# Stochastic Dynamic Causal Modelling

**J. Daunizeau**

*Wellcome Trust Centre for Neuroimaging, London, UK*
*UZH – Foundations of Human Social Behaviour, Zurich, Switzerland*

# Overview

1 Dynamic Causal Modelling (DCM)

2 System’s states dynamics

3 Stochastic DCM: a variational Bayesian approach

4 VB evaluation

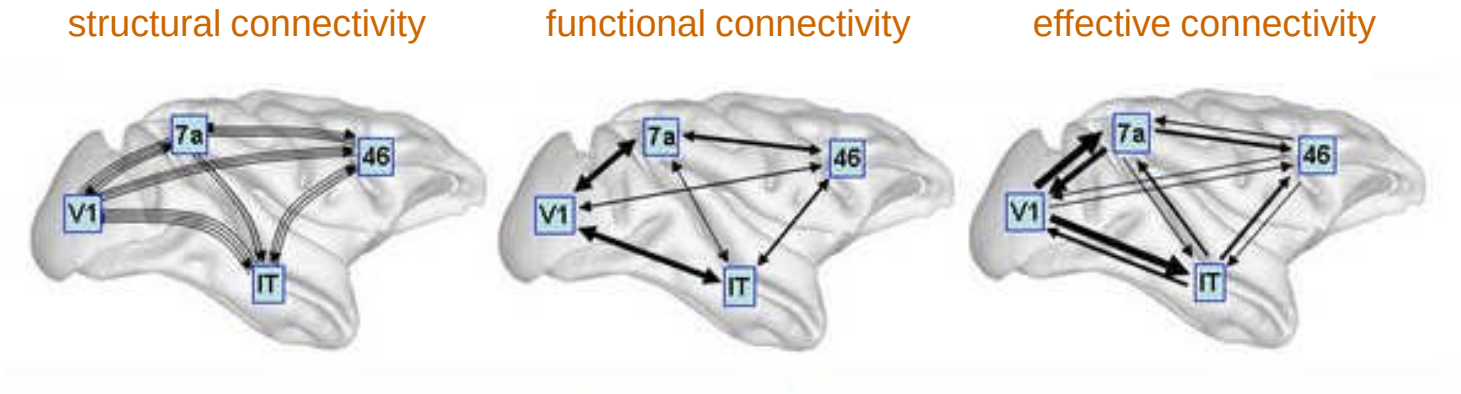
5 Stochastic DCM: example

6 Conclusion

# Overview

# Dynamic Causal Modelling

## *structural*, *functional* and *effective* connectivity




O. Sporns 2007, *Scholarpedia*

- ***structural*** **connectivity**
  = presence of axonal connections

- ***functional*** **connectivity**
  = statistical dependencies between regional time series

- ***effective*** **connectivity**
  = causal (directed) influences between neuronal populations

! connections are recruited in a *context-dependent* fashion

# Dynamic Causal Modelling

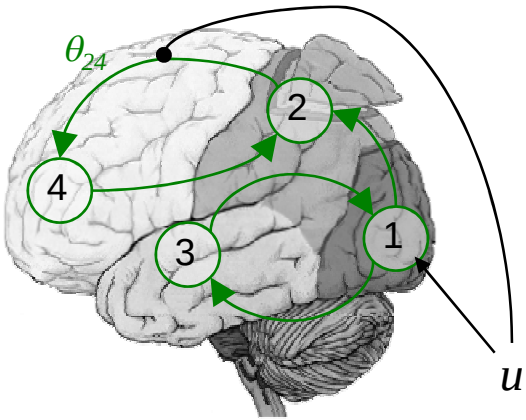
*parametric statistical approach*

- DCM: model structure

$$\begin{cases} y = g(x,\varphi) + \varepsilon \\ \dot{x} = f(x,u,\theta) \end{cases}$$

likelihood

$$\Rightarrow p(y|\theta,\varphi,m)$$




- DCM: Bayesian inference

priors on parameters

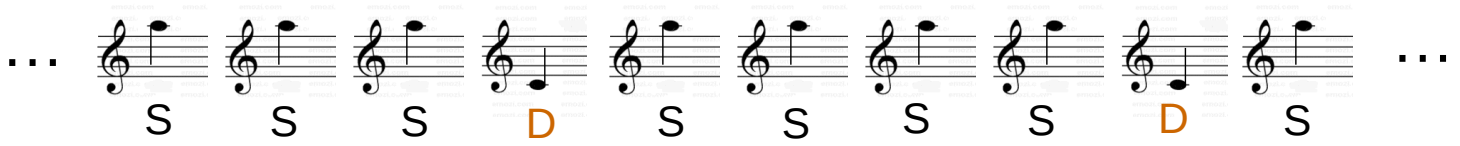
marginal posterior density: $$p(\theta|y,m) \propto \int p(y|\theta,\varphi,m)\, p(\theta|m)\, p(\varphi|m)\, d\varphi$$

model evidence: $$p(y|m) = \int p(y|\theta,\varphi,m)\, p(\theta|m)\, p(\varphi|m)\, d\varphi d\theta$$

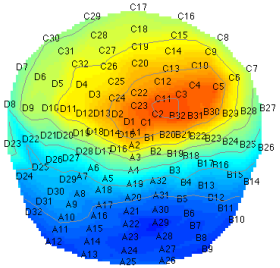
# Dynamic Causal Modelling

## *DCM for EEG-MEG: auditory mismatch negativity*

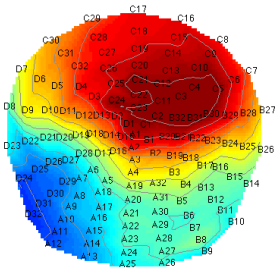
sequence of auditory stimuli




standard condition (S)

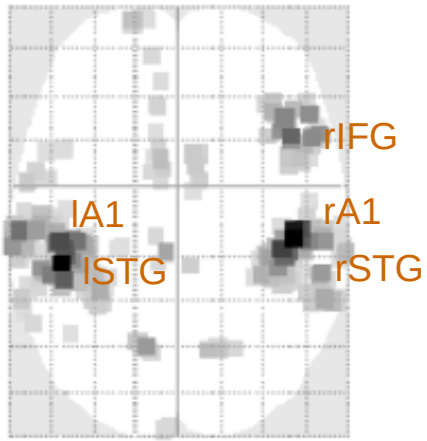


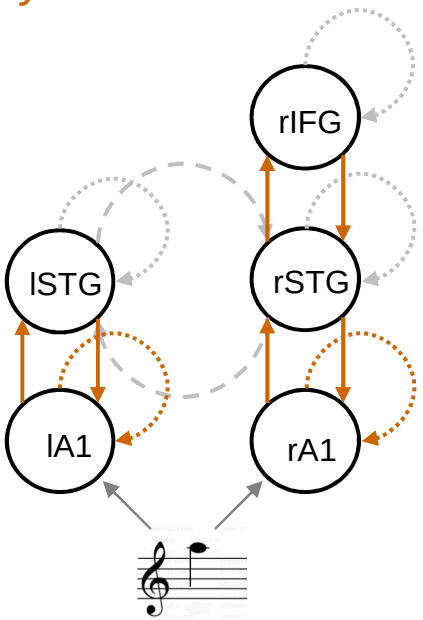
deviant condition (D)



