional connectivity**
  = statistical dependencies between regional time series

- **effective connectivity**
  = causal (directed) influences between neuronal populations

"connections are recruited in a context-dependent fashion"
Does network XYZ explain my data better than network XY?
Does network XYZ explain my data better than network XY?

Which XYZ connectivity structure best explains my data?
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Which XYZ connectivity structure best explains my data?

Are X & Y linked in a bottom-up, top-down or recurrent fashion?
Does network XYZ explain my data better than network XY?

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Are X & Y linked in a bottom-up, top-down or recurrent fashion?

Is my effect driven by extrinsic or intrinsic connections?
Does network XYZ explain my data better than network XY?
Which XYZ connectivity structure best explains my data?
Are X & Y linked in a bottom-up, top-down or recurrent fashion?
Is my effect driven by extrinsic or intrinsic connections?
Which neural populations are affected by contextual factors?
Does network XYZ explain my data better than network XY?
Which XYZ connectivity structure best explains my data?
Are X & Y linked in a bottom-up, top-down or recurrent fashion?
Is my effect driven by extrinsic or intrinsic connections?
Which neural populations are affected by contextual factors?
Which connections determine observed frequency coupling?
Does network XYZ explain my data better than network XY?

Which XYZ connectivity structure best explains my data?

Are X & Y linked in a bottom-up, top-down or recurrent fashion?

Is my effect driven by extrinsic or intrinsic connections?

Which neural populations are affected by contextual factors?

Which connections determine observed frequency coupling?

How changing a connection/parameter would influence data?
DCM for EEG/MEG

Physiological

Neurophysiological model

• DCM for event-related potentials
• DCM for cross-spectral density

Phenomenological

Models a particular data feature

• DCM for Induced Responses
• DCM for Phase Coupling

Electromagnetic forward model included
States $x$ different from data $y$

Source locations not optimized
States $x$ and data $y$ in the same “format”
Evolution and observation mappings

Hemodynamic observation model: temporal convolution

Electromagnetic observation model: spatial convolution

\[ \dot{x} = f(x, u, \theta) \]

- Simple neuronal model
- Realistic observation model

- Realistic neuronal model
- Simple observation model

fMRI

EEG/MEG

Inputs
Forward models and their inversion

Forward model (measurement)
\[ y = g(x, \theta) + \varepsilon \]

Observed data

Model inversion
\[ p(x, \theta | y, u, m) \]

Forward model (neuronal)
\[ \dot{x} = f(x, u, \theta) + \omega \]

Input \( u(t) \)
Model specification and inversion

**Neural dynamics**

\[ u(t) \]

\[ \dot{x} = f(x, u, \theta) \]

**Observer function**

\[ y = g(x, \theta) + \varepsilon \]

**Define likelihood model**

\[ p(y \mid \theta, m) = \mathcal{N}(g(\theta), \Sigma(\theta)) \]

\[ p(\theta, m) = \mathcal{N}(\mu_\theta, \Sigma_\theta) \]

**Specify priors**

**Invert model**

\[ p(y \mid m) = \int p(y \mid \theta, m) p(\theta) d\theta \]

**Inference on parameters**

\[ p(\theta \mid y, m) = \frac{p(y \mid \theta, m) p(\theta, m)}{p(y \mid m)} \]

**Design experimental inputs**

**Inference**
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\[ \dot{x} = -k \cdot x \]

**Analytic solution**

\[ x(t) = x_0 e^{-kt} \]

**Numerical solution**

\[ x(0) = x_0 \]
\[ x(0 + \Delta t) = x(0) - k \cdot x(0) \cdot \Delta t \]
\[ x(0 + 2\Delta t) = x(0 + \Delta t) - k \cdot x(0 + \Delta t) \cdot \Delta t \]
\[ \cdots \]
‘Neural’ equation (exponential decay)

\[ \dot{x} = -k \cdot x + U \]

Observation equation

\[ y = G \cdot x \]

\[ U = a \cdot e^\frac{(t-t_0)^2}{\Delta t^2} \]

\[ \int \dot{x} \, dt \]

