Posterior Probability Maps

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Advanced fMRI Course

Human Brain Mapping

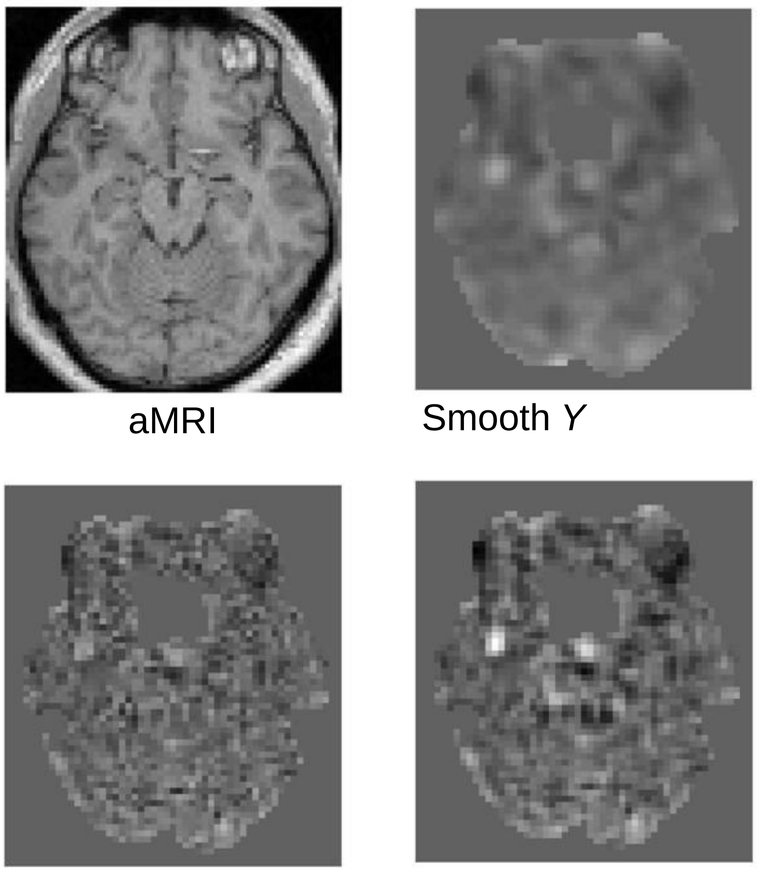
June 2014

Overview

- Parameter Inference
- Model Inference
- Nested Model Inference
- Nonlinear Models

Smoothness Priors

$$Y = X\beta + \varepsilon$$



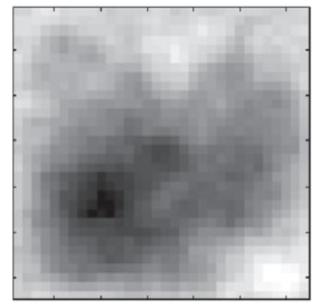
aMRI

Smooth Y

ML

Posterior

$$p(\beta) = N\left(0, \alpha^{-1}L\right)$$



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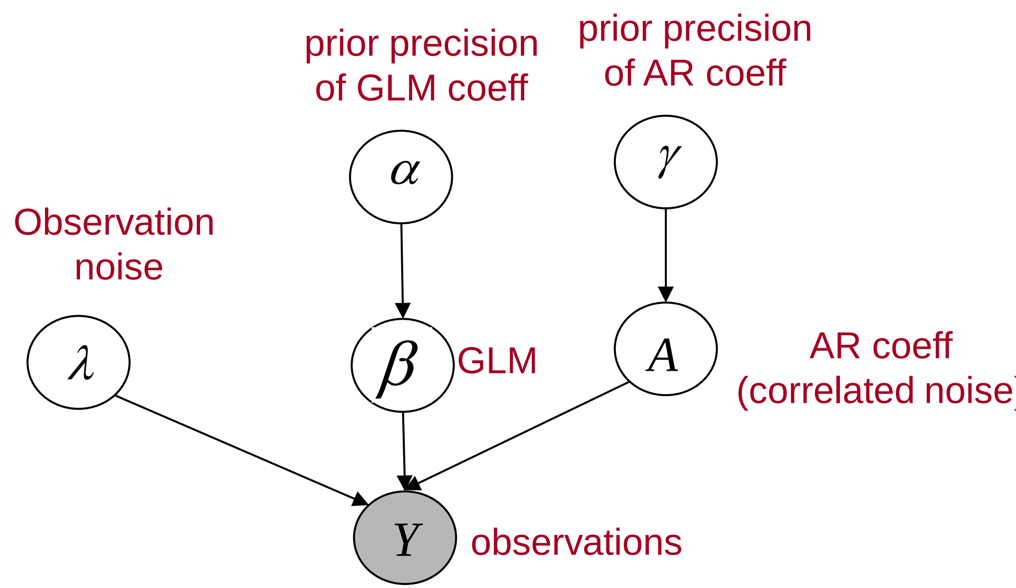
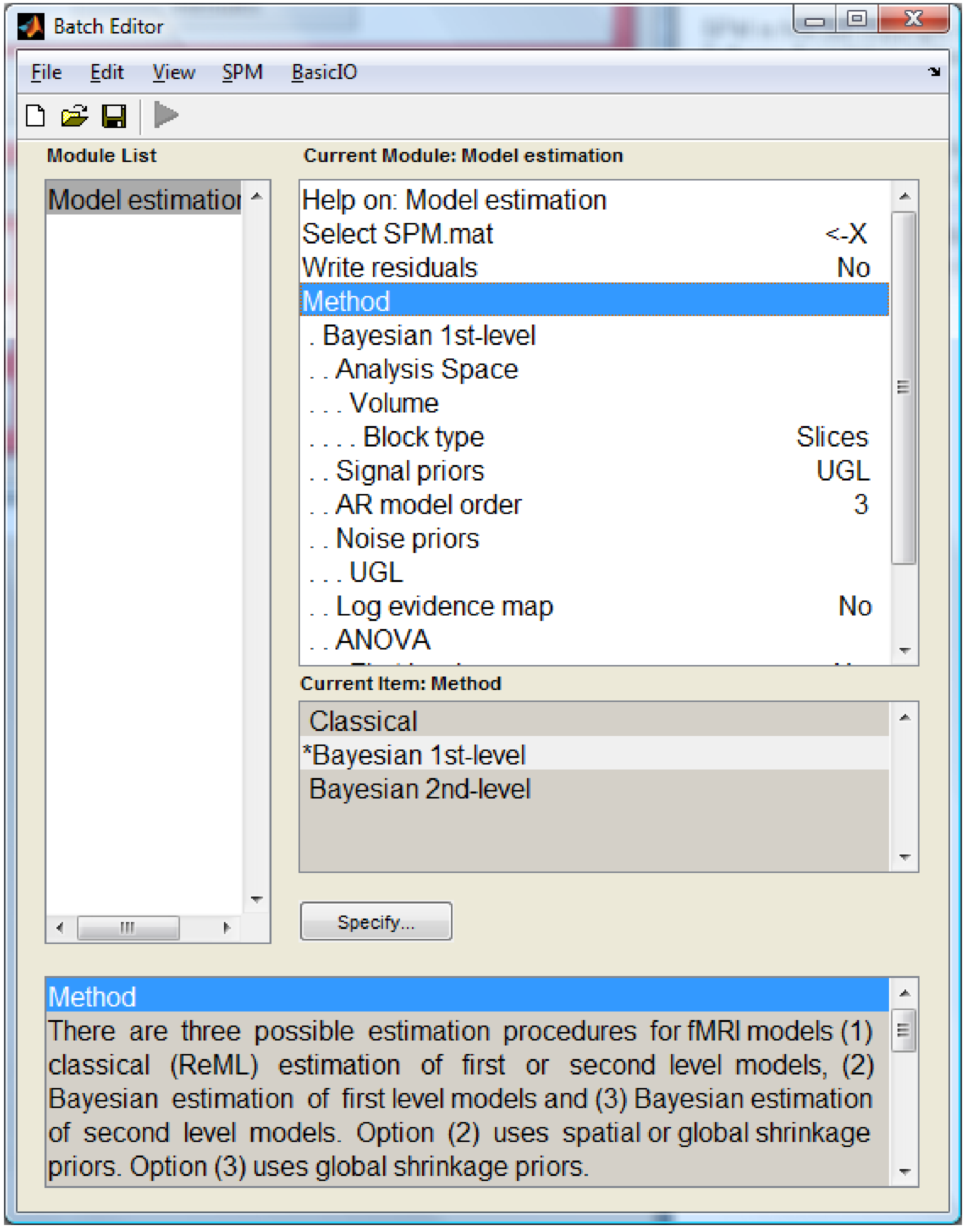
	2	-8	2	
1	-8	20	-8	1
	2	-8	2	
		1		



SPM


Batch Editor
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Module List
Model estimatio
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Method
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. . Analysis Space
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Current Item: Method
Classical
*Bayesian 1st-level
Bayesian 2nd-level
Specify...
Method
There are three possible estimation procedures for fMRI models (1) classical (ReML) estimation of first or second level models, (2) Bayesian estimation of first level models and (3) Bayesian estimation of second level models. Option (2) uses spatial or global shrinkage priors. Option (3) uses global shrinkage priors.

