Comparing Families of Dynamic Causal Models

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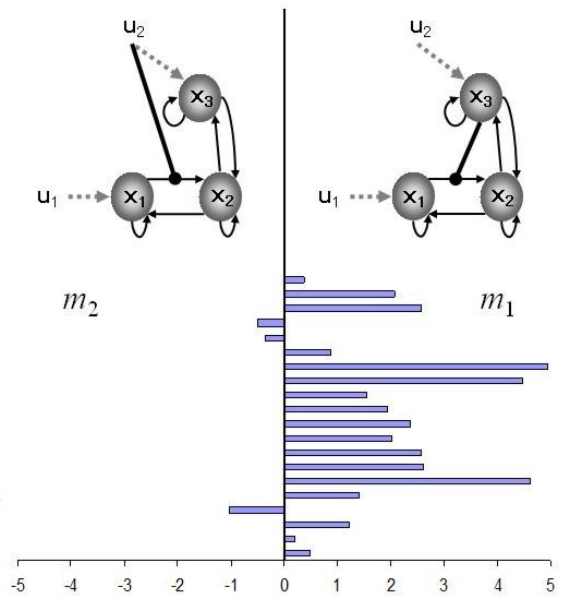
Figure 1: **Fixed Effects Model Inference** Two models, twenty subjects.
$\log p(Y|m) = \sum_{n=1}^{N} \log p(y_n|m)$

Bayesian inference at the model level can then be implemented using Bayes rule

$$p(m|Y) = \frac{p(Y|m)p(m)}{\sum_{m=1}^{M} p(Y|m)p(m)}$$

Under uniform model priors, $p(m)$, the comparison of a pair of models, $m = i$ and $m = j$, can be implemented using the Bayes Factor which is defined as the ratio of model evidences

$$BF_{ij} = \frac{p(Y|m = i)}{p(Y|m = j)}$$

This is known as the Group Bayes Factor (GBF). For the example, $BF_{12} = 10^{14}$, $p(m = 1|Y) \approx 1$.

Family related to model level by

$$p(f_k) = \sum_{m \in f_k} p(m)$$

To avoid any unwanted bias in our inference we wish to have a uniform prior at

the family level

$$p(f_k) = \frac{1}{K}$$

Can be implemented by setting

$$p(m) = \frac{1}{KN_k} \forall m \in f_k$$

The posterior distribution over families is then given by summing up the relevant

posterior model probabilities
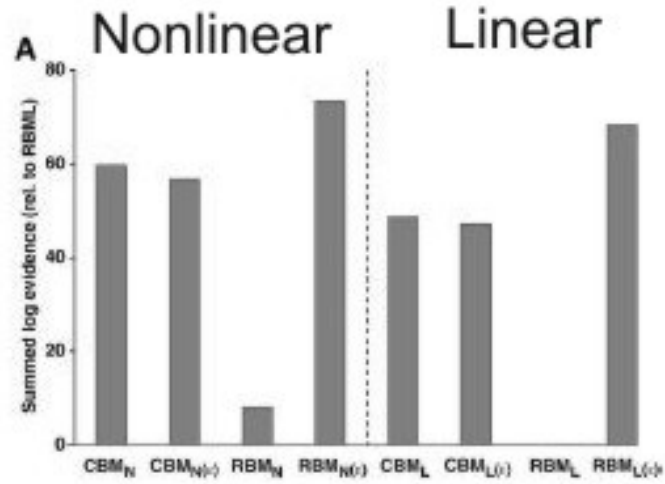
$$p(f_k|Y) = \sum_{m \in f_k} p(m|Y)$$