Numerical integration

\[ U \]

\[ a \]

\[ t_0 \]

\[ x \]
Even simpler \[ U \times x \times \frac{-1}{2} = 2.47 \]

**Neural** equation (exponential decay)

\[ \dot{x} = -2.47 \cdot x + U \]

\[ U = 750 \cdot e^{-\frac{(t-t_0)^2}{\Delta t^2}} \]

**Observation equation**

\[ y = 1 \cdot x \]

\[ \int \dot{x} dt \]

\[ t_0 - \text{the only free parameter} \]

\[ U \]

\[ x \]
Optimization scheme for fitting the parameters to the data

- The objective function for optimization is the free energy which approximates the (log) model evidence:

\[ p(y|m) = \int p(y|\mathcal{G},m) p(\mathcal{G}|m) \, d\mathcal{G} \]

- There are many possible schemes based on different assumptions. Present DCM implementations in SPM use variational Bayesian scheme.

- Once the scheme converges it yields
  - The highest value of free energy the scheme could attain
  - Posterior distribution of the free parameters
  - Simulated data as similar to the original data as the model could generate
Neural ensembles dynamics

DCM for M/EEG: extrinsic connections between brain regions

\[ \dot{x} = -k \cdot x \]
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Neural ensembles dynamics

DCM for M/EEG: systems of neural populations

macro-scale

meso-scale

micro-scale

mean-field firing rate

synaptic dynamics

Golgi

Nissl

external granular layer

external pyramidal layer

internal granular layer

internal pyramidal layer
Neural ensembles dynamics

DCM for M/EEG: from micro- to meso-scale

\[ x_j(t) : \text{post-synaptic potential of } j^{th} \text{ neuron within its ensemble} \]

\[
\frac{1}{N-1} \sum_{j \neq j} H(x_j(t) - \theta) \xrightarrow{N \to \infty} \int H(x(t) - \theta) p(x(t)) \, dx
\]

\approx S(\mu) \text{ mean-field firing rate}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{neural_ensembles_dynamics.png}
\end{figure}
Neural ensembles dynamics
DCM for M/EEG: synaptic dynamics

\begin{align*}
\dot{\mu}_1 &= \mu_2 \\
\dot{\mu}_2 &= \kappa_{ile}^2 S(\mu_0) - 2\kappa_{ile} \mu_2 - \kappa_{ile}^2 \mu_1
\end{align*}

post-synaptic potential

[Graph showing membrane depolarization over time with EPSP and IPSP]
Neural ensembles dynamics
DCM for M/EEG: intrinsic connections within the cortical column

\[ \begin{align*}
\dot{\mu}_1 &= \mu_6 \\
\dot{\mu}_2 &= \mu_5 \\
\dot{\mu}_3 &= \gamma_3 \kappa_3^2 S(\mu_0) - 2\kappa_2 \mu_5 - \kappa_1^2 \mu_7 \\
\dot{\mu}_4 &= \gamma_4 \kappa_4^2 S(\mu_6) - 2\kappa_3 \mu_6 - \kappa_2^2 \mu_1 \\
\dot{\mu}_5 &= \gamma_5 \kappa_5^2 S(\mu_5) - 2\kappa_4 \mu_4 - \kappa_3^2 \mu_2 \\
\dot{\mu}_6 &= \gamma_6 \kappa_6^2 S(\mu_7) - 2\kappa_5 \mu_7 - \kappa_4^2 \mu_3 \\
\dot{\mu}_7 &= \gamma_7 \kappa_7^2 S(\mu_1) - 2\kappa_6 \mu_1 - \kappa_5^2 \mu_0 \\
\end{align*} \]
Neural ensembles dynamics
DCM for M/EEG: extrinsic connections between brain regions

\[ \begin{align*}
\mu_s &= \mu_s \\
\mu_b &= \kappa_4^2 (\gamma_B + \gamma_L + \gamma_I) S(\mu_b) - 2\kappa_1 \mu_b - \kappa_2^2 \mu_s
\end{align*} \]

\[ \begin{align*}
\mu_s &= \mu_s \\
\mu_b &= \kappa_4^2 (\gamma_B + \gamma_L + \gamma_I) S(\mu_b) + \gamma_B S(\mu_s) - 2\kappa_1 \mu_b - \kappa_2^2 \mu_s
\end{align*} \]