Choice of Priors

Stationary smoothness:

W.D. Penny, N. Trujillo-Barreto, and K.J. Friston. **Bayesian fMRI time series analysis with spatial priors**. *NeuroImage*, 24(2):350-362, 2005.

Nonstationary smoothness:

L M Harrison, W Penny, J Daunizeau, and K J Friston.
Diffusion-based spatial priors for functional magnetic resonance images. *Neuroimage*, 41(2):408-23, 2008.

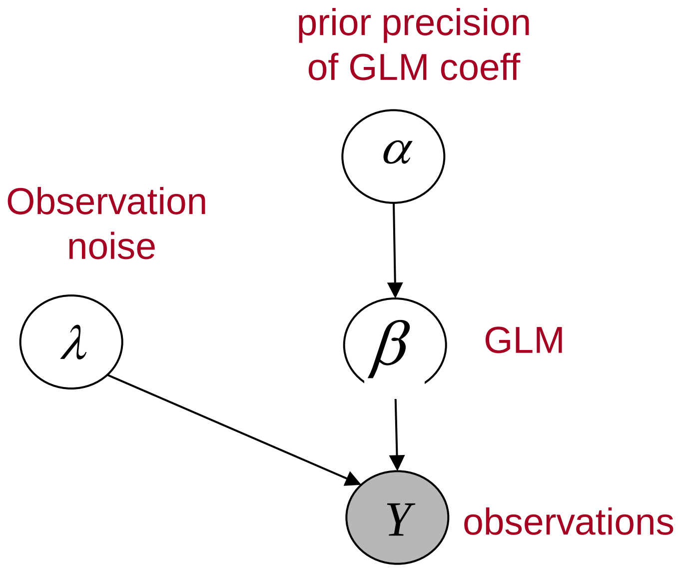
Global Shrinkage:

K.J. Friston and W.D. Penny. **Posterior probability maps and SPMs**. *NeuroImage*, 19(3):1240-1249, 2003.

Global Shrinkage Priors

$$p(\beta) = N\left(0, \alpha^{-1} I\right)$$

$$Y = X\beta + \varepsilon$$



K.J. Friston and W.D. Penny. **Posterior probability maps and SPMs**. *NeuroImage*, 19(3):1240-1249, 2003.

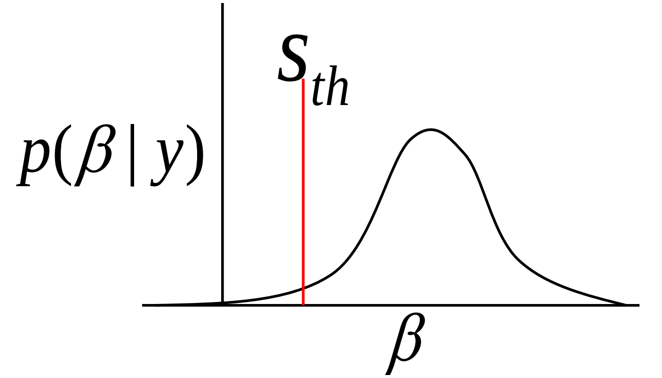
Posterior

Posterior distribution: probability of the effect given the data

$$\boxed{p(\beta \mid y)}$$

Posterior Probability Map: images of the probability that an activation exceeds some specified threshold s_{th}, given the data y

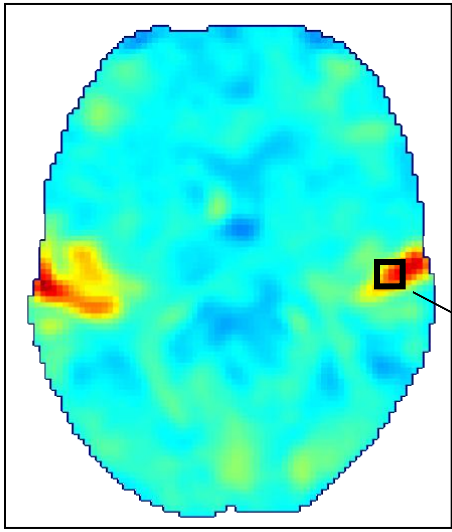
$$\boxed{p(\beta > s_{th} \mid y) > p_{th}}$$



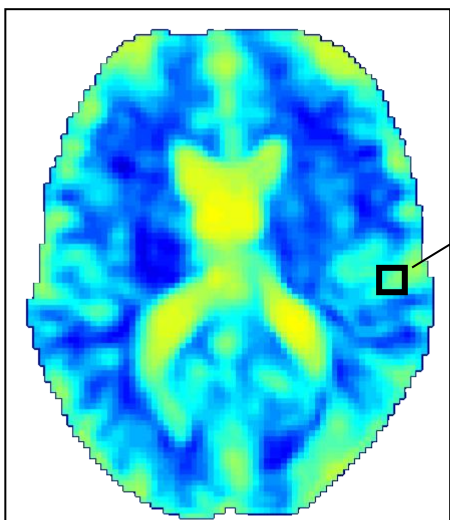
Two thresholds:

- activation threshold s_{th} : percentage of whole brain mean signal (physiologically relevant size of effect)
- probability p_{th} that voxels must exceed to be displayed (e.g. 95%)

