Elements of thought

Can mathematics help us find elegant order behind the apparent pandemonium of our minds, asks Colin Barras

How could an equation or formula ever hope to capture something as complex and beautiful as the human mind? In a sense we've long been describing the brain with numbers – 86 billion neurons, 1200 cubic centimetres, 1400 grams. But you might expect that more ambitious attempts to explain the brain with mathematics would be doomed to failure.

Yet over the last few years, neuroscientists have built a mathematical framework for understanding many aspects of the brain. In the same way that Newton's laws of motion describe the dance of the stars and planets in the night sky, mathematical principles are now revealing telling patterns in the melee of our minds. What's surprising is just how often the brain's dynamics mimic other natural phenomena, from earthquakes and avalanches to the energy flow in a steam engine.

The equations we end up with describe everything from the brain's structure to the generation of our thoughts and feelings. They may even help us begin to understand the nature of consciousness itself. Join us as we explore the five laws that rule the mind.

SMALL WORLD, BIG CONNECTIONS

If you stretched out all the nerve fibres in the brain, they would wrap four times around the globe. Crammed into the skull, this wiring looks like a tangled mess, but in fact mathematicians know its structure well – it is a form of the "small-world network".

The hallmark of a small-world network is the relatively short path between any two nodes. You've probably already heard of the famous "six degrees of separation" between you and anyone else in the world, which reflects the small-world structure of human societies. The average number of steps between any two brain regions is similarly small, and slight variations in this interconnectivity have been linked to measures of intelligence.

That may be because a small-world structure makes communication between different areas of a network rapid and efficient. Relatively few long-range connections are involved – just 1 in 25 nerve fibres connect distant brain regions, while the rest join neurons in their immediate vicinity. Long nerve fibres are costly to build and maintain, says Martijn van den Heuvel at the University Medical Center in Utrecht, the Netherlands, so a small-world-network architecture may be the best compromise between the cost of these fibres and the efficiency of messaging.

The brain's long-range connections aren't distributed evenly over the brain, though. Last year van den Heuvel and Olaf Sporns of Indiana University Bloomington discovered that clusters of these connections form a strong "backbone" that shuttles traffic between a dozen principal brain regions (see diagram, page 37). The backbone and these brain regions are together called a "rich club", reflecting the abundance of its interconnections.

No one knows why the brain is home to a rich club, says van den Heuvel, but it is clearly important because it carries so much traffic. That makes any problems here potentially very serious. "There's an emerging idea that perhaps schizophrenia is really a problem with integrating information within these rich-club hubs," he says. Improving rich-club traffic flow might be the best form of treatment, though it is not easy to say how that might be achieved.

What is clear for now is that this highly interconnected network is the perfect platform for our mental gymnastics, and it forms a backdrop for many of the other mathematical principles behind our thoughts and behaviour. ›

TEETERING ON THE EDGE OF CHAOS

The familiar chords of our favourite song reach the ear, and moments later a neuron fires. Because that neuron is linked into a highly connected small-world network, the signal can quickly spread far and wide, triggering a cascade of other cells to fire. Theoretically it could even snowball chaotically, potentially taking the brain offline in a seizure.

Thankfully, the chances of this happening are slight. "Perhaps 1 per cent of the population will experience a seizure at one time in their lives," says John Beggs at Indiana University Bloomington. This suggests there is a healthy balance in the brain – it must inhibit neural signals enough to prevent a chaotic flood without stopping the traffic altogether.

The sweet spot

An understanding of how the brain hits that sweet spot emerged in the 1970s, when Jack Cowan, now at the University of Chicago, realised that this balance represents a state known as the critical point or "the edge of chaos" that is well known to theoretical physicists. Cascades of firing neurons – or "neural avalanches" – are the moments when brain cells temporarily pass this critical point, before returning to the safe side, he said.