Figure 2: **Fixed Effects Family Inference** Are hemodynamics linear or nonlinear ? (not interested in whether parameters should be 'revised or classic' or whether ϵ - ratio of intra-to-extra vascular signal changes - should be fixed or estimated). So integrate these factors out ie. add up evidences for family, f_k. Sum of log model evidences for linear $\approx 50 + 50 + 0 = 70$. Sum of log model evidence for nonlinear $\approx 60 + 50 + 10 + 75 = 195$. So, $\log BF_{f_{nonlinear}, f_{linear}} = 25$, $p(f_{nonlinear}|Y) \approx 1$.
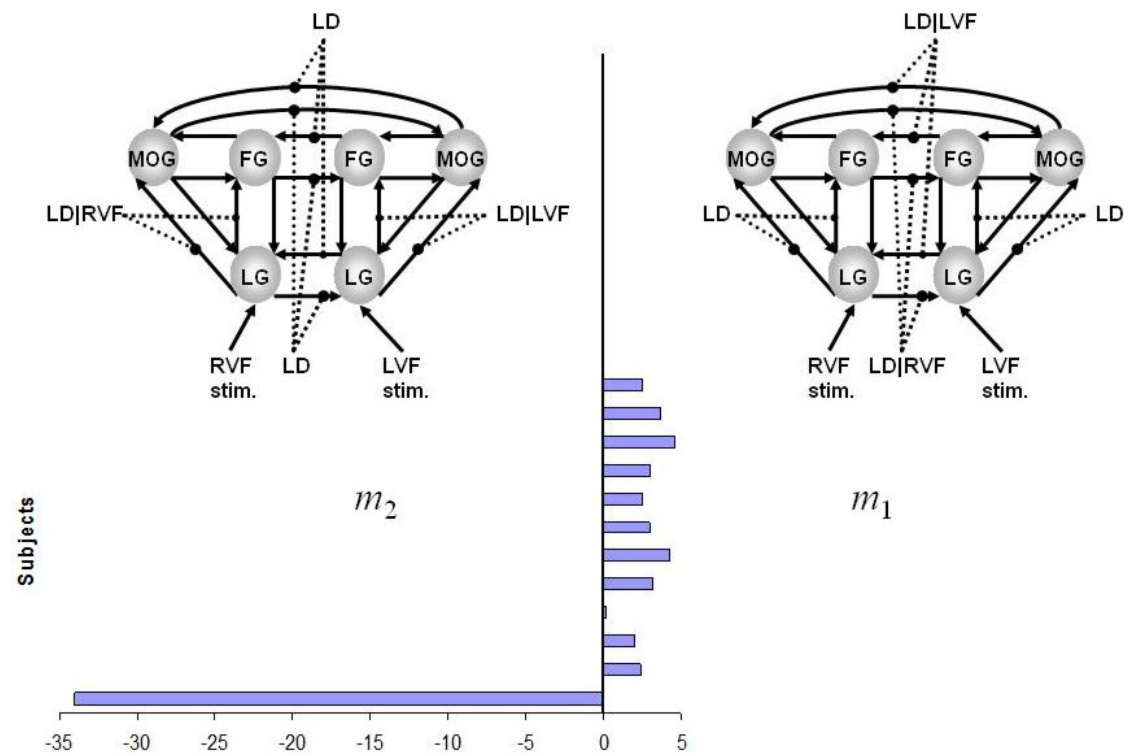
Figure 3: **Random Effects Model Inference ?** First $11/12 = 0.92$ subjects favour model 1. Subject 12 data favours model 2 (by 10 a factor of 10 more than the others favour model 1). Make inference about proportion of subjects, r_m, that use model m.
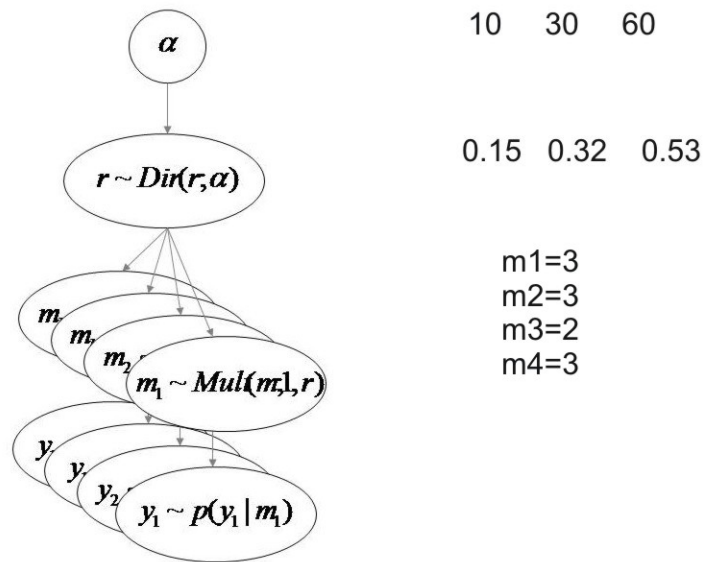
Figure 4: **Random Effects Model Inference** Make inference about proportion of subjects, r_m, that use model m.