t ~ 200 ms

S-D: **reorganisation**
of the connectivity structure



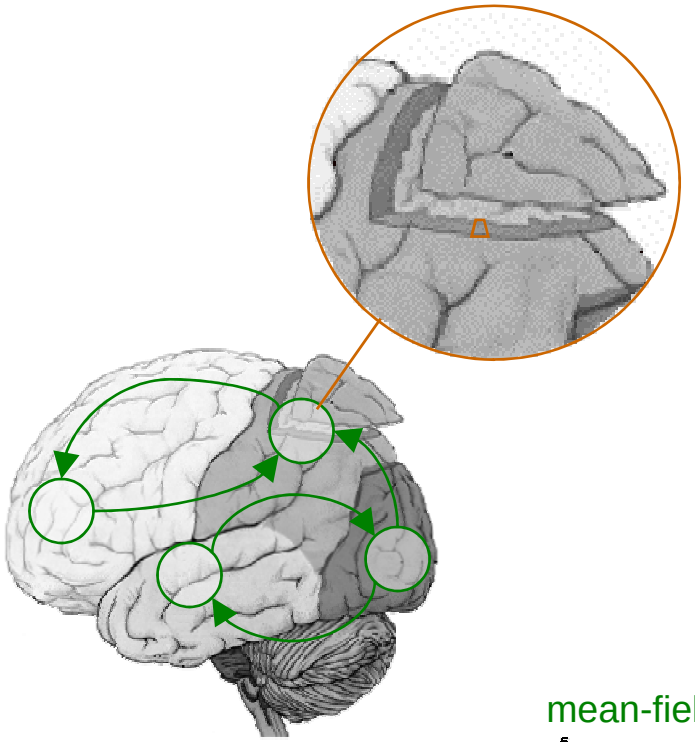

Daunizeau, Kiebel et al., Neuroimage, 2009

# Overview

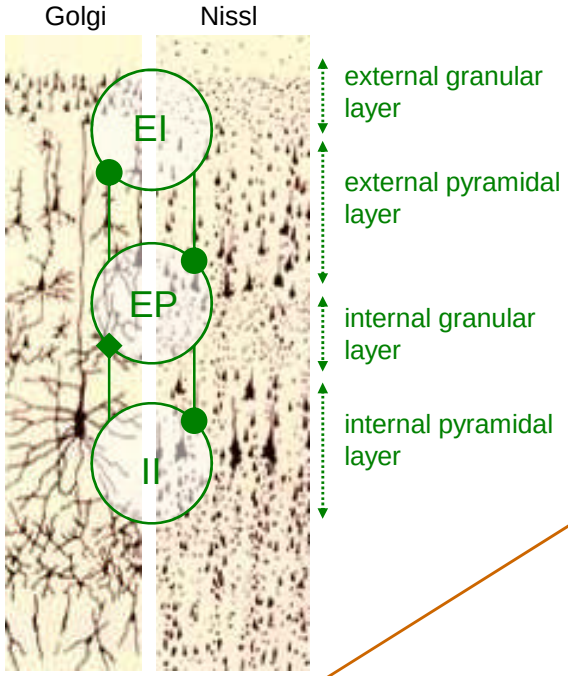
# System's states dynamics

*DCM for EEG/MEG: neural ensembles dynamics*

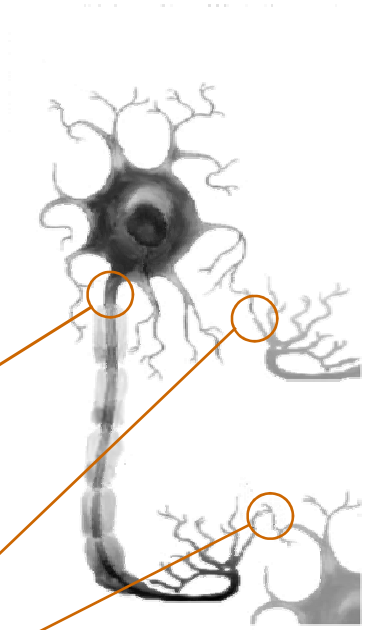
macro-scale



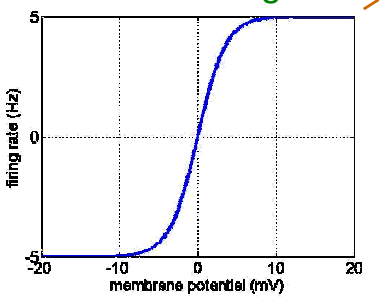
meso-scale



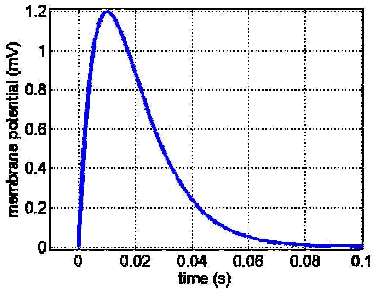

micro-scale



mean-field firing rate

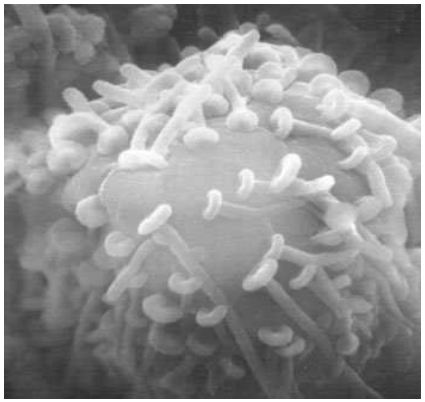



synaptic dynamics



# System's states dynamics

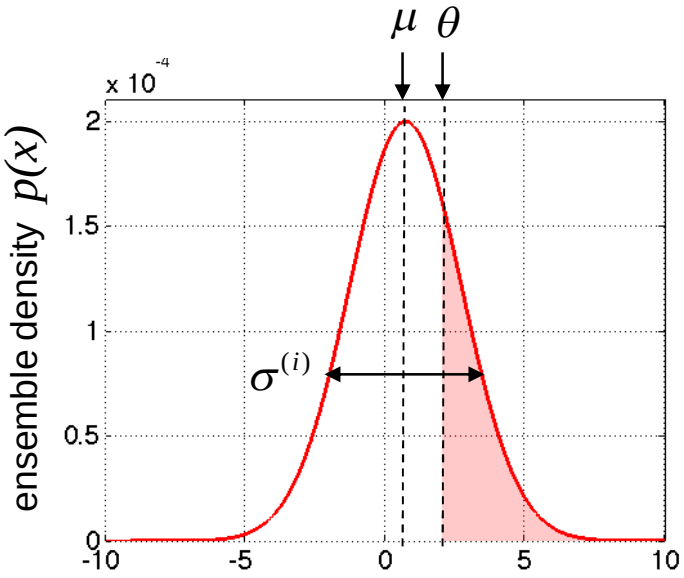
*DCM for EEG/MEG: from micro- to meso-scale*

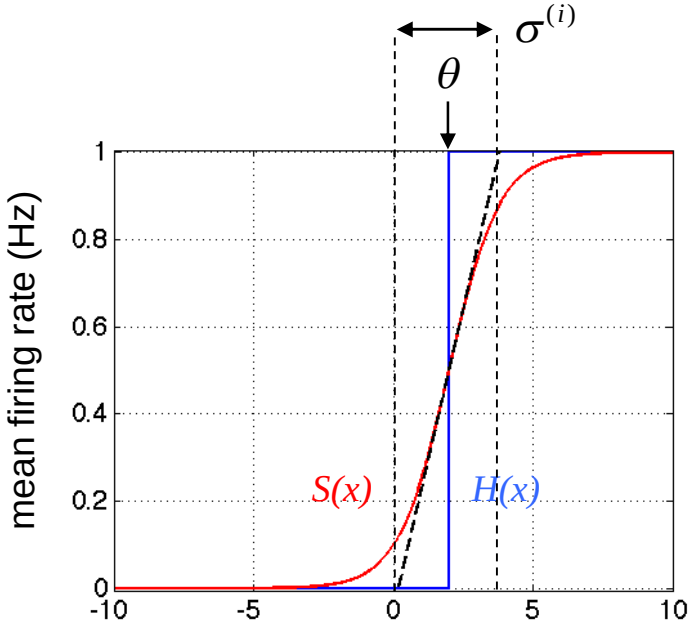


$x_j(t)$ : post-synaptic potential of $j^{th}$ neuron within its ensemble

$$\frac{1}{N-1}\sum_{j'\neq j} H\left(x_{j'}^{(i)}(t)-\theta\right) \xrightarrow{N\to\infty} \int H\left(x^{(i)}(t)-\theta\right) p\left(x^{(i)}(t)\right) dx^{(i)}$$