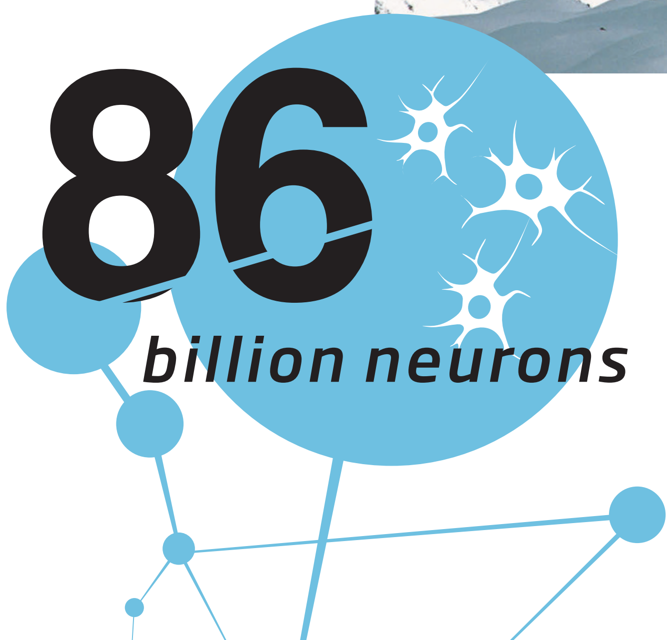
Avalanches, forest fires and earthquakes also result from systems lying at the critical point, and they all share certain mathematical characteristics. Chief among them is the so-called "power law" distribution, which means that bigger earthquakes or forest fires happen less often than smaller ones according to a strict mathematical ratio; an earthquake that is 10 times as strong as another quake is also just one-tenth as likely to happen, for instance.

How does the brain compare? In 2003 Beggs and Dietmar Plenz, both then at the National Institute of Mental Health in Bethesda, Maryland, checked whether neural activity matches Cowan's theory by using a grid-like array of electrodes hooked to a chunk of rat cortex. Sure enough, they found that an excited neuron passed its signal to just one neighbour on average, which is exactly what you would expect of a system on the edge of chaos: any more and the system would lie in permanent, full-blown disorder. Importantly, larger neural avalanches do occur, but they are much rarer. Like earthquakes and forest fires, their frequency drops with size according to the precise ratio predicted by a power law.

Since Beggs's initial work, further functional MRI scans suggest that the same kind of edge-of-chaos activity can be found at much larger scales, across the whole human brain; indeed, computer models suggest it might be a result of the small-world structure of the brain (*New Scientist*, 27 June 2009, p 34).

Balancing on the edge of chaos may seem risky, but the critical state is thought to give the brain maximum flexibility – speeding up the transmission of signals and allowing it to quickly coordinate its activity in the face of a changing situation. Some of the researchers are beginning to wonder whether certain disorders might arise when the brain veers away from this delicate balance. "There's now some evidence that people with epilepsy are not at this critical point," says Beggs. "Just as there's a healthy heart rate and a healthy blood pressure, this may be what you need for a healthy brain."

"Avalanches, forest fires and cascades of firing neurons all share certain mathematical characteristics"

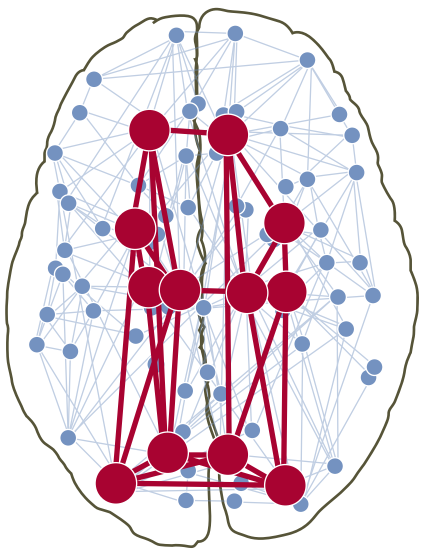


12 hyper-connected hubs that help direct traffic flow

COLIN MONTEATH/HEDGEHOG HOUSE/MINDEN/GETTY

The rule of the rich

The brain's wiring allows for the rapid transmission of information, with a set of particularly well-connected hubs, known as the **rich club**, directing much of the traffic between different parts of the brain



This group may be crucial for integrating all the thoughts and feelings that make up our conscious experience

KNACK FOR THE FUTURE

From its crackling electrical storm of activity, the brain needs to predict the surrounding world in a trustworthy way, whether that be working out which words are likely to crop up next in a conversation, or calculating if a gap in the traffic is big enough to cross the road. What lies behind its crystal-ball gazing?