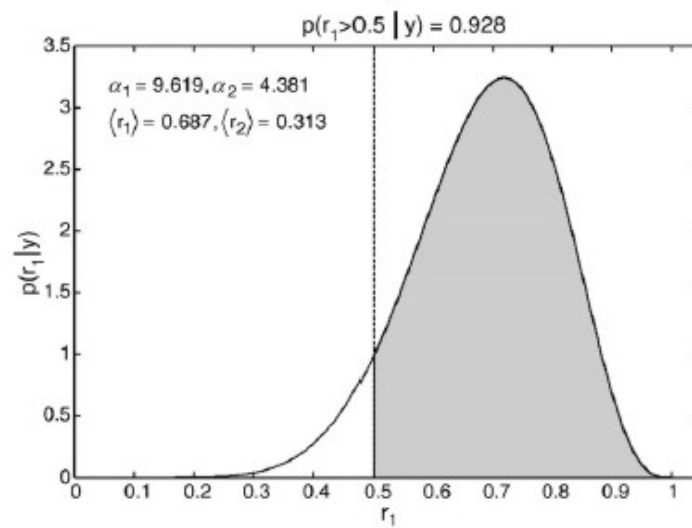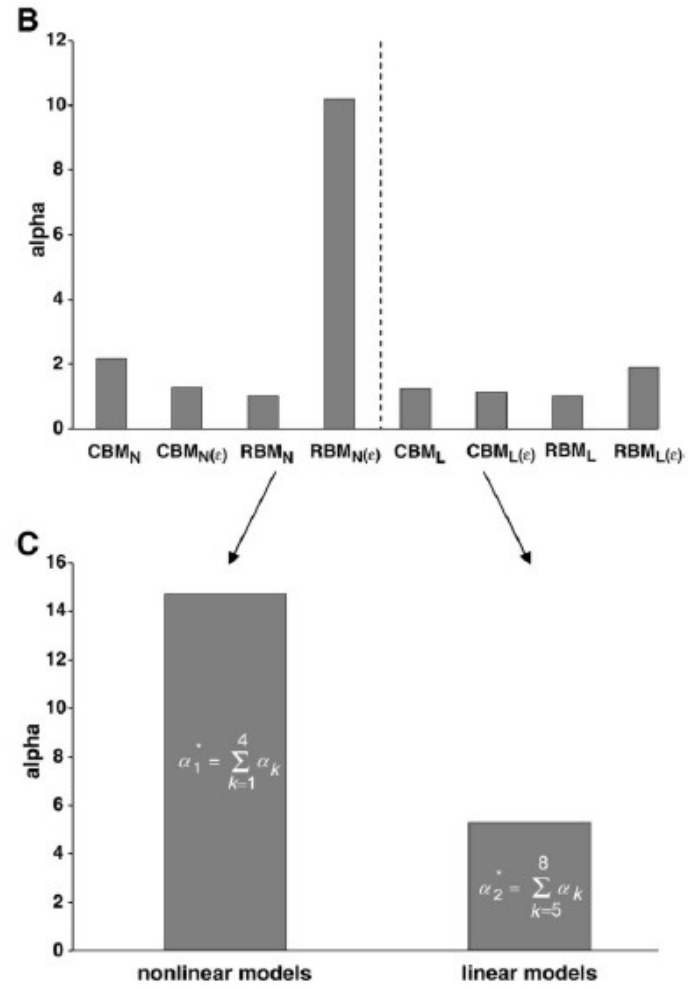Figure 5: **Random Effects Model Inference**