$$\approx S\left(\mu^{(i)}\right)$$ mean-field firing rate

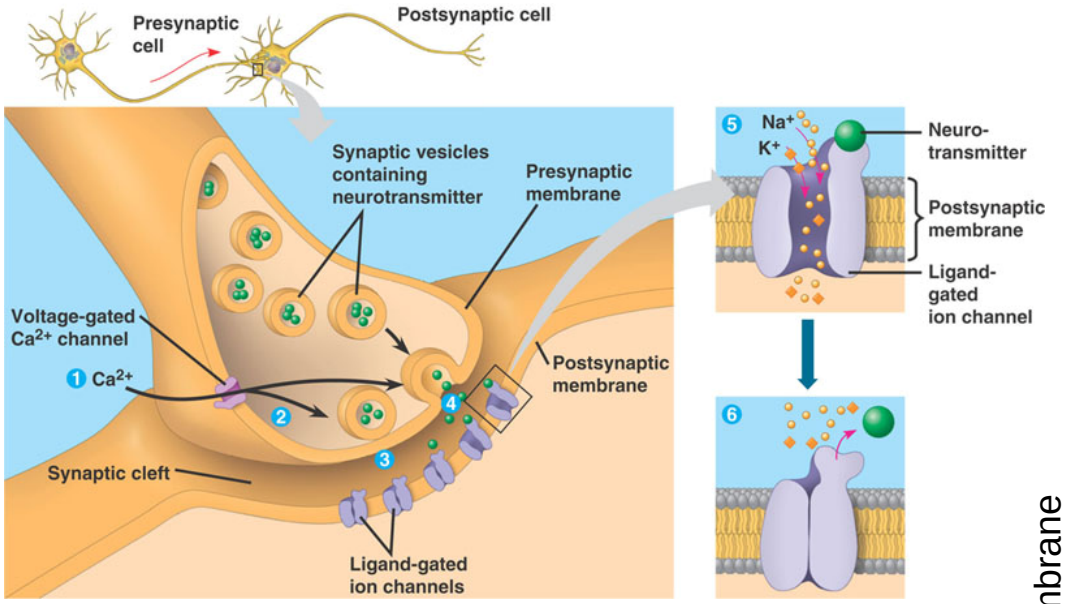


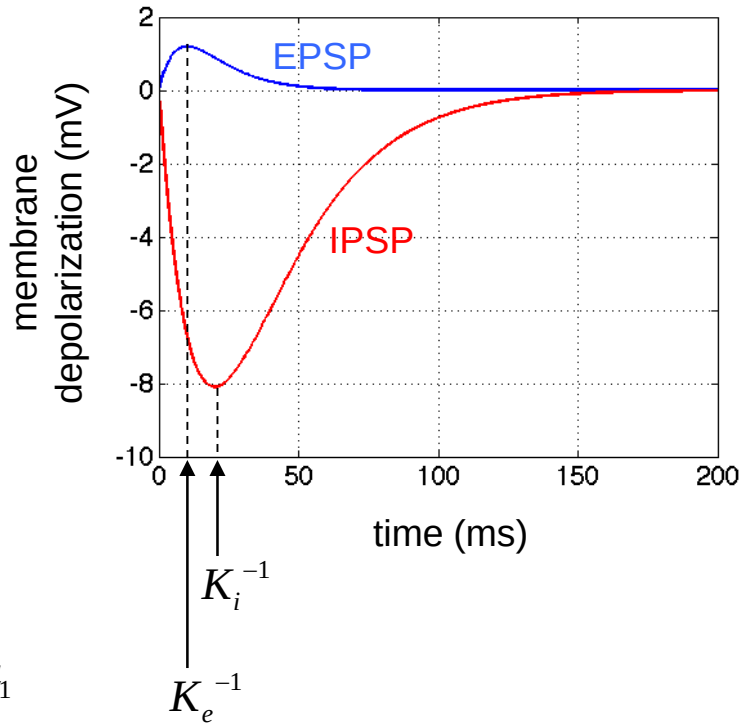

Marreiros, Daunizeau et al., Neuroimage 2008

# System's states dynamics

*DCM for EEG/MEG: synaptic dynamics*



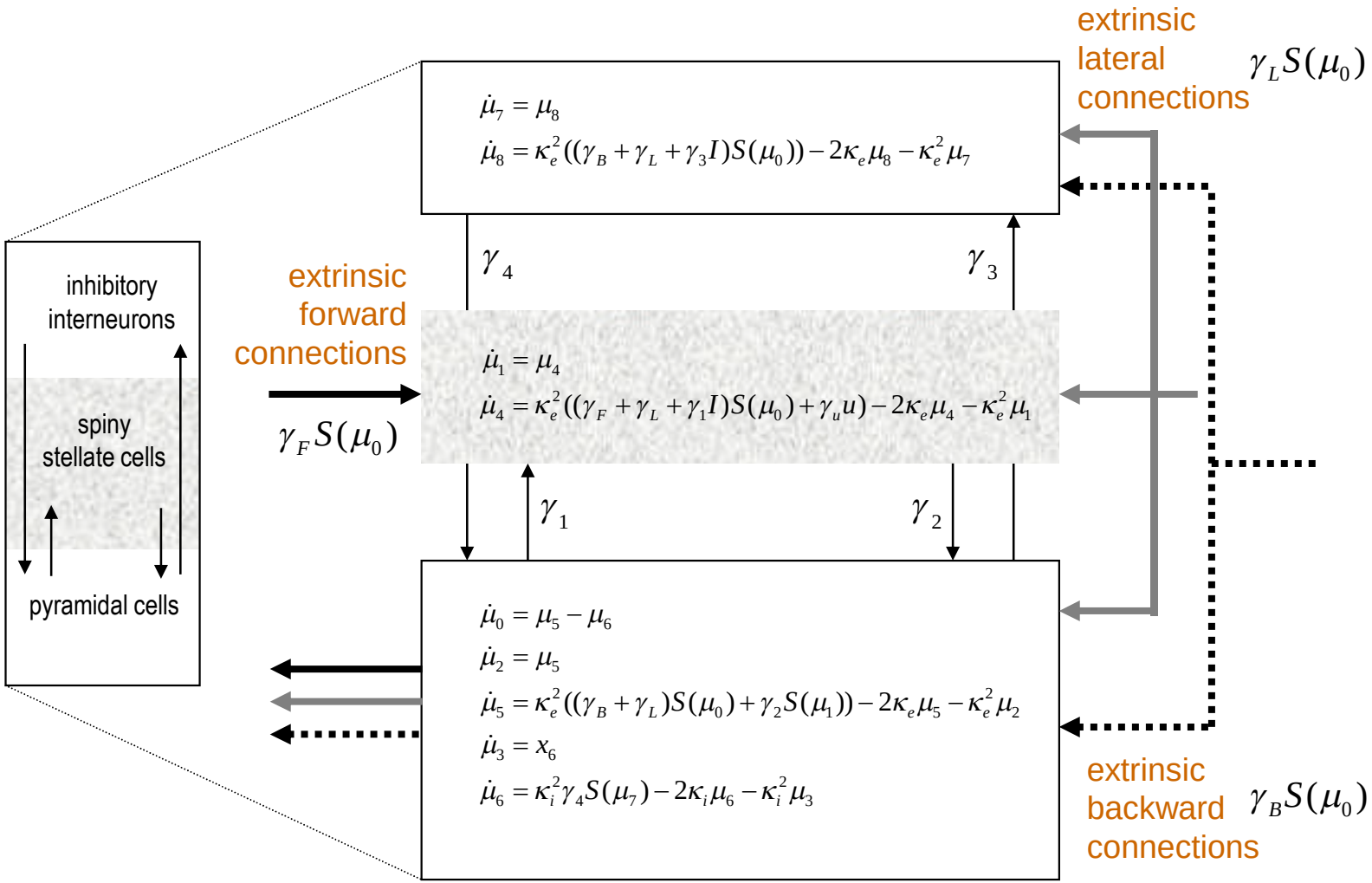

post-synaptic potential




$$\begin{cases} \dot{\mu}_1 = \mu_2 \\ \dot{\mu}_2 = \kappa_{i/e}^2 \, S(\bullet) - 2\kappa_{i/e} \, \mu_2 - \kappa_{i/e}^2 \, \mu_1 \end{cases}$$

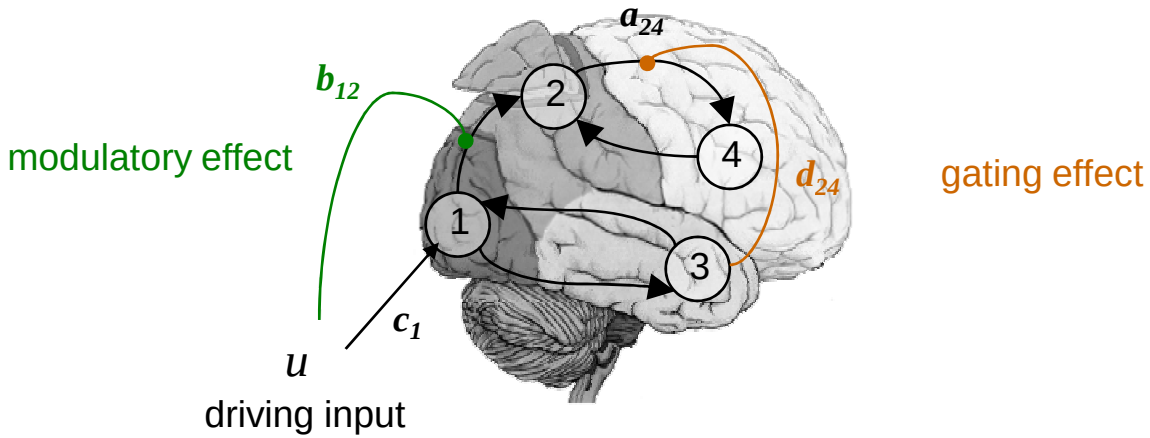
# System's states dynamics

*DCM for EEG/MEG: extrinsic connections*



# System's states dynamics

*DCM for fMRI: neuronal states dynamics*




$$\dot{x} = f(x,u) \approx f(x_0,0) + \frac{\partial f}{\partial x}x + \frac{\partial f}{\partial u}u + \frac{\partial^2 f}{\partial x \partial u}ux + \frac{\partial^2 f}{\partial x^2}\frac{x^2}{2} + \ldots$$

bilinear state equation:

$$\frac{dx}{dt} = \left( A + \sum_{i=1}^{m} u_i B^{(i)} \right) x + Cu$$

nonlinear state equation:

$$\frac{dx}{dt} = \left( A + \sum_{i=1}^{m} u_i B^{(i)} + \sum_{j=1}^{n} x_j D^{(j)} \right) x + Cu$$

Stephan, Harrison et al., Neuroimage 2008

# System's states dynamics

*DCM for fMRI: the neuro-vascular coupling*


experimentally controlled stimulus

$u$

$$\dot{x} = \left( A + \sum_{j=1}^{m} u_j B^{(j)} + \sum_{j=1}^{n} x_j D^{(j)} \right) x + Cu$$

neural states dynamics

*vasodilatory signal*

$$\dot{s} = x - \kappa s - \gamma (f - 1)$$

*flow induction* (rCBF)

$$\dot{f} = s$$

Balloon model

*changes in volume*

$$\tau \dot{v} = f - v^{1/\alpha}$$

*changes in dHb*

$$\tau \dot{q} = f\, E(f, E_0)/E_0 - v^{1/\alpha} q / v$$

hemodynamic states dynamics

$$\theta^h = \{\kappa, \gamma, \tau, \alpha, E_0, \varepsilon\}$$

$$\theta^n = \{A, B^{(i)}, C, D^{(j)}\}$$

$$\lambda(q,v) = \frac{\Delta S}{S_0} \approx V_0 \left[ k_1 (1-q) + k_2 \left( 1 - \frac{q}{v} \right) + k_3 (1-v) \right]$$

$$k_1 = 4.3 \vartheta_0 E_0 TE$$

$$k_2 = \varepsilon r_0 E_0 TE$$

$$k_3 = 1 - \varepsilon$$

BOLD signal change observation

# Overview

# stochastic DCM

## *stochastic processes: basics*

stochastic differential equation:

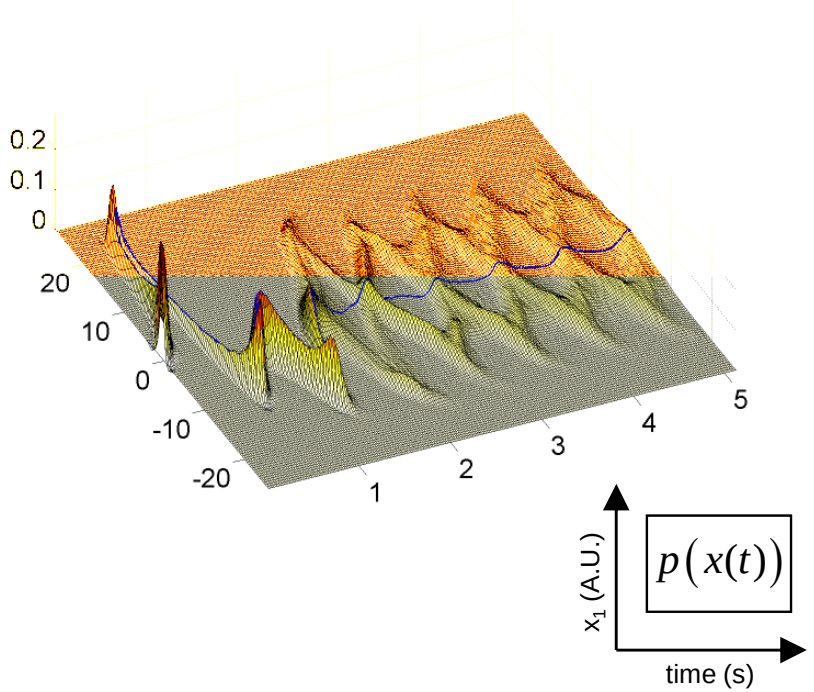
$$dx_t = \underbrace{f\left(x_t, \theta, u_t\right)}_{\text{drift}} dt + \underbrace{b\left(x_t, \theta, u_t\right)}_{\text{diffusion}} d\omega_t$$

→ limit of a discrete Markov process:

$$x_{t+1} \approx \tilde{f}\left(x_t\right) + b\sqrt{\Delta t}\,\eta_t$$
$$\eta_t \sim N\left(0, I\right)$$