One answer comes from an area of mathematics known as Bayesian statistics. Named after an 18th-century mathematician, Thomas Bayes, the theory offers a way of calculating the probability of a future event based on what has gone before, while constantly updating the picture with new data. For decades neuroscientists had speculated that the brain uses this principle to guide its predictions of the future, but Karl Friston at University College London took the idea one step further.

Friston looked specifically at the way the brain minimises the errors that can arise from these Bayesian predictions; in other words, how it avoids surprises. Realising that he could borrow the mathematics of thermodynamic systems like a steam engine to describe the way the brain achieves this, Friston called his theory "the free energy principle". Since prediction is so central to almost everything the brain does, he believes the principle could offer a general law for much, if not all, of our neural activity – the brain's equivalent of $E=mc^2$ in terms of its descriptive power and elegance.

So far, Friston has successfully used his free energy principle to describe the way neurons send signals backwards and forwards in the visual cortex in response to incoming sights. He believes the theory could also explain some of our physical actions. For instance, he has simulated our eye movements as we take in familiar or novel images, suggesting the way the brain builds up a picture with each sweep of our gaze to minimise any errors in its initial perception. In another paper he turned his attention to the delicate control of our arm as we reach for an object, using the free energy principle to describe how we update the muscle movements by combining internal signals from the turning joints with visual information (*Biological Cybernetics*, vol 102, p 227).

Others are using the concept to explain some of the brain's more baffling behaviours. Dirk De Ridder at the University of Otago's Dunedin School of Medicine in New Zealand, for instance, has used the principle to explain the phantom pains and sounds people experience during sensory deprivation. He suggests they come from the neural processes at work as the brain casts about wildly to predict future events when there is little information to help guide its forecasts (*Neuroscience and Biobehavioural Reviews*, doi.org/j9q).

Friston points out that the brain's ability to update its thoughts and make predictions about the world depends on a finely tuned system. "Signals in the brain decay," he says, and if the decay is too fast, an important hypothesis may disappear by the time the brain makes its next observation and generates a new prediction." For this reason, the free energy principle relies on the brain's ability to hang in that "critical state" on the edge of chaos. "Criticality is almost mandated by the Bayesian brain," says Friston. ›

170,000 kilometres of nerve fibres

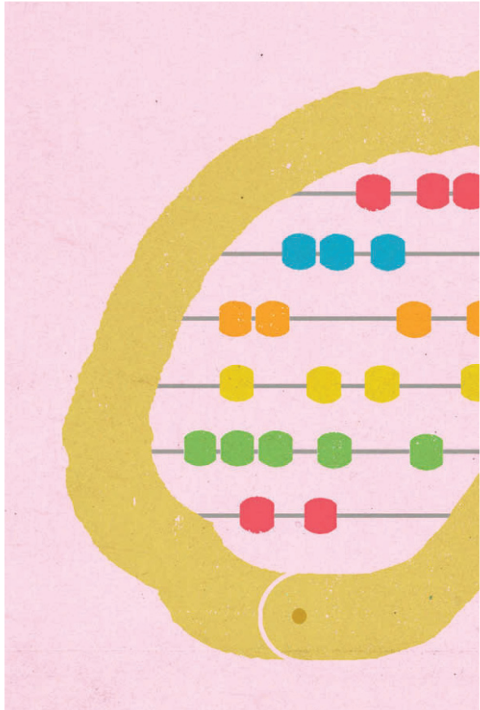
PREYING ON YOUR MIND

As your mind flits from thought to thought, it may seem as if dozens of sensations and ideas are constantly fighting for your attention. In fact, that's surprisingly close to the mark; the way different neural networks compete for dominance echoes the battle for survival between a predator species and its prey, and the result may be your wandering mind.