PPM

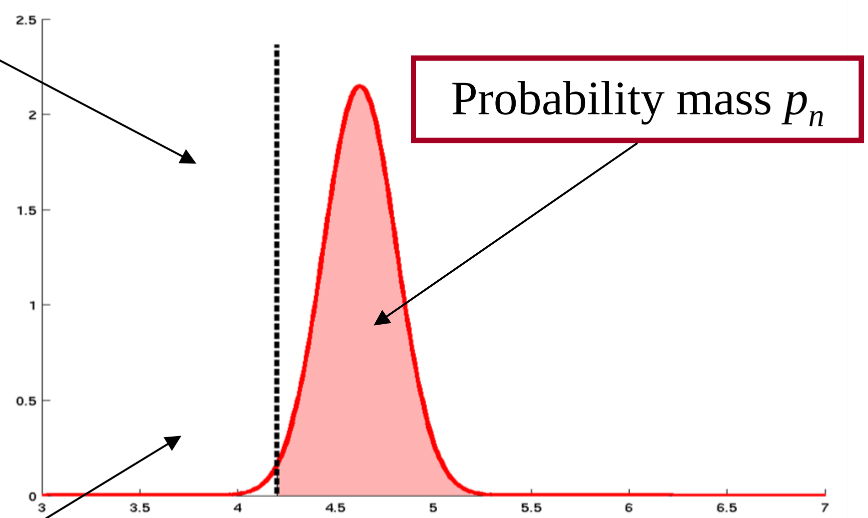


Mean *(Cbeta_*.img)*



Std dev *(SDbeta_*.img)*

activation threshold s_{th}



Posterior density $q(\beta_n)$

probability of getting an effect, given the data

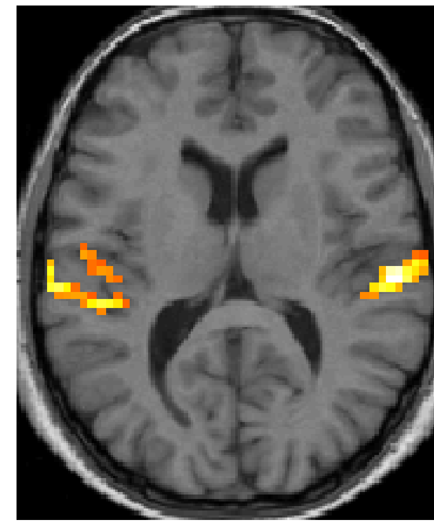
$$q(\beta_n) = N(\mu_n, \Sigma_n)$$

mean: *size of effect*
covariance: *uncertainty*

Display only voxels that exceed e.g. 95%

$$p > p_{th}$$

$$p = q(\beta > s_{th})$$



PPM *(spmP_*.img)*

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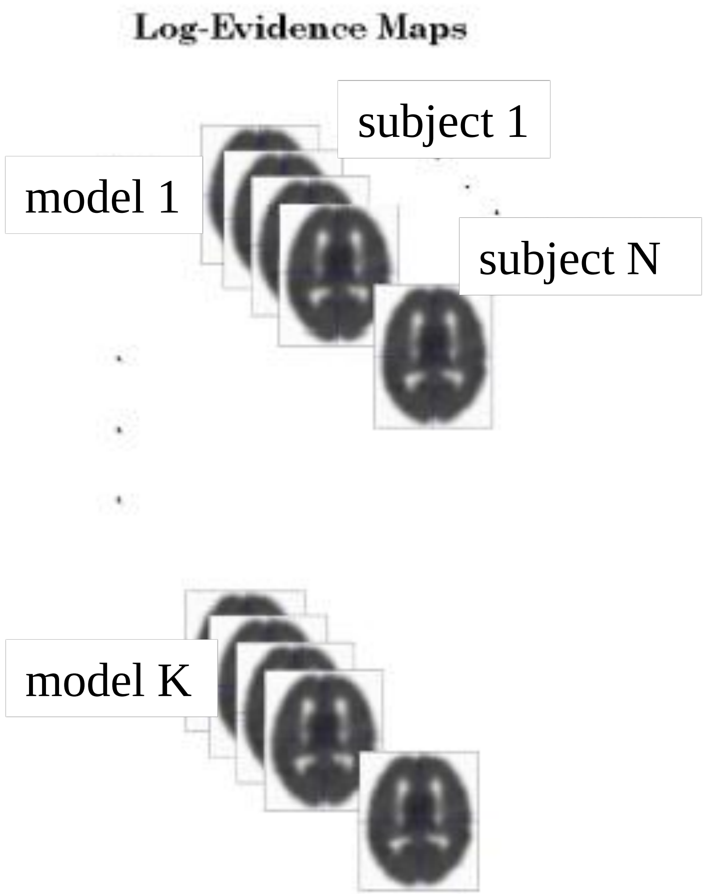
Model Inference

M Rosa, S.Bestmann, L. Harrison, and W Penny. **Bayesian model selection maps for group studies**. *Neuroimage*, Jan 1 2010; 49(1):217-24.



Compute log-evidence map for each model/subject

Model Inference

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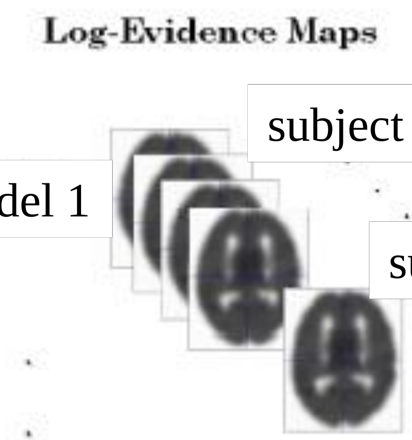


subject 1

model 1

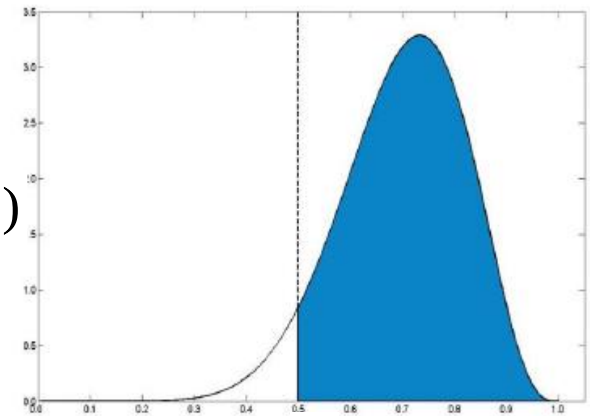
subject N

model K

Compute log-evidence map for each model/subject

$q(r_k > 0.5) = 0.941$

$q(r_k)$

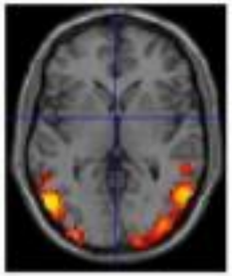


r_k

Probability that model *k* generated data

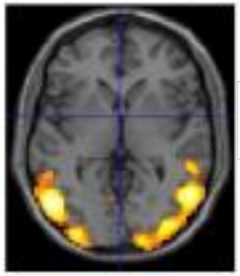
BMS maps

$\langle r_k \rangle > \gamma$



PPM

$\varphi_k > \gamma$



EPM

model *k*

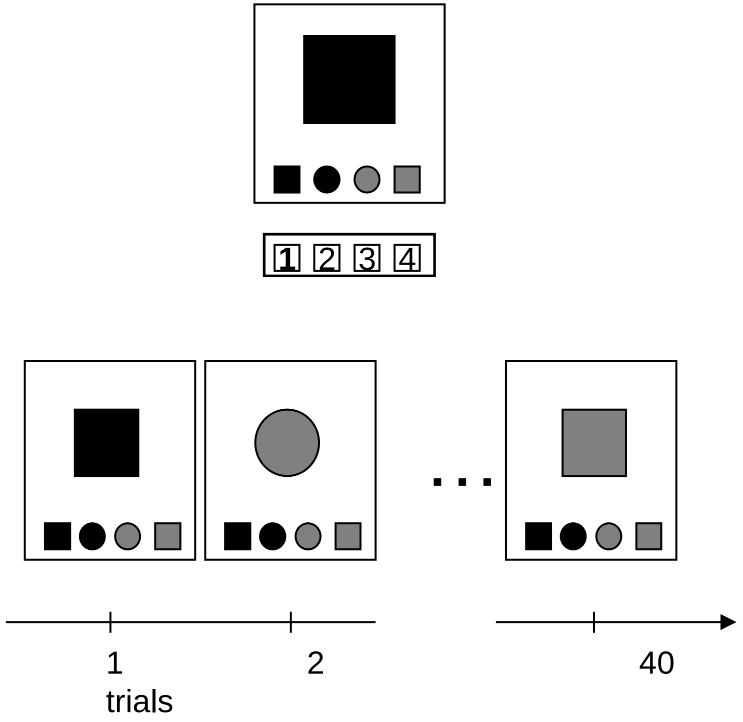
Computational fMRI

Subjects pressed 1 of 4 buttons depending on the category of visual stimulus.