\[ \begin{align*}
\mu_b &= \mu_b - \mu_b \\
\mu_s &= \mu_s \\
\mu_b &= \kappa_4^2 (\gamma_B + \gamma_L) S(\mu_b) + \gamma_2 S(\mu_s) - 2\kappa_1 \mu_b - \kappa_2^2 \mu_s
\end{align*} \]

\[ \gamma_L S(\mu_0) \]

extrinsic lateral connections

\[ \gamma_B S(\mu_0) \]

extrinsic backward connections

\[ \gamma_F S(\mu_0) \]

extrinsic forward connections
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Bayesian inference

*forward and inverse problems*

\[ p(y|\mathcal{G}, m) \]

\[ p(\mathcal{G}|y, m) \]

- **forward problem**
- **likelihood**
- **posterior distribution**
- **inverse problem**
Bayesian inference

the electromagnetic forward problem

\[ y(t) = \sum_i L^{(i)} w_0^{(i)} \sum_j \beta_j \mu^{(ij)}(t) + \varepsilon(t) \]
Bayesian paradigm  
deriving the likelihood function

- Model of data with unknown parameters:
  \[ y = f(\theta) \quad \text{e.g., GLM: } f(\theta) = X\theta \]

- But data is noisy:
  \[ y = f(\theta) + \varepsilon \]

- Assume noise/residuals is ‘small’:
  \[
p(\varepsilon) \propto \exp\left(-\frac{1}{2\sigma^2} \varepsilon^2\right)
\]

\[ P(|\varepsilon| > 4\sigma) \approx 0.05 \]

\[
\rightarrow \text{Distribution of data, given fixed parameters:}
\]

\[
p(y|\theta) \propto \exp\left(-\frac{1}{2\sigma^2}(y - f(\theta))^2\right)
\]
Bayesian paradigm

likelihood, priors and the model evidence

Likelihood: \( p(y|\theta, m) \)

Prior: \( p(\theta|m) \)

Bayes rule: \( p(\theta|y, m) = \frac{p(y|\theta, m) p(\theta|m)}{p(y|m)} \)
Bayesian inference

*model comparison*

**Principle of parsimony:**
« plurality should not be assumed without necessity »

Model evidence:

$$p(y|m) = \int p(y|\mathcal{G},m) p(\mathcal{G}|m) \, d\mathcal{G}$$

“Occam’s razor”:
Bayesian inference
the variational Bayesian approach

$$\ln p(y|m) = \langle \ln p(\mathcal{G}, y|m) \rangle_q + S(q) + D_{KL}(q(\mathcal{G}); p(\mathcal{G}|y,m))$$

free energy : functional of $q$

mean-field: approximate marginal posterior distributions: $\{q(\mathcal{G}_1), q(\mathcal{G}_2)\}$
Bayesian inference

DCM: key model parameters

\[(\theta_{21}, \theta_{32}, \theta_{13})\] state-state coupling

\[\theta_{3}^{u}\] input-state coupling

\[\theta_{13}^{u}\] input-dependent modulatory effect
Bayesian inference
model comparison for group studies

\[ \ln p(y|m_1) - \ln p(y|m_2) \]

fixed effect
assume all subjects correspond to the same model

random effect
assume different subjects might correspond to different models
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Conclusions

The main principle of DCM is the use of data and generative models in a Bayesian framework to infer parameters and compare models.

Implementation details may vary – e.g. variational Bayes vs. sampling methods.

Model inversion is an optimization procedure where the objective function is the free energy which approximates the model evidence.

Model evidence is the goodness of fit expected under the prior parameter values.

The best model is the one with precise priors that yield good fit to the data.

Different models can be compared as long as they were fitted to the same data.

Models and priors can be gradually refined from one study to the next, making it possible to use DCM as an integrative framework in neuroscience.
DCM for EEG/MEG: variants

- mean-field DCM for evoked responses
- second-order mean-field DCM

- DCM for steady-state responses

- DCM for induced responses

- DCM for phase coupling
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