Mikhail Rabinovich at the University of California in San Diego and Gilles Laurent at the California Institute of Technology in Pasadena were the first to notice this strange dynamic. They were studying the neuronal activity in the antennal lobe – the insect equivalent of the olfactory bulb in the mammalian brain – as locusts experienced different odours. Rabinovich expected the activity to flatline when they got used to each smell, but he was wrong. "Even when the scent stimulus was constant, the activity of the principal neurons in the antennal lobe changed with time," he says.

Looking closely, Rabinovich noticed that the pattern of activity was not random, but similar to the form described by mathematicians Alfred Lotka and Vito Volterra in the early 20th century. The Lotka-Volterra equations, also known as predator-prey equations, are a key ecological tool for predicting fluctuations in populations of interacting species. A predator near-exhausts its supply of prey, and so starves while its prey recovers, and the cycle starts again.

Rabinovich dubs such perpetual fights "winnerless competitions" and he says they occur in the brain as well. Here, though, the fight is not between just two competitors, but between multitudes of cognitive patterns. None ever manages to gain more than a fleeting supremacy, which Rabinovich thinks might explain the familiar experience of the wandering mind. "We can all recognise that thinking is a process," he says. "You are always shifting your attention, step-by-step, from one thought to another through these temporary stable states."

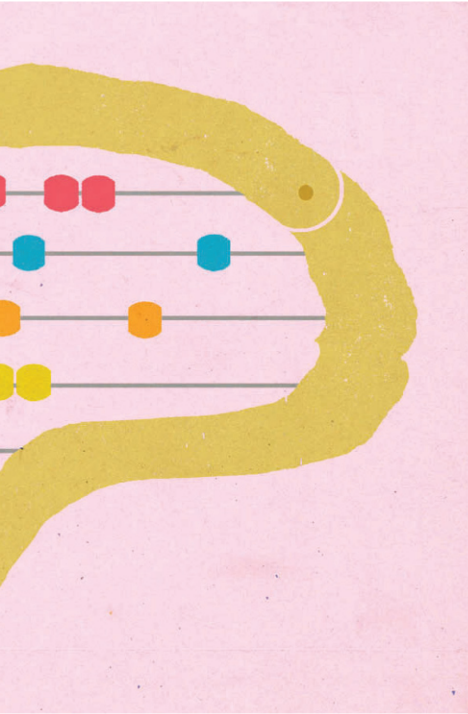


DAN PAGE

People with psychiatric conditions might benefit from the work. In the past, conditions like attention-deficit hyperactivity disorder were studied by looking at the quick snapshots of neural activity. But Rabinovich's work gives neuroscientists a tool to make sense of the brain's responses as they evolve with time, potentially explaining why the attention drifts in unusual ways. Working with Alexander Bystritsky at the University of California in Los Angeles, Rabinovich has already shown that his equations can accurately describe the neuronal activity associated with both ADHD and obsessive compulsive disorder (*Journal of Psychiatric Research*, vol 46, p 428). "They are very convenient for diagnosing the disorders," he says.

ANUP SHAH/NATUREPL.COM

"The competing activity between brain regions resembles the perpetual fight between predator and prey"



THE SUM OF CONSCIOUSNESS

Getting to grips with consciousness may seem like a step into the unknown, or even the unknowable, but Giulio Tononi at the University of Wisconsin-Madison was not daunted.

The first challenge was to find a good definition of consciousness by boiling it down to its most essential elements. He reasoned that each moment of awareness is a fusion of information from all of our senses. An experience's colours, smells and sounds are impossible to isolate from one another, except through deliberate actions such as closing your eyes. At the same time, each conscious experience is a unique, never-to-be-repeated event. In computational terms, this means that a seat of consciousness in the brain does two things: it makes sense of potentially vast amounts of information and, just as importantly, it internally binds this information into a single, coherent picture that differs from everything we have ever – or will ever – experience.