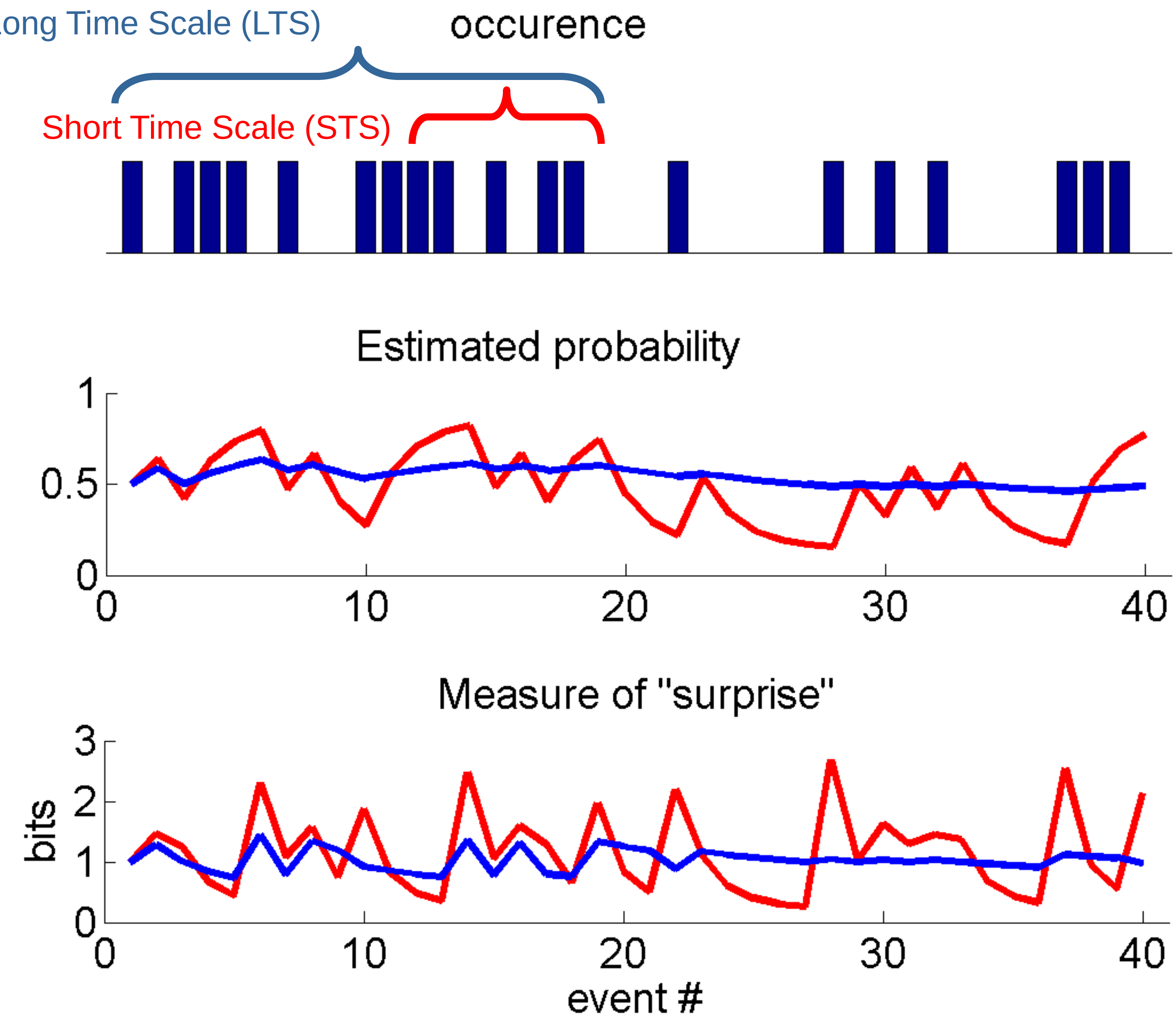
The 4 categories of stimuli occurred with different frequencies over a session.

Brain responses are then hypothesised to be proportional to the surprise, *S*, associated with each stimulus where *S=log(1/p)*.

But over what time scale is the probability *p* estimated ? And do different brain regions use different time scales ?

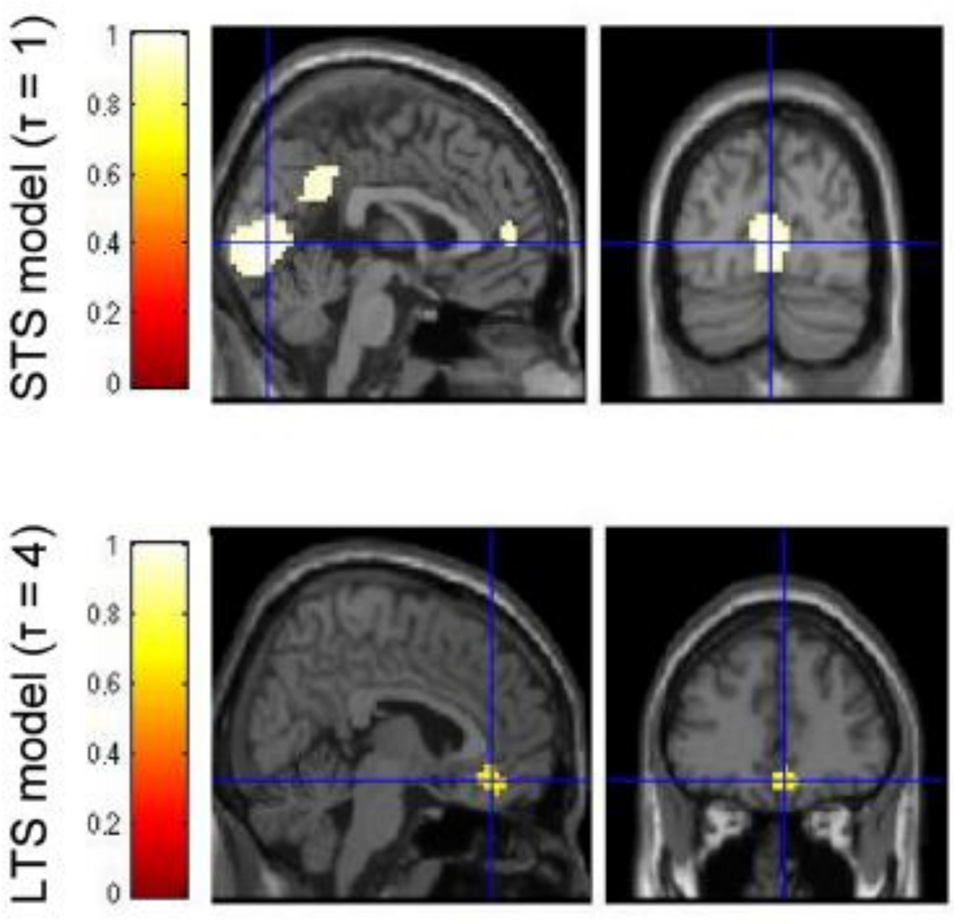


L. Harrison, S Bestmann, M. Rosa, W. Penny and G. Green (2011). **Time scales of representation in the human brain: weighing past information to predict future events.** *Frontiers in Human Neuroscience*, 5, 00037.



Enter surprise as a Parametric Modulator in first level GLM analysis.
Which surprise variable (STS or LTS) underlies the best model of fMRI responses?

Exceedance Probability Maps



L. Harrison, S Bestmann, M. Rosa, W. Penny and G. Green (2011). **Time scales of representation in the human brain: weighing past information to predict future events.** *Frontiers in Human Neuroscience*, 5, 00037.

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Nested Model Inference

Bayesian equivalent of inference using F-tests implemented using Savage-Dickey approximations to the log Bayes Factor.

