To be Bayesian or Frequentist or Not: A Debate on Functional Imaging Analyses

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I. Inferences about Parameters

Model, $m$

Parameters, $\beta$

Data, $y$

Bayes rule:

\[
p(\beta | y, m) = \frac{p(y | \beta, m) p(\beta | m)}{p(y | m)}
\]

Likelihood \hspace{1cm} Prior

Model Evidence

Efficient Computation using \textit{Approximate Inference} methods (Bishop, 2007)

For example, a GLM: $y = X \beta + e$
fMRI Analysis with Spatial Priors

Spatial Priors:

1. Empirical Bayes: parameters of prior estimated from data

2. Spatial scale of effects can be automatically estimated

3. Can be different for eg. main effect versus interaction

4. Can be different in eg. visual cortex vs. amygdala

Increased sensitivity
Posterior Probability Maps

Effect size threshold

\( S_{th} \)

Probability of getting an effect, given the data

\[ p(w_n \mid y, m) \]

Active > Rest

Overlay of effect sizes at voxels where we are 99% sure that the effect size is greater than 2% of the global mean

II. Inferences about Models

• In **model-based fMRI** (O’Doherty etc.) signals derived from a computational model for a specific cognitive process are correlated against fMRI data from subjects performing a relevant task to determine brain regions showing a response profile consistent with that model.

• For example, reinforcement learning models fitted to behavioural data producing subjective estimates of ‘value’, ‘prediction error’ …

• But which models are correct?
II. Inferences about Models

Bayes Rule (parameter level):

\[ p(\beta \mid y, m) = \frac{p(y \mid \beta, m) p(\beta \mid m)}{p(y \mid m)} \]

Model evidence:

\[ p(y \mid m) = \int p(y \mid \beta, m) p(\beta \mid m) d\beta \]

\[ \log p(y \mid m) = \text{Accuracy}(m) - \text{Complexity}(m) \]

Bayes Rule (model level):

\[ p(m \mid y) = \frac{p(y \mid m) p(m)}{p(y)} \]
Bayesian Model Selection Maps for Group Studies

Evidence maps

subject 1

model 1

subject N

model K

Compute evidence for each model/subject

Activity best predicted by Ideal Observer model in 70% subjects in these regions

PPM: 'Ideal Observer' model

Maria Joao Rosa et al, 2009: Poster 357, Sunday AM
Summary

- Bayesian inference is computationally efficient
- There are many sources of prior information e.g. spatial smoothness
- Empirical Bayes framework allows parameters of prior to be estimated
- PPMs for inference about effect size
- Model-based fMRI enhanced by model inference
- Same algorithms used for e.g. M/EEG source reconstruction
- Neurobiological priors on connectivity models