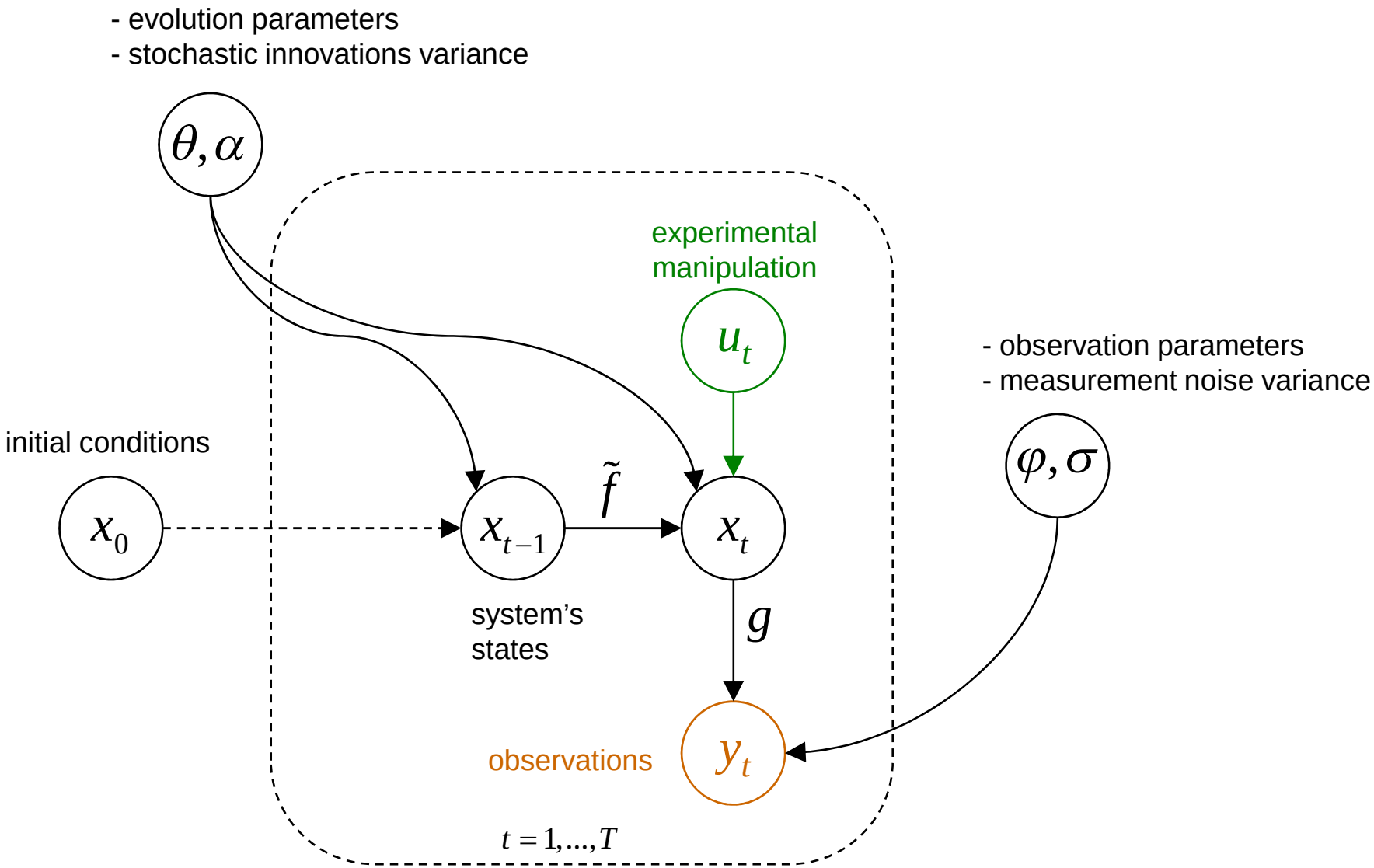
$$\tilde{f}\left(x_t\right) = J^{-1}\left(\exp\left[J\Delta t\right] - I_n\right) a\left(x_t\right)$$
$$\xrightarrow[\Delta t \to 0]{} x_t + \Delta t\, f\left(x_t\right),$$

DCM for fMRI with gating (nonlinear) effects:



# stochastic DCM

*the generative model*

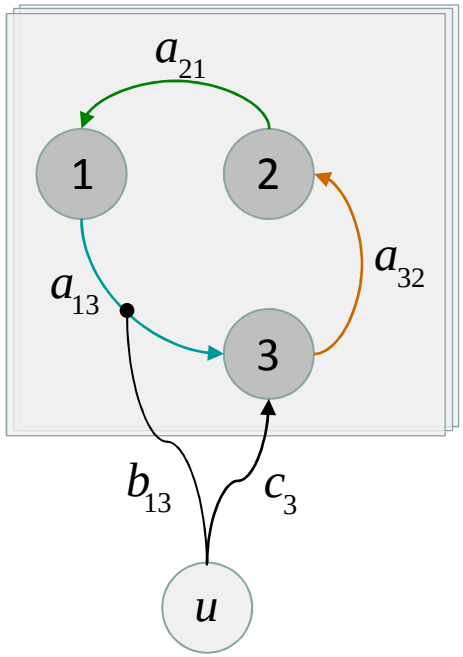


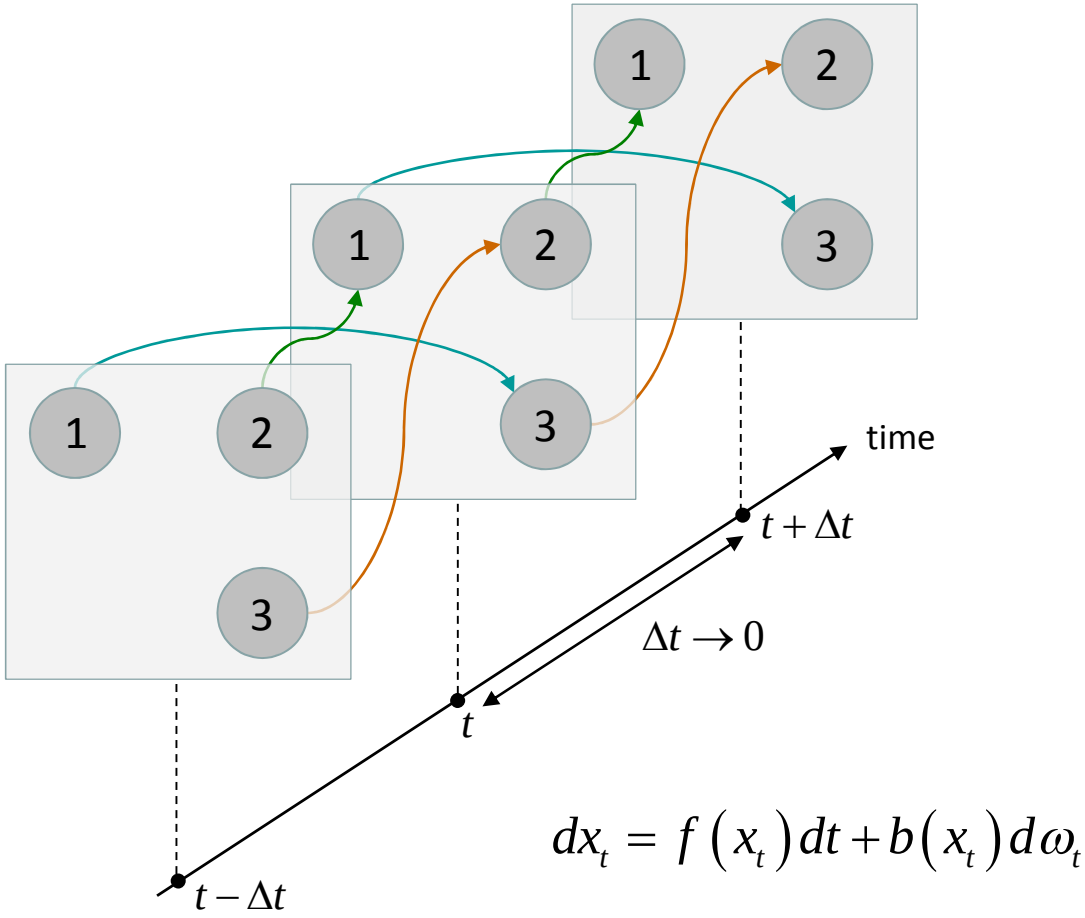

Daunizeau, Friston et al., Physica D, 2009

# stochastic DCM

*a note on causality*

$$u \xrightarrow{\theta} x \xrightarrow{\varphi} y$$




$$dx_t = f(x_t)\,dt + b(x_t)\,d\omega_t$$

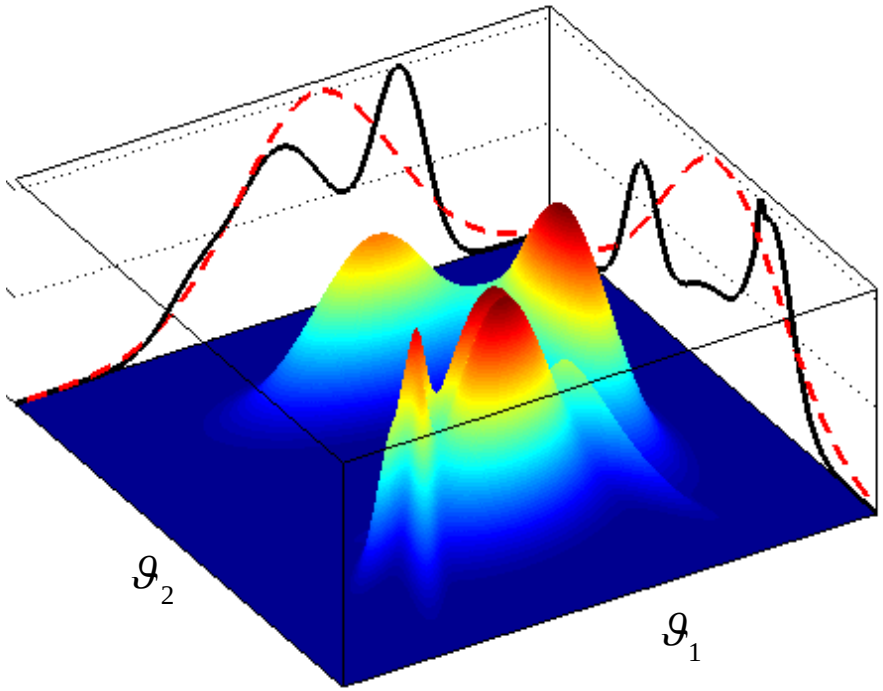
# stochastic DCM

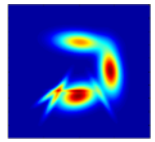
*VB: mean field - Laplace approximation*

$$\ln p(y|m) = \underbrace{\left\langle \ln p(\vartheta, y|m) \right\rangle_q + S(q)}_{\text{free energy}} + D_{KL}\left(q(\vartheta); p(\vartheta|y,m)\right)$$

free energy : functional of $q$

approximate (marginal) posterior probability density functions: $\{q(\vartheta_1), q(\vartheta_2)\}$