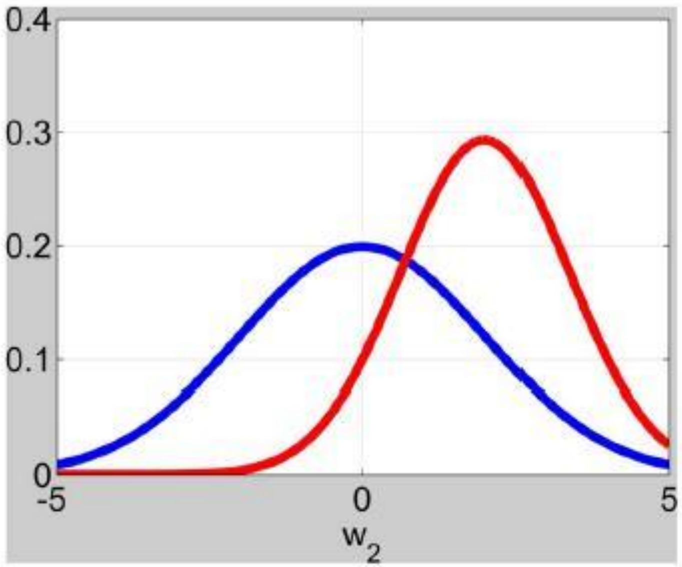
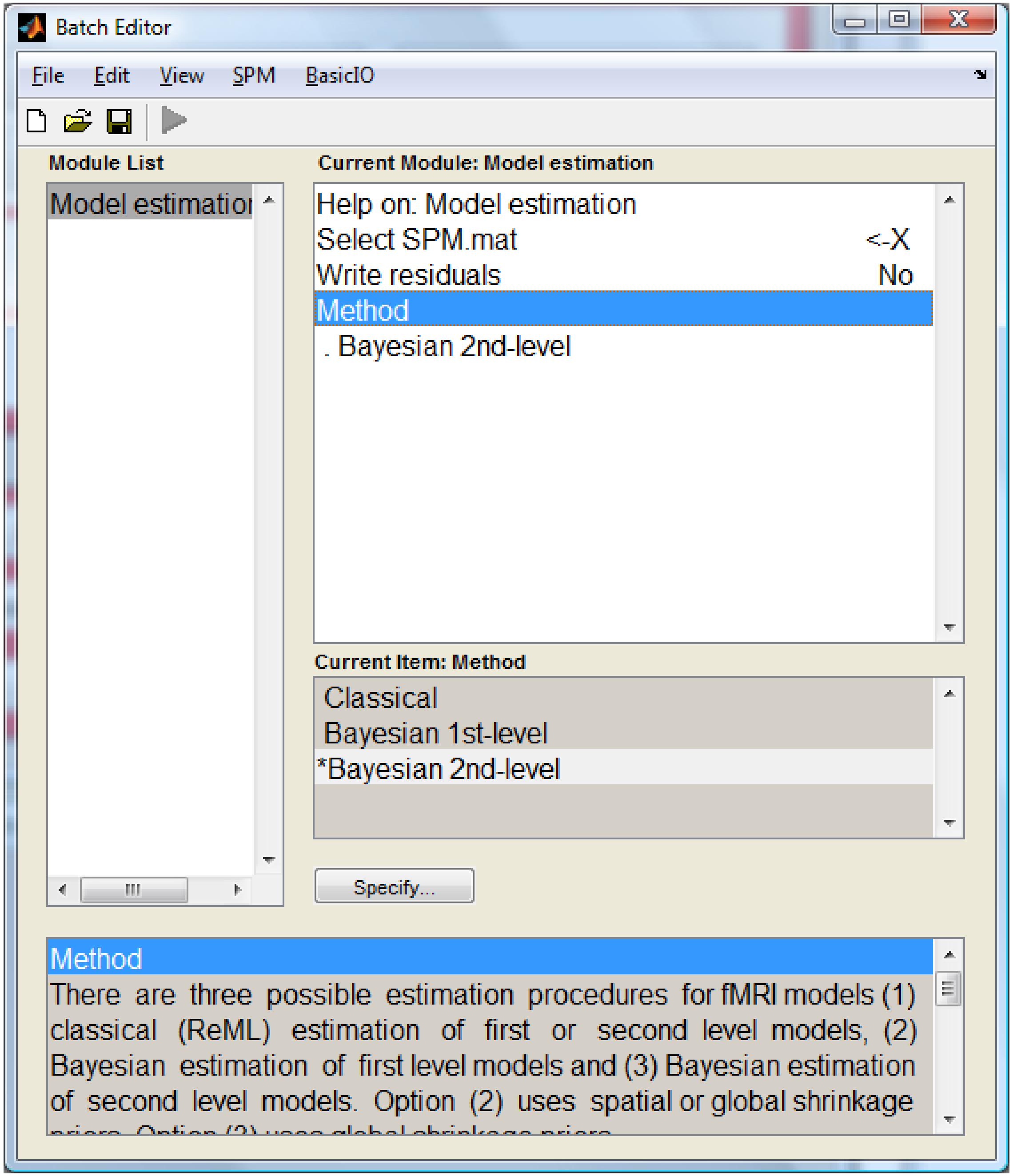


Figure 1. The figure shows the prior density $p(w_2|m_2)$ in blue and the posterior density $p(w_2|m_2,y)$ in red. Here $BF_{12}=0.5$, weakly favouring the more complex model m_2, since the parameter w_2 is half as likely to be zero after seeing the data than before.

W. Penny and G. Ridgway (2013). **Efficient Posterior Probability Mapping using Savage-Dickey Ratios.** *PLoS One* 8(3), e59655


Batch Editor
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Module List
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Help on: Model estimation
Select SPM.mat
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Write residuals
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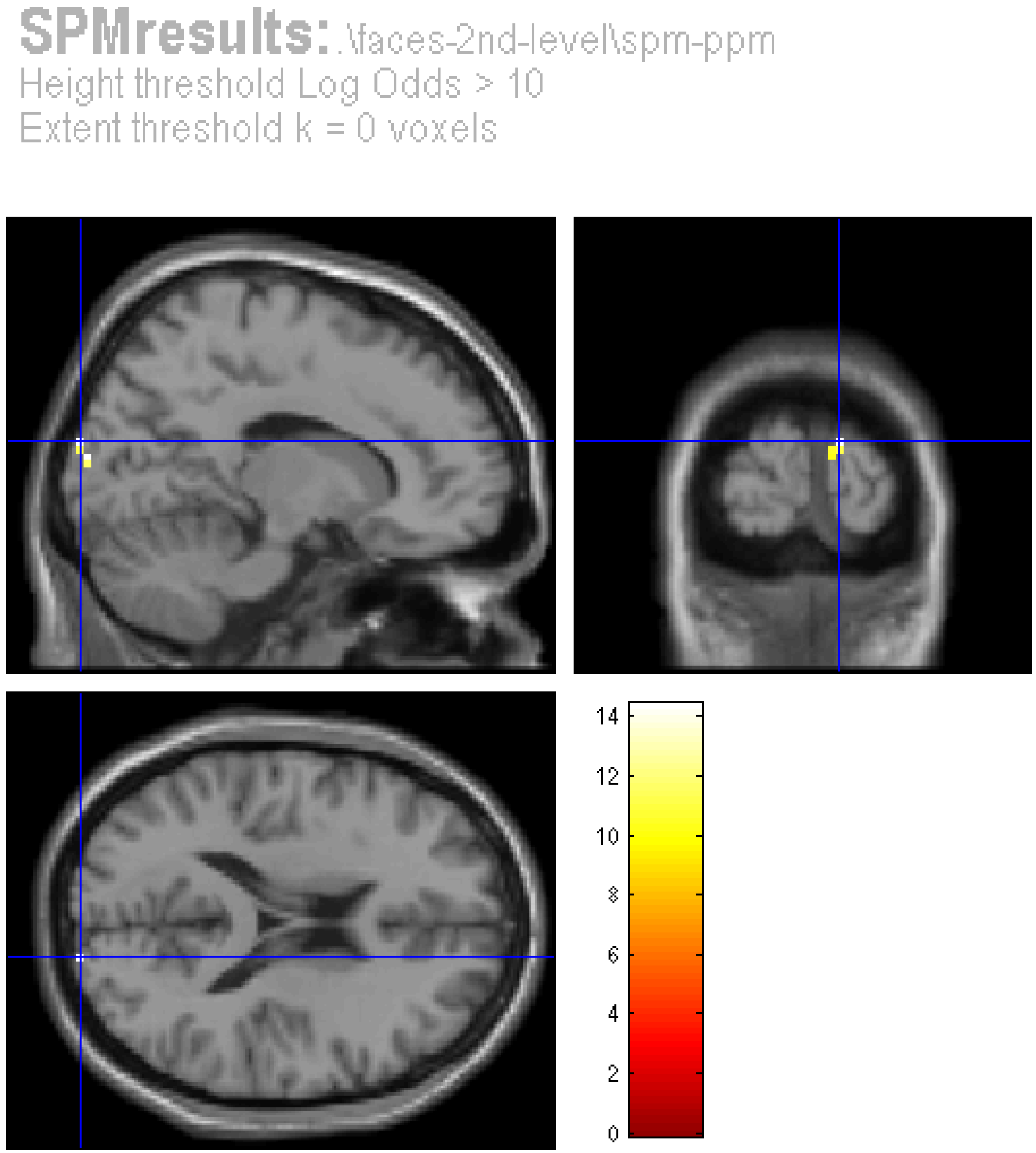
Method
. Bayesian 2nd-level
Current Item: Method
Classical
Bayesian 1st-level
*Bayesian 2nd-level
Specify...
Method
There are three possible estimation procedures for fMRI models (1) classical (ReML) estimation of first or second level models, (2) Bayesian estimation of first level models and (3) Bayesian estimation of second level models. Option (2) uses spatial or global shrinkage