Figure 6: **Random Effects Family Inference**

The family probabilities are given by

$$s_k = \sum_{m \in f_k} r_m$$

where s_k is the frequency of the family of models in the population. We define a prior distribution over this probability using a Dirichlet density

$$p(s) = \mathsf{Dir}(\gamma)$$

A uniform prior over family probabilities can be obtained by setting $\gamma_k = 1$ for all k. This can be achieved by setting
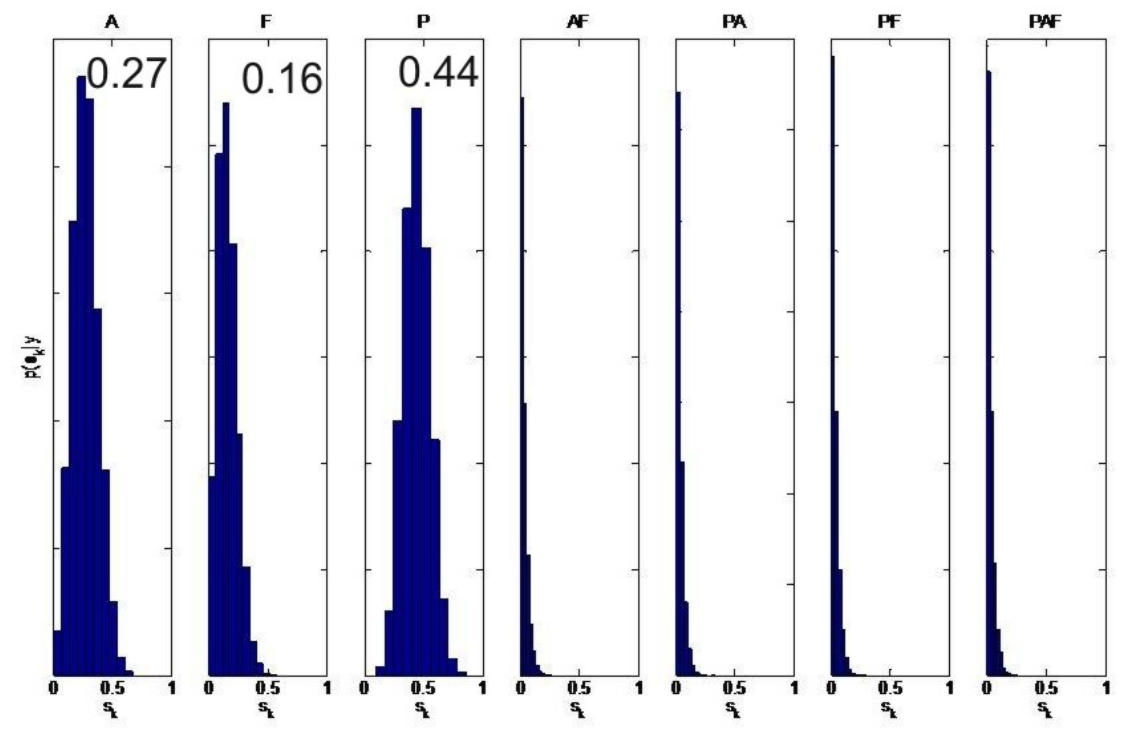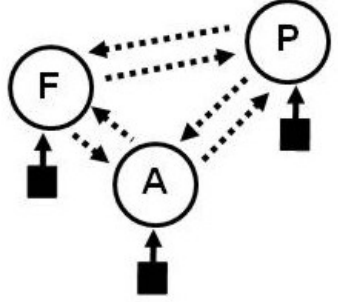
$$\alpha_{prior}(m) = \frac{1}{N_k} \forall m \in f_k$$

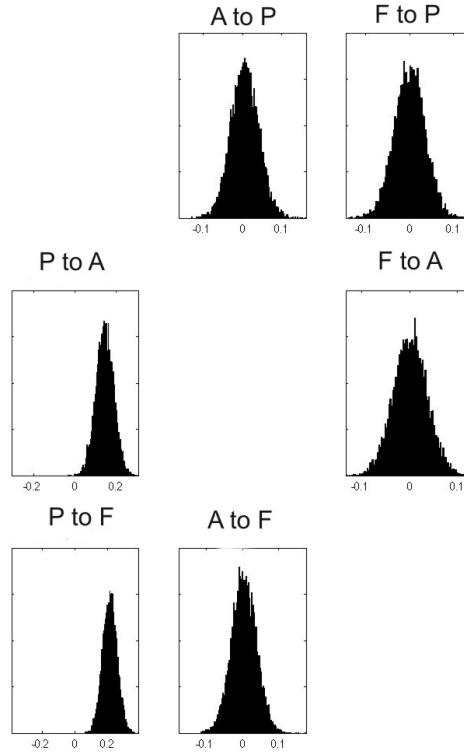Figure 7: **Random Effects Family Inference**