$p(\vartheta_1, \vartheta_2|y,m)$

$p(\vartheta_{1 \text{ or } 2}|y,m)$

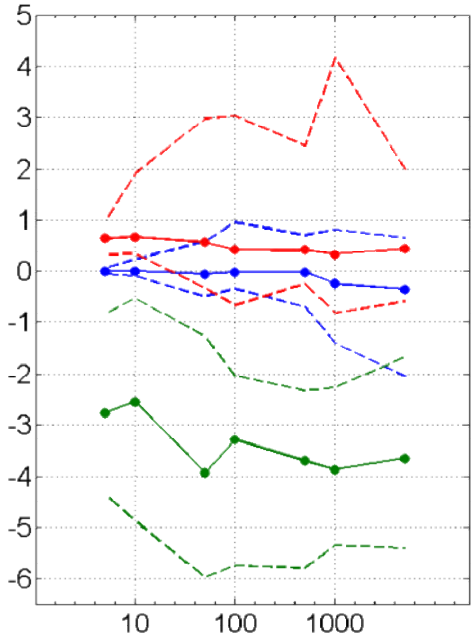
$q(\vartheta_{1 \text{ or } 2})$

# Overview

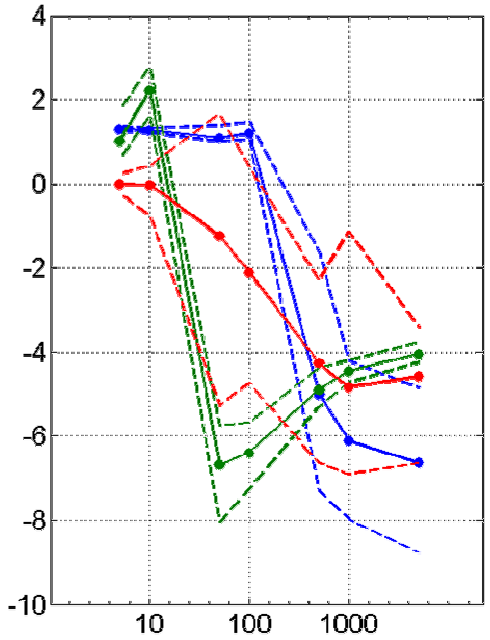
# VB evaluation

*asymptotic efficiency*

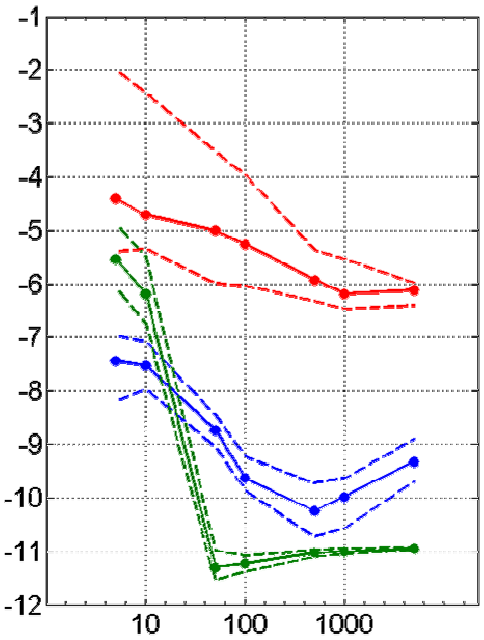
initial conditions ($x_0$)

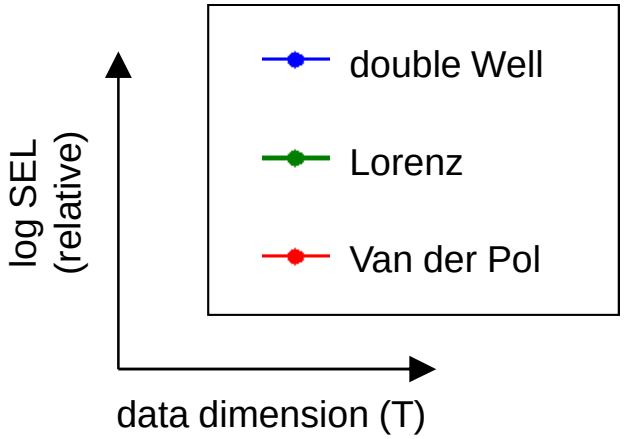



evolution parameters (θ)




hidden states ($x_t$)

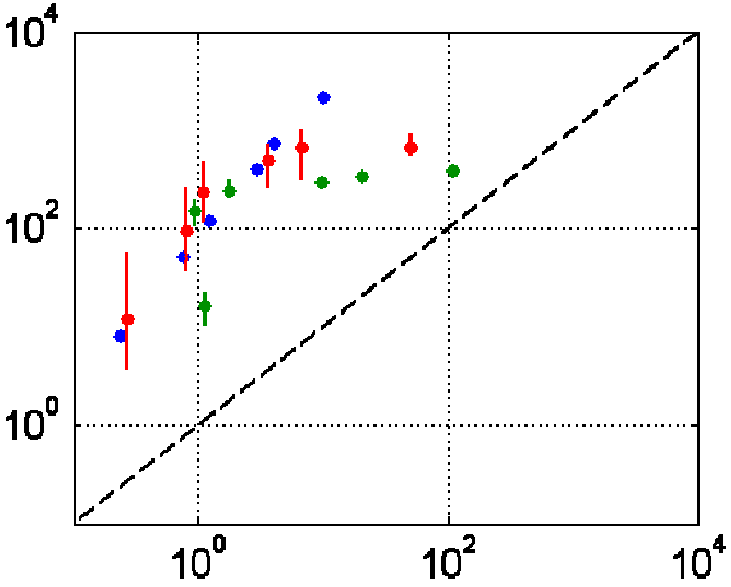


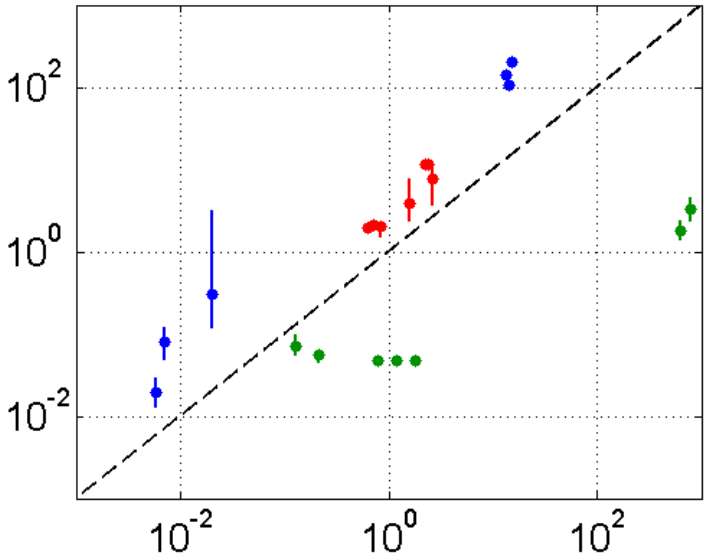
# VB evaluation

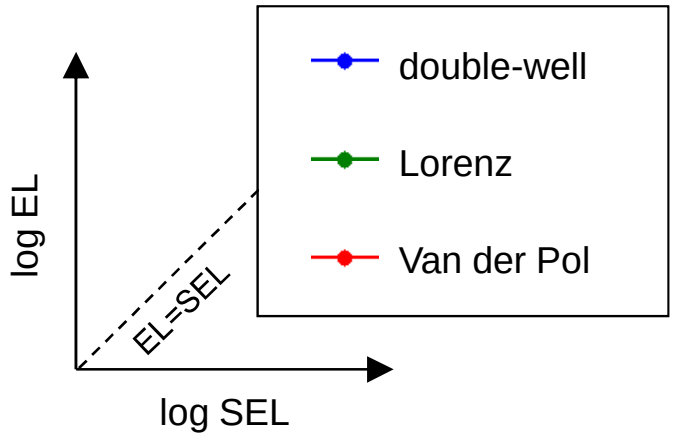
*posterior confidence intervals*

stochastic innovations ($\eta$)




evolution parameters ($\theta$)
+ initial conditions ($x_0$)