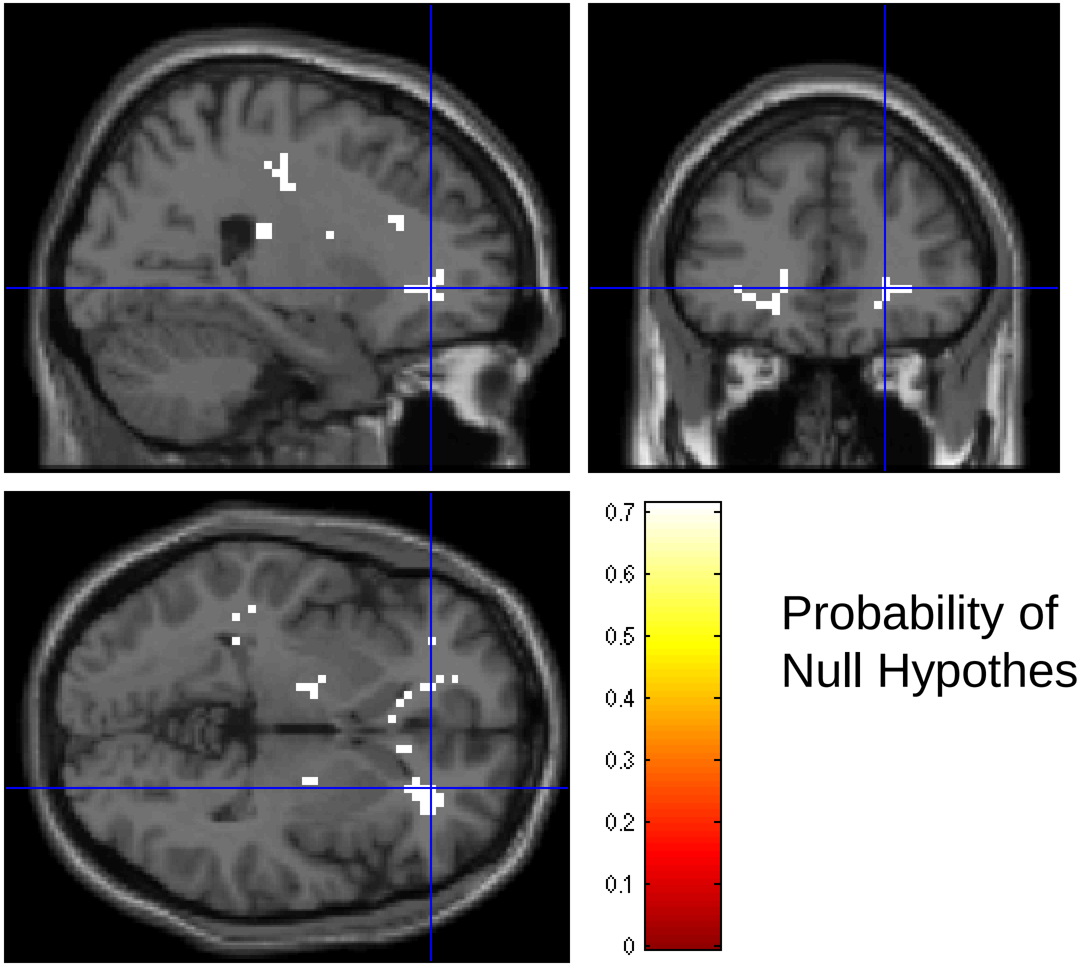
Faces versus scrambled faces



RFX analysis on 18 subjects.

Data from Rik Henson.

Faces versus scrambled faces: Evidence for Null



Probability of
Null Hypothesis

Using command line call to *spm_bms_test_null.m*

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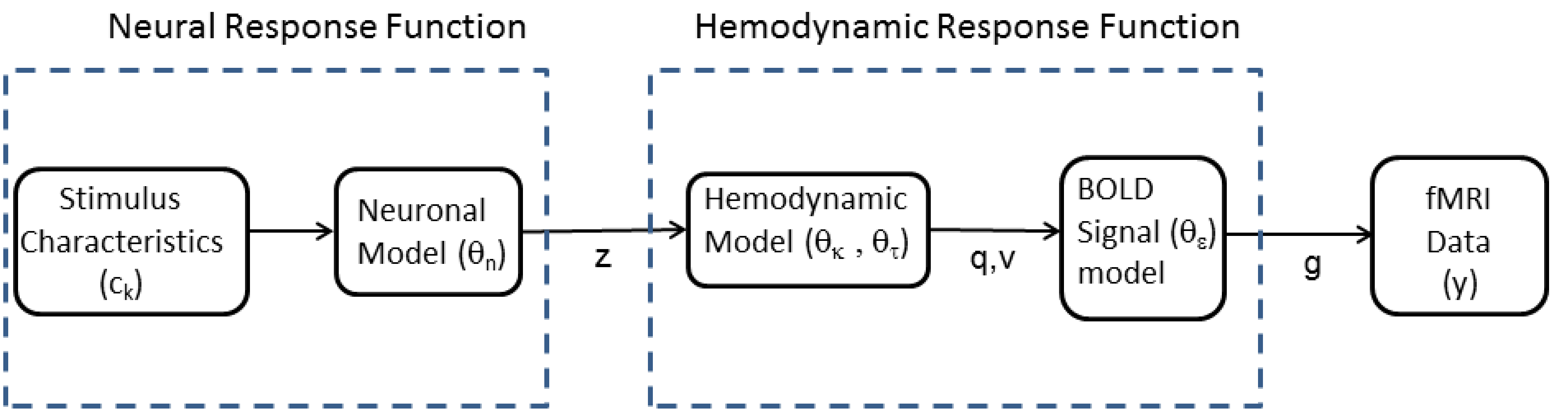
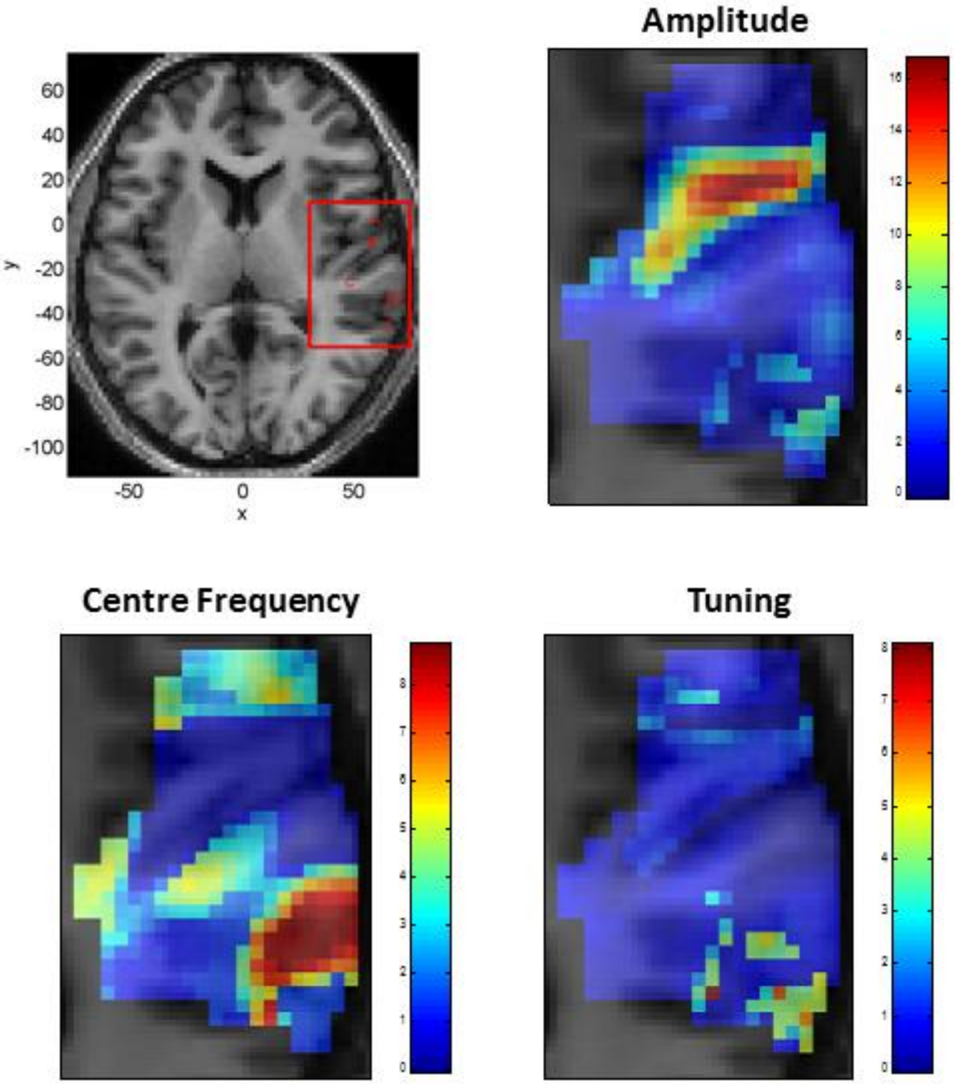
Nonlinear Models