Perhaps the best way to understand this is to consider the difference between the brain and a digital camera. Although the screen seems to show complete image to our eyes, the camera just treats the image as a collection of separate pixels, which work completely independently from one another; it never combines the information to find links or patterns. For this reason, it has very low "integration", and so according to Tononi's theory, it isn't conscious. The brain, on the other hand, is constantly drawing links between every bit of information that hits our senses, which allows us to be aware of what we see.

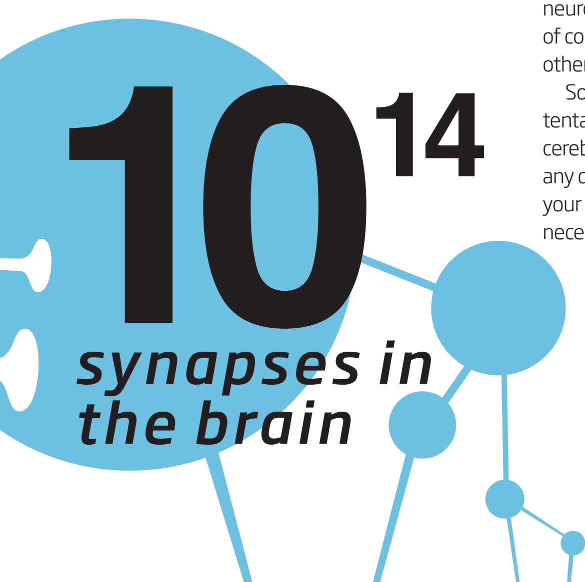
Physicists haven't paid much attention to measuring how much information a physical system can hold on to and integrate, so Tononi worked out the equations himself. The result is a quantity known as "phi". "Now I could go back to neurobiology with this tentative theory: any seat of consciousness must have a high level of phi, and other systems must not," says Tononi.

Some accepted anatomical findings gel with this tentative theory. For instance, we know that the cerebral cortex is crucial for conscious experience – any damage to the brain here will have an effect on your mental life. Conversely, the cerebellum is not necessary for conscious awareness, which was something of a puzzle given that it contains more than twice as many neurons as the cerebral cortex.

When Tononi analysed the two regions using his theory, it all made sense: the cerebral cortex may have fewer neurons, but the cells are very well connected to one another. They can hold large amounts of information and also integrate it to generate a single coherent picture – the level of phi is very high. The cerebellum is more like the digital camera: it may contain more neurons than the cerebral cortex, but there are fewer interconnections and so no coherent picture – the level of phi is low, in other words.

"I've been studying consciousness for 25 years, and Giulio's theory is the most promising," says Christof Koch at the California Institute of Technology in Pasadena. "It's unlikely to be the final word but it goes in the right direction – it makes predictions. It moves consciousness away from the realm of speculative metaphysics."

"An experience's colours, smells and sounds are impossible to isolate from one another"



Lights out

Tononi's theory can also explain what happens when we fall asleep or are given an anaesthetic – through experiments he has shown that the level of phi in the cerebral cortex drops as our consciousness fades away.

This makes sense when we consider all of the ideas emerging from the field of computational neuroscience. The cerebral cortex is home to many of the highly interconnected "rich club" hubs, which may explain why it is so good at integrating incoming information. Neural signals zip freely through these interconnections to generate conscious experiences. Fall asleep, though, and the neural signals within the cerebral cortex slip further away from the critical point vital for neural communication. The physical interconnections remain, but traffic no longer flows through them. The Bayesian brain loses its ability to make sense of the world around it – all of the thoughts engaged in the brain's winnerless competitions fade to black.

The various strands of the computational neuroscience story come together powerfully. Are they the final word in our understanding of the brain? "They're undoubtedly flawed in some way – no one is being naive," says Beggs. Nevertheless, he and others think neuroscience is poised to become a numbers game. "We'll find out in a few years," he says. "In the meantime, it's certainly a fun journey." ■

Colin Barras is a writer based near Ann Arbor in Michigan