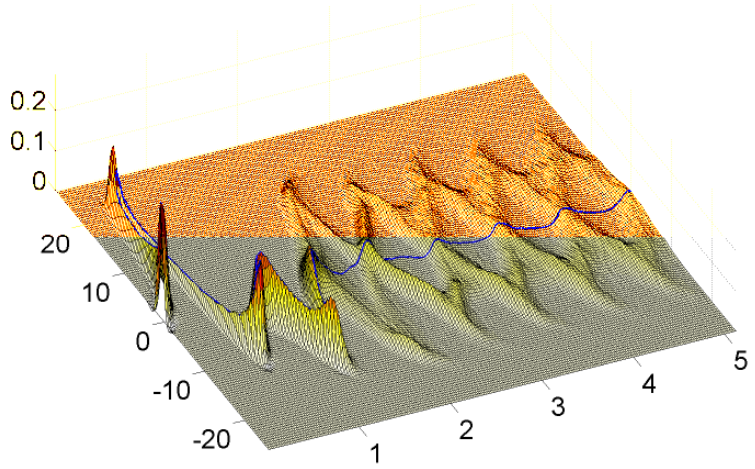



# VB evaluation

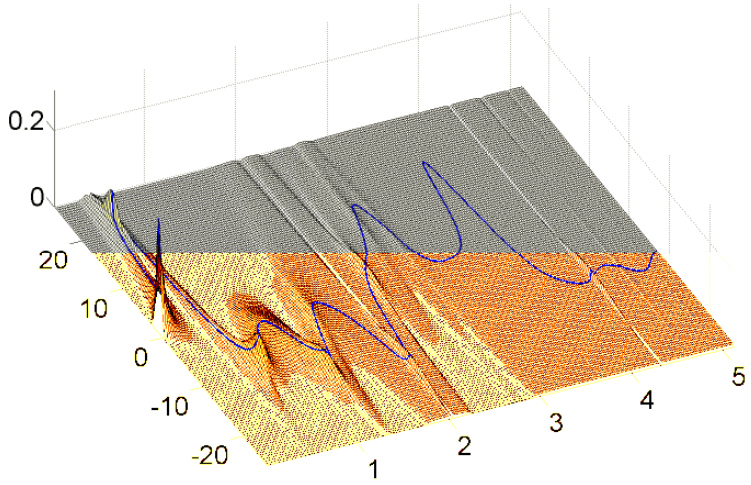
*prediction ability*

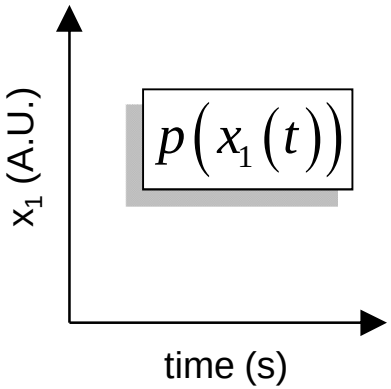
Lorenz system: predictive density ($x_1$)