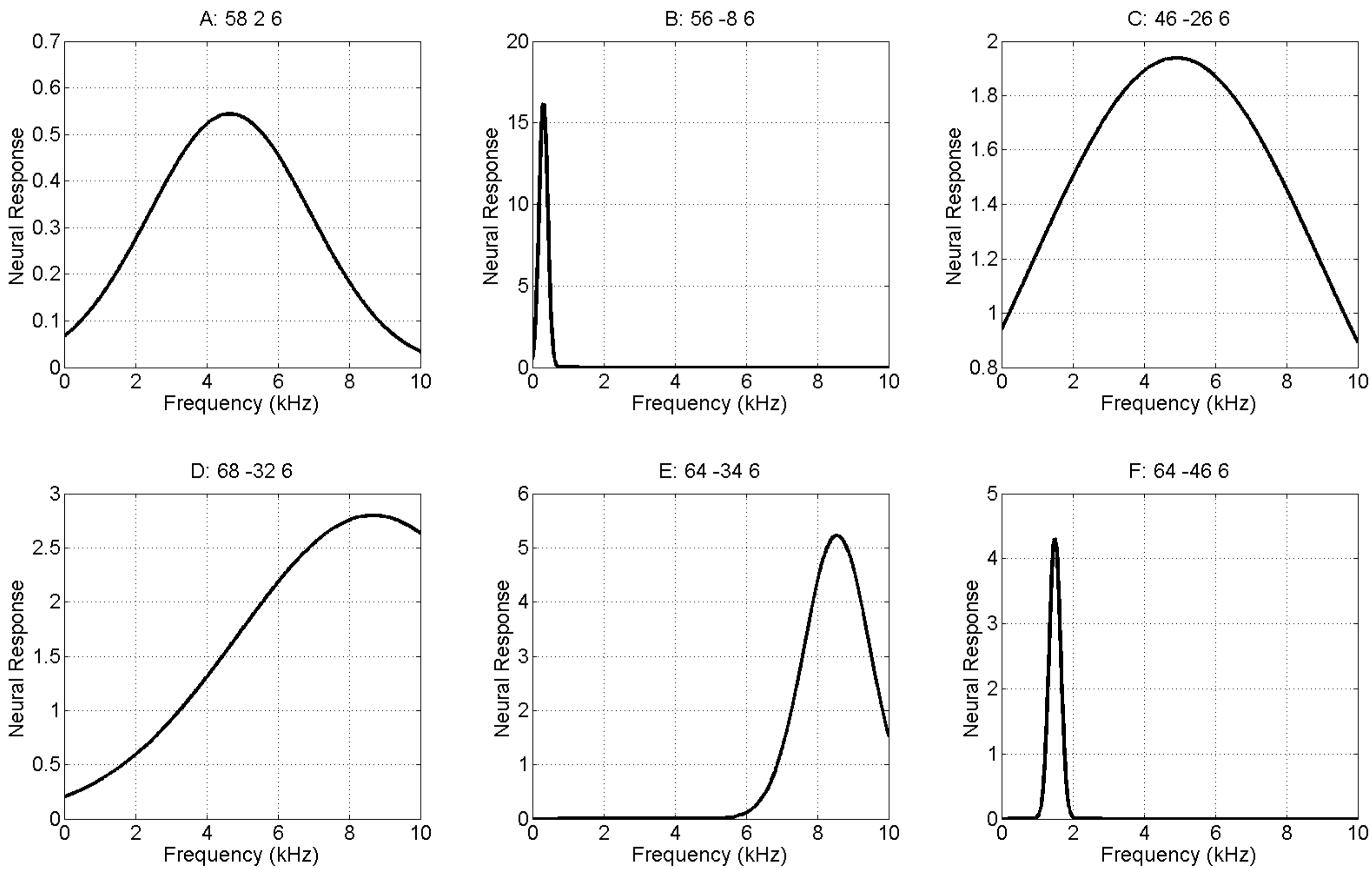


S. Kumar and W. Penny (2014).
Estimating Neural Response Functions from fMRI. *Frontiers in Neuroinformatics, 8th May, doi: 10.3389/fninf.2014.00048.*