Figure 8: **Bayesian Model Averaging over Input Family P (and subjects)** $p(\theta_n|Y, m \in f_k) = \sum_{m \in f_k} q(\theta_n|y_n, m)p(m_n|Y)$ where $q(\theta_n|Y, m) \approx p(\theta_n|Y, m)$ is our variational approximation to the model specific posterior and $p(m_n|Y)$ is the posterior probability that subject n uses model m. We could take this to be $p(m_n|Y) = p(m|Y)$ under the FFX assumption that all subjects use the same model, or $p(m_n|Y) = g_{nm}$ under the RFX assumption that each subject uses their own model.

Occam's window

Models with low probability contribute little to the estimate of the marginal density. This property can be made use of to speed up the implementation of BMA by excluding low probability models from the summation. This can be implemented by including only models for which

$$\frac{p(m_{MAP}|Y)}{p(m|Y)} \leq \pi_{OCC}$$

where π_{OCC} is the maximal posterior odds ratio. Models satisfying this criterion are said to be in Occam's window. The number of models in the window, N_{OCC}, is a useful indicator as smaller values correspond to peakier posteriors. In this paper we use $\pi_{OCC} = 20$. We emphasise that the use of Occam's window is for computational expedience only.
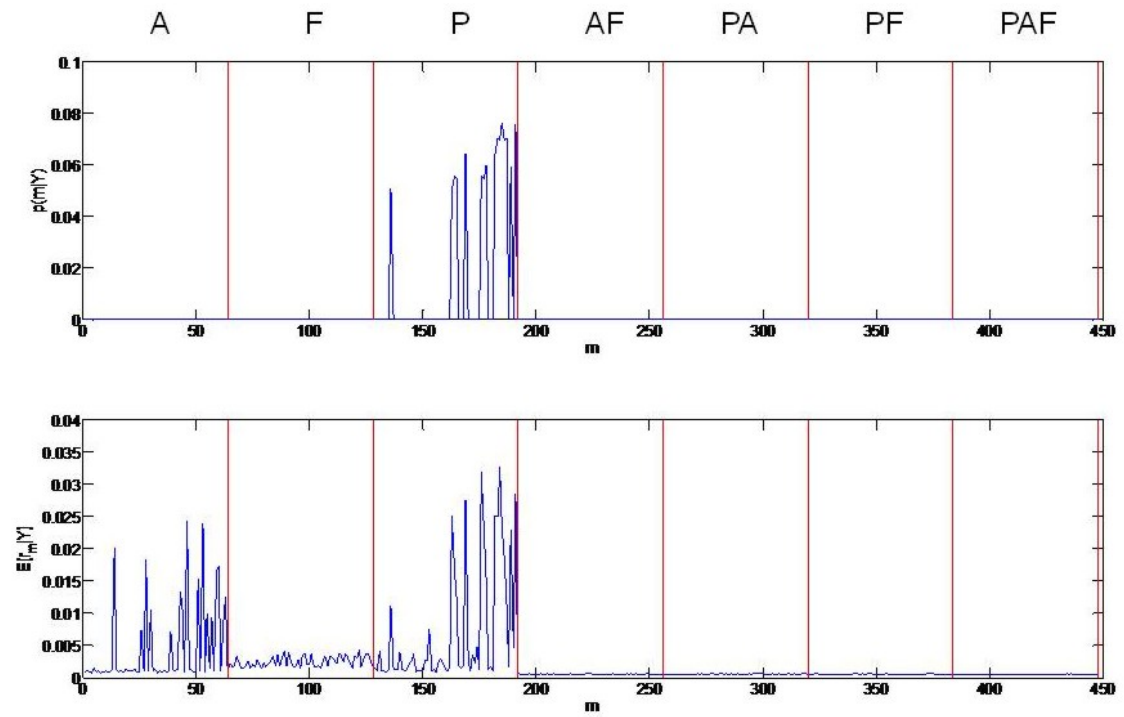
Figure 9: **Model Posteriors**