MCMC




Laplace approximation



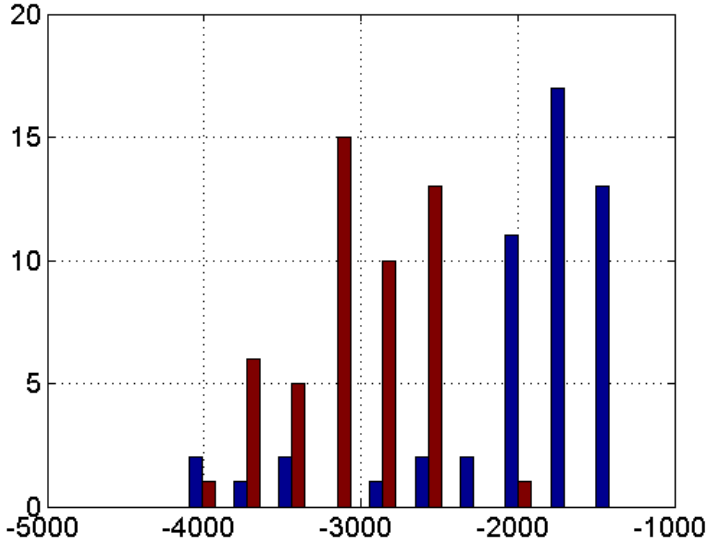




$$p\left(x_1(t)\right)$$

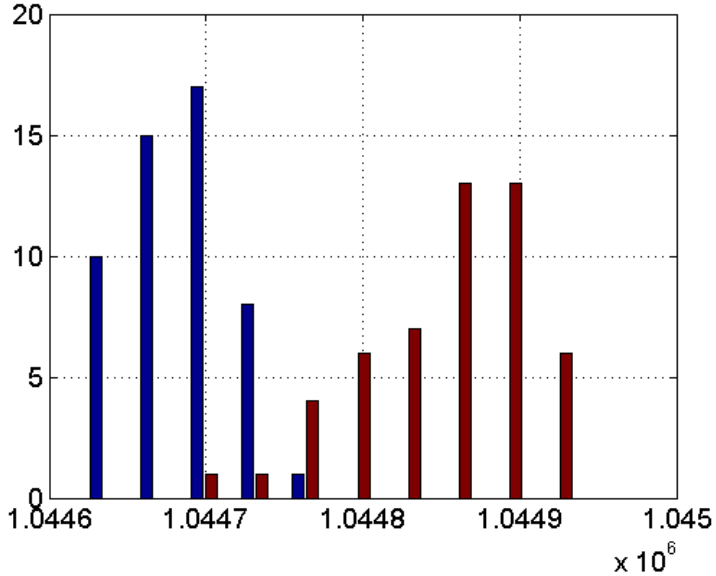
# VB evaluation

*model comparison (I)*

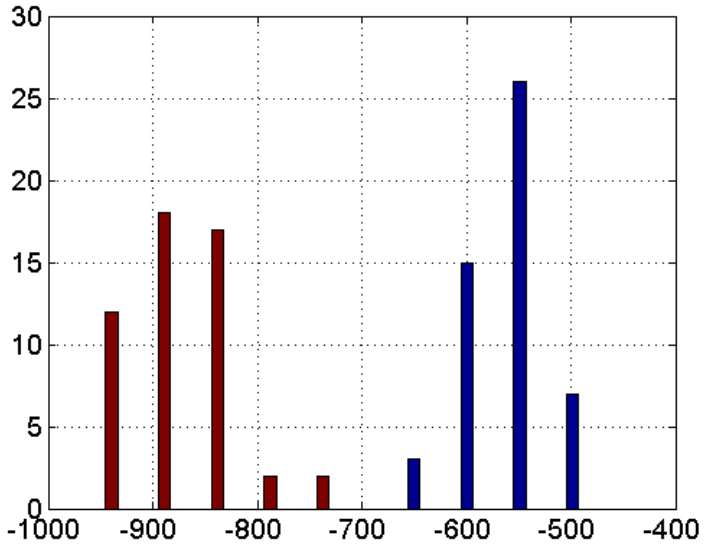
double-well

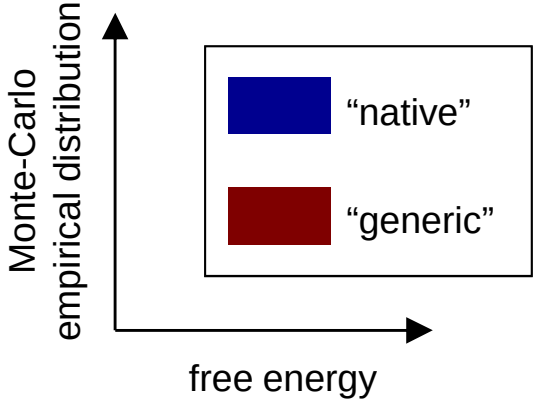



Lorenz



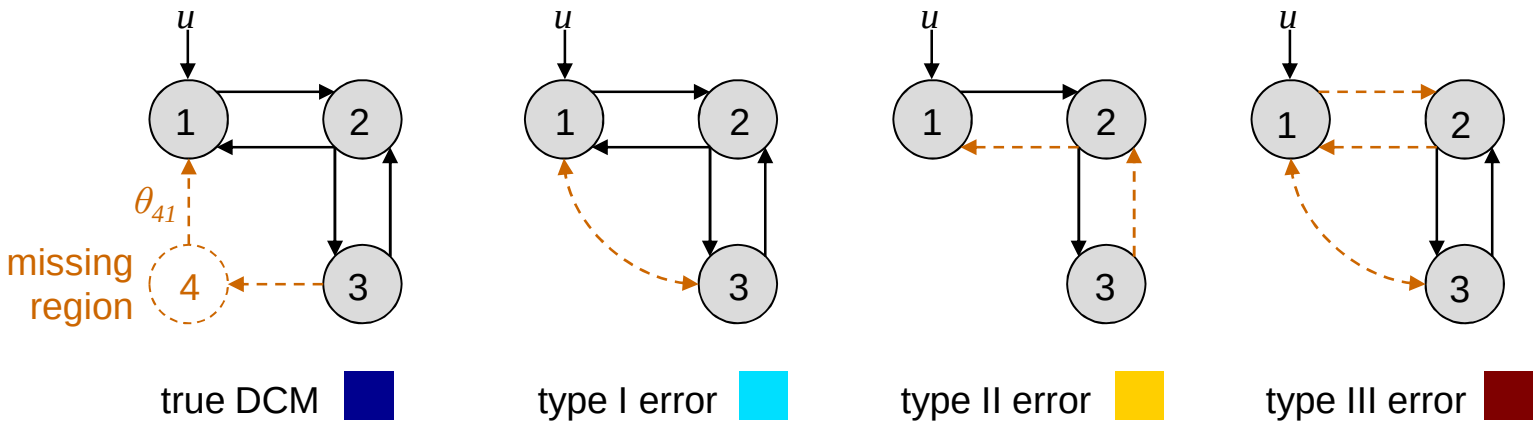

Van Der Pol



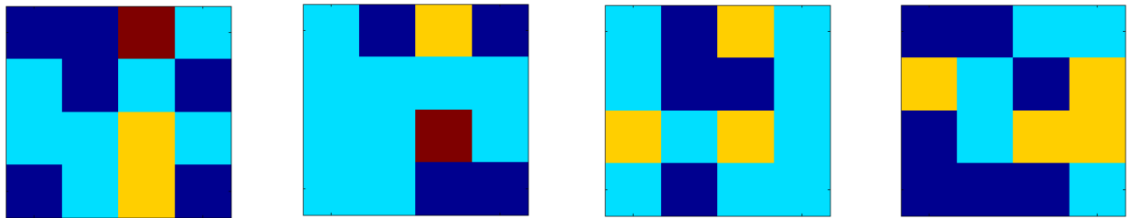



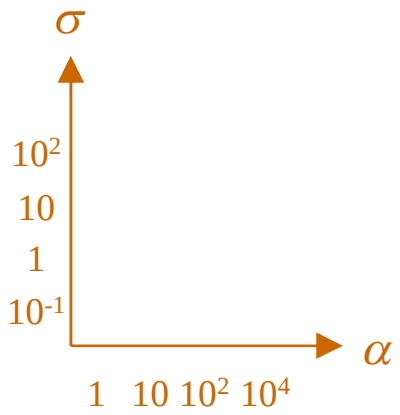
# VB evaluation

*model comparison (II): robustness to missing regions*

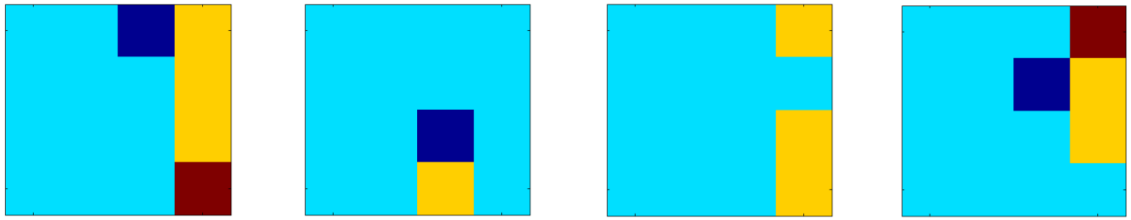



VB-Laplace for stochastic DCM




VB-Laplace for DCM

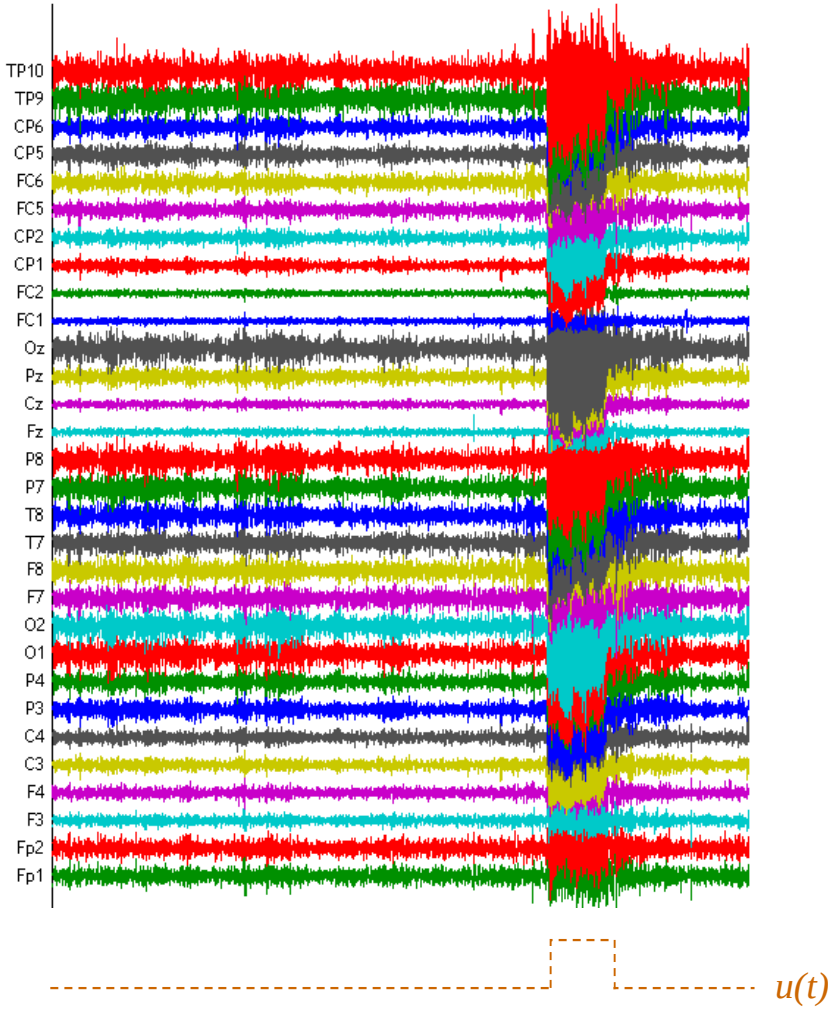


# Overview

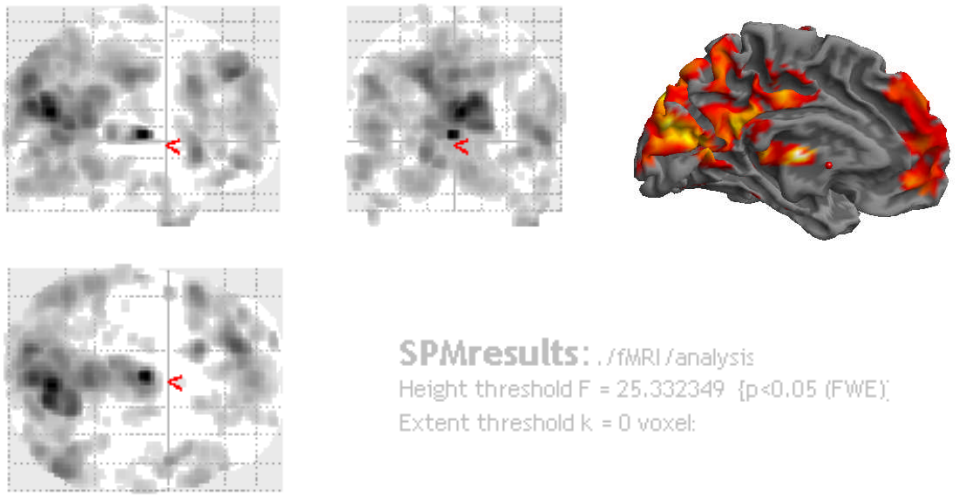
# sDCM: example

*epileptogenic network mapping*

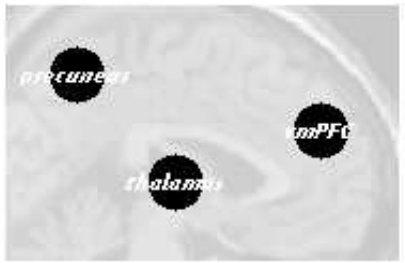
EEG: ictal activity




fMRI: GLM activation results

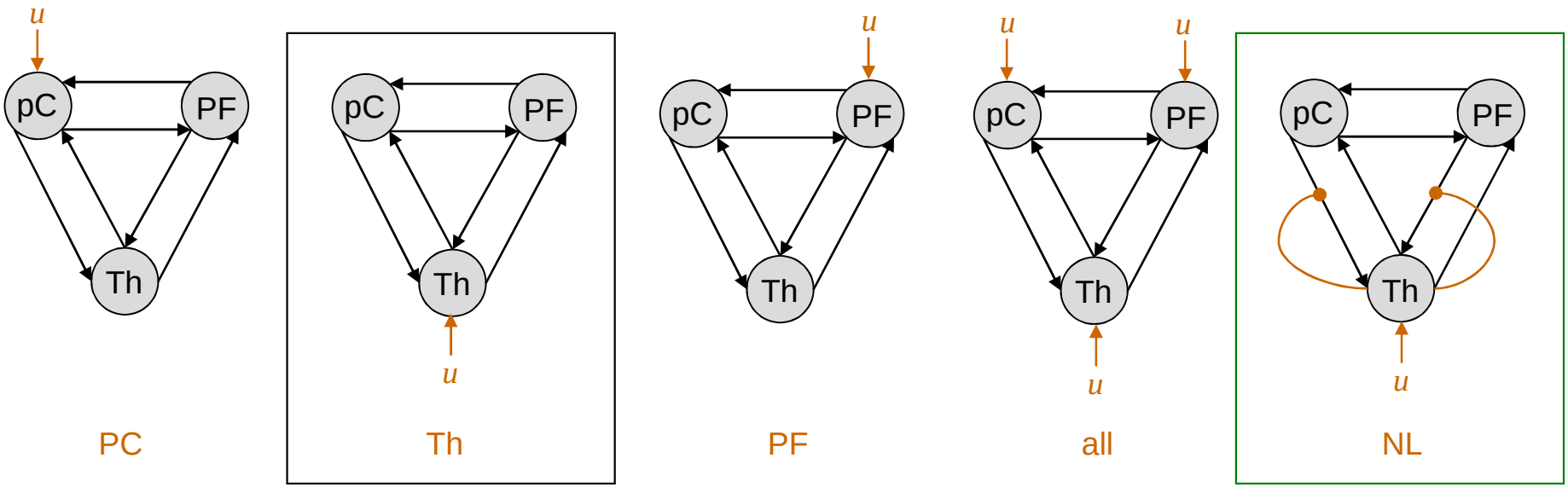



group-level consistent ROIs

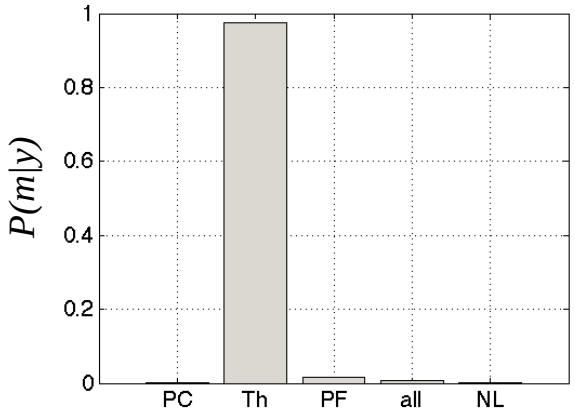


# sDCM: example

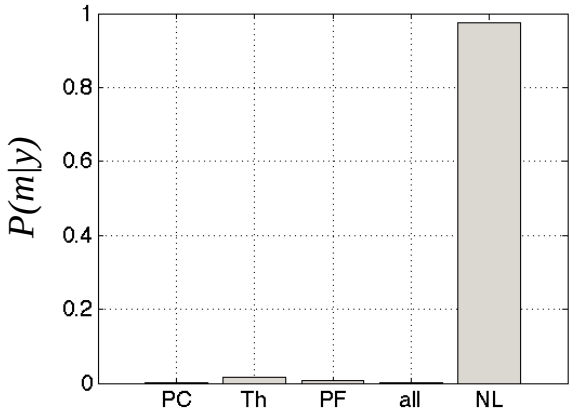
*Bayesian model comparison*




deterministic DCMs

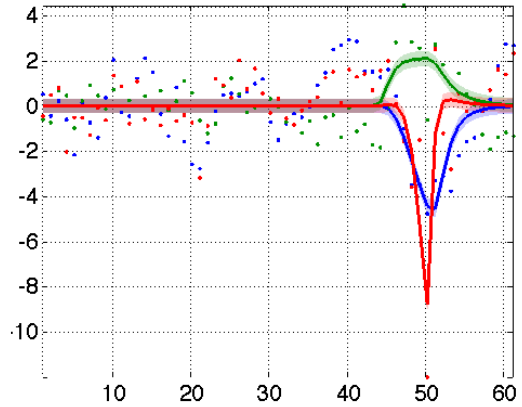
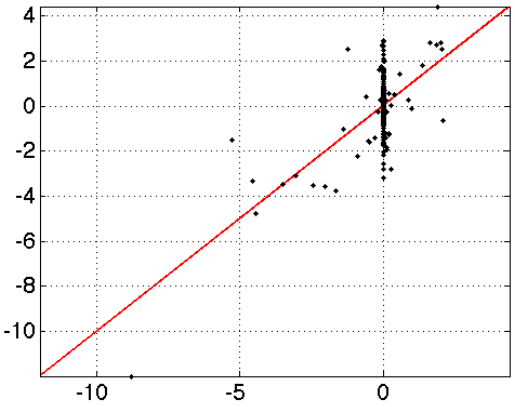
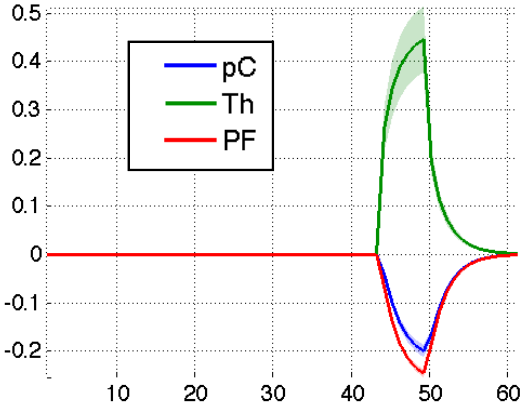
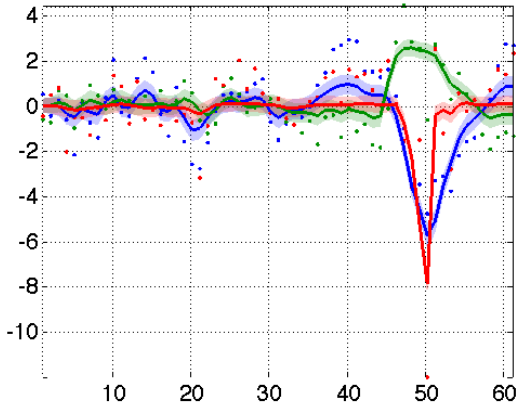
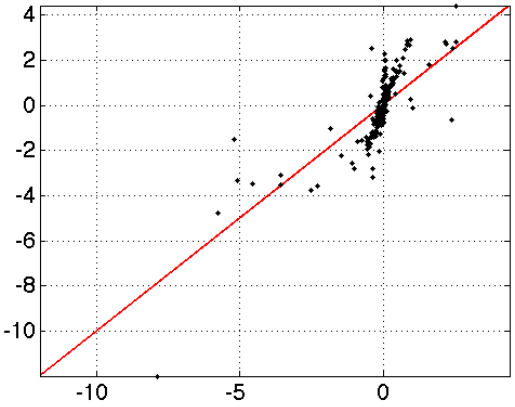
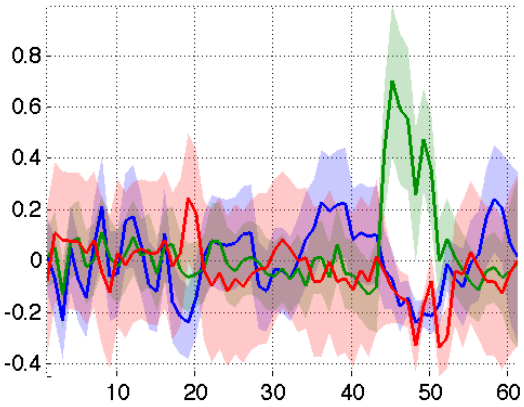



stochastic DCMs