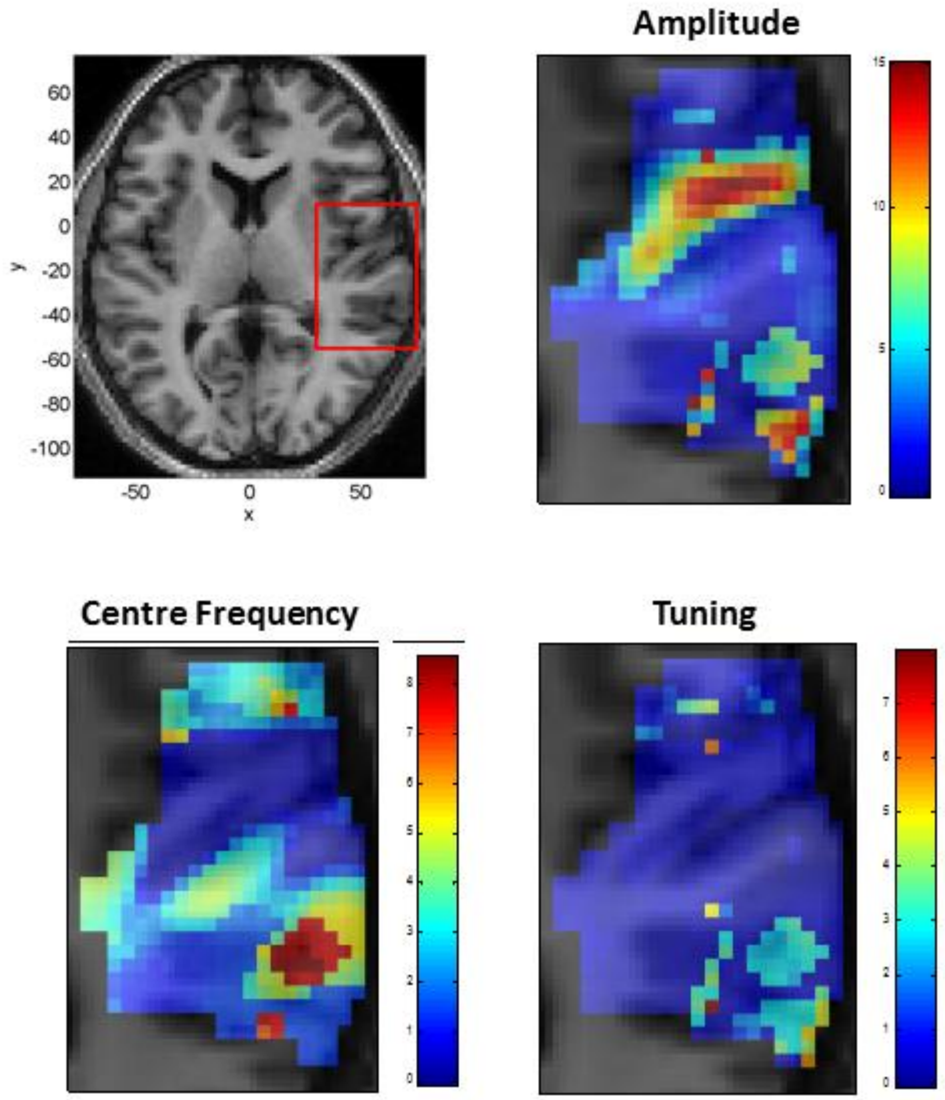
Gaussian Population Receptive Fields



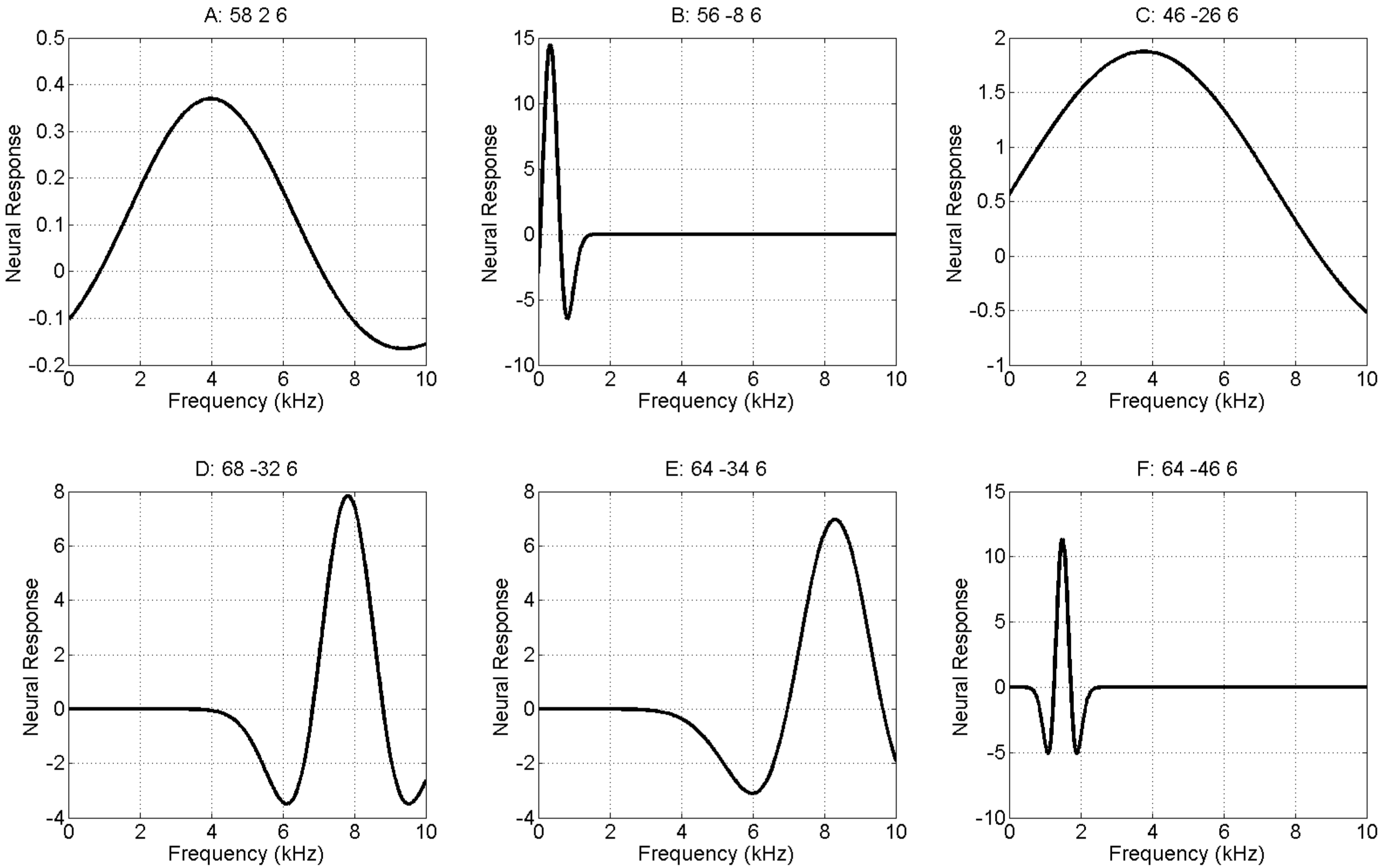
Gaussian Population Receptive Fields



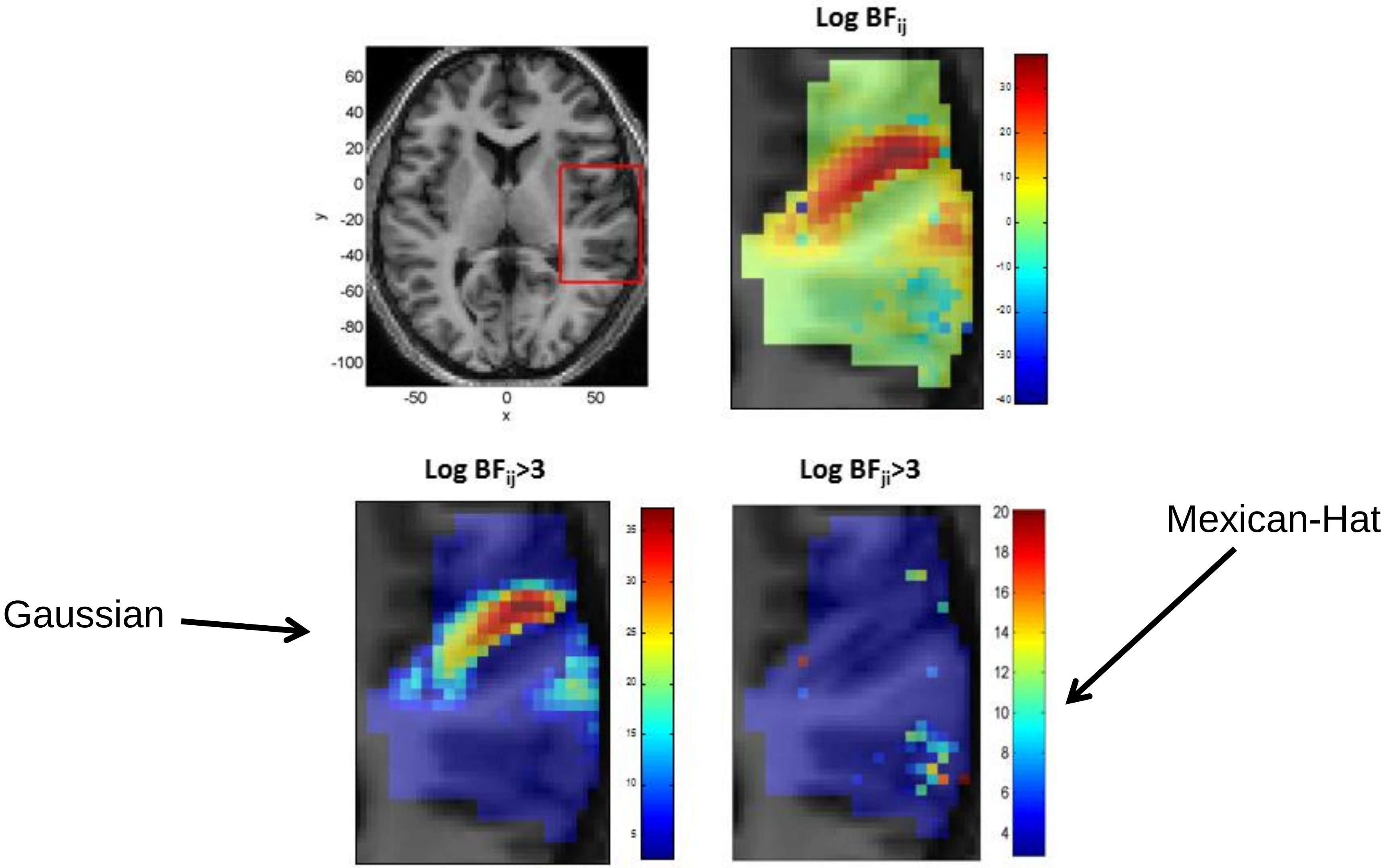
Mexican-Hat Population Receptive Fields



Mexican-Hat Population Receptive Fields



Which Parametric Function is a Better Descriptor ?



Summary

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