# sDCM: example

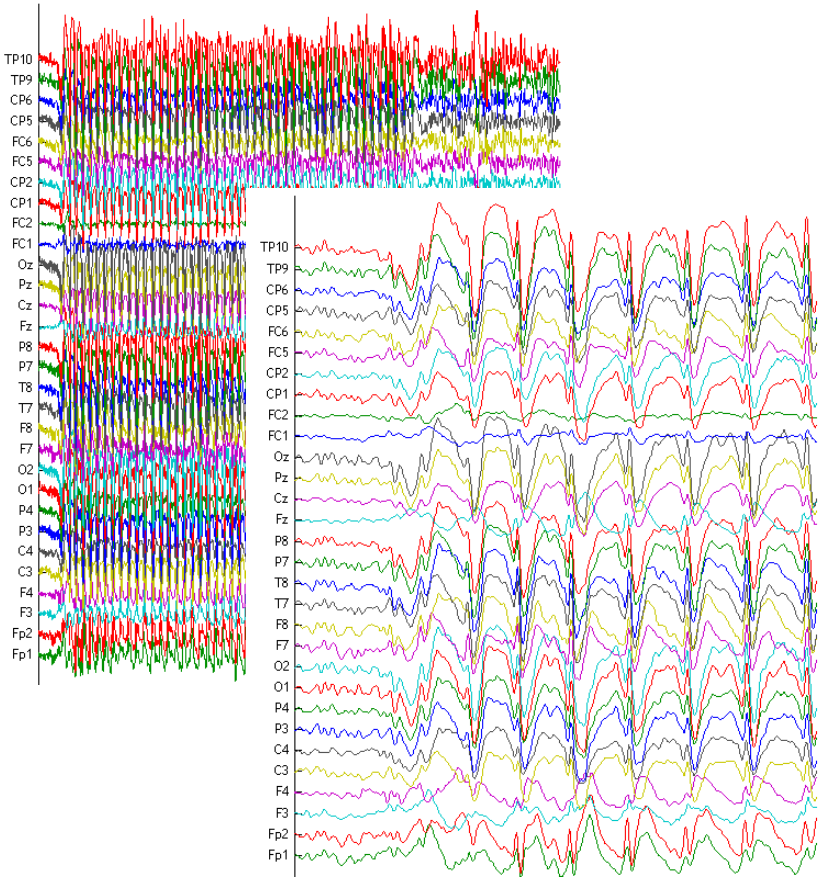
*stochastic versus deterministic (most likely) DCM*

| | posterior predictive density $p(g(x)\|y,m)$ as a function of time | model fit: $\langle g(x) \rangle$ versus y | neural states dynamics: $\langle x \rangle$ as a function of time |
|---|---|---|---|
| DCM |  |  |   |
| sDCM |  |  |  |

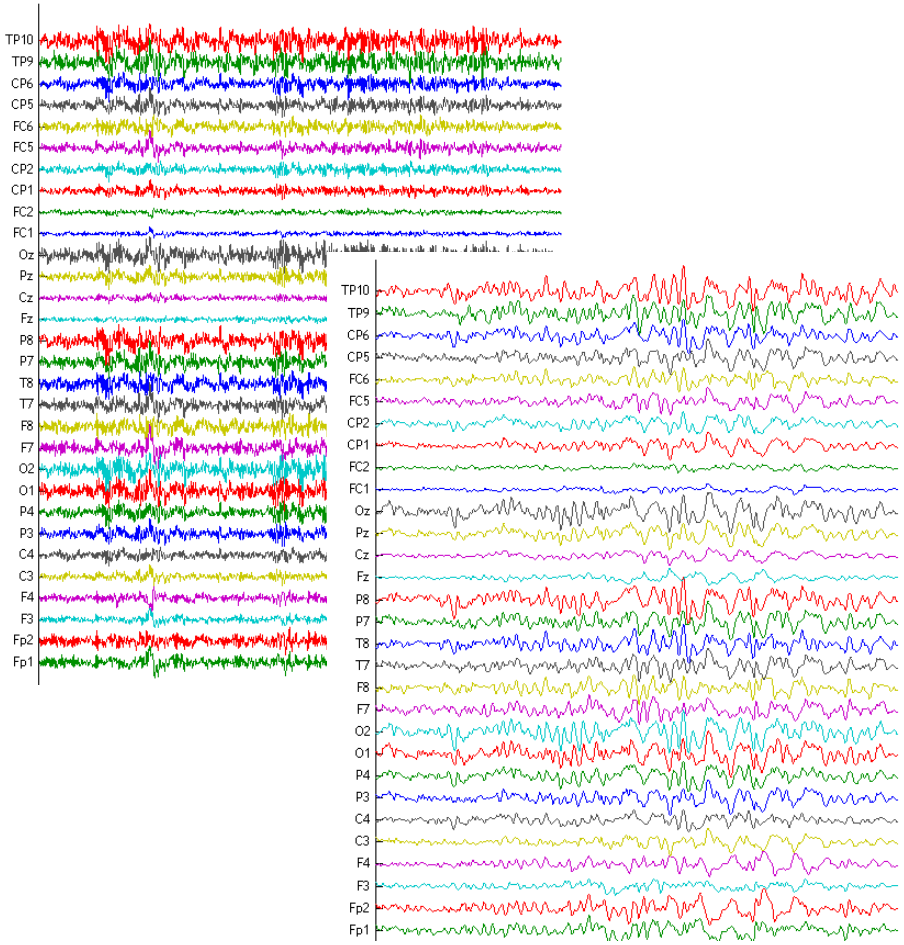
# sDCM: exemple

*back to the EEG*

ictal activity (absence seizure)




interictal activity (no clinical sign)



# Overview

# Conclusion

→ Motivations:

- model imperfections
- endogenous non-controlled activity
- robustness to the “missing region” problem

→ VB: balanced computational load/accuracy (curse of dimensionality):

- asymptotic efficiency
- posterior confidence intervals
- model comparison

→ Limitations:

- prediction: lack of robustness to violation of *quasideterminism*
- non-stationarity? (volatile models)
- estimation of “neural noise”: robustness of the backward pass?
- potential conflicts between deterministic and stochastic effects

→ What about *continuous* stochastic systems?

- augmented Kushner-Stratonovitch-Pardoux equations?
- Dynamic Expectation-Maximization (Friston K.; Neuroimage 2008)

many thanks to:

Karl J. Friston (London, UK)
Stefan J. Kiebel (Leipzig, Germany)
Klaas E. Stephan (Zurich, Switzerland)
Louis Lemieux (Chalfon, UK)