1st level analysis
Basis functions, parametric modulation and correlated regressors
First Level Analysis

- Bold impulse response
- Temporal Basis Functions
- Parametric modulation
- Correlated regressors
Blocked design vs. event-related design

Block/epoch designs examine responses to series of similar stimuli

Event-related designs account for response to each single stimulus
Hemodynamic Response Function (HRF)

- Function of blood oxygenation, flow, volume
- Peak (max. oxygenation) 4-6s poststimulus; baseline after 20-30s
- Initial undershoot can be observed
- Similar across V1, A1, S1… but possible differences across:
  - other regions
  - individuals
Hemodynamic Response Function (HRF)

- Long SOA $\rightarrow$ BOLD response returns to baseline, no overlap
- Overlap can be accommodated if the BOLD response is explicitly modelled (linear superposition)
- Short SOAs are more sensitive
General Linear (convolution) model

GLM for a single voxel:

\[ y(t) = u(t) \ast h(\tau) + \epsilon(t) \]

\( u(t) = \) neural causes (stimulus train)

\[ u(t) = \sum \delta(t - nT) \]

\( h(\tau) = \) hemodynamic (BOLD) response

\[ h(\tau) = \sum \beta_i f_i(\tau) \]

\( f_i(\tau) = \) temporal basis functions

\[ y(t) = \sum \sum \beta_i f_i(t - nT) + \epsilon(t) \]

\[ y = X\beta + \epsilon \]
General linear model

Stimulus every 20s

Gamma functions $f_i(\tau)$ of peristimulus time $\tau$ (Orthogonalised)

Sampled every TR = 1.7s
Design matrix, $X$

$[x(t) \otimes f_1(\tau) \mid x(t) \otimes f_2(\tau) \mid \ldots]$
Temporal basis functions
Temporal basis functions

• Fourier Set
  Windowed sines & cosines
  Any shape (up to frequency limit)
  Inference via F-test

• Finite Impulse Response
  Mini “timebins” (selective averaging)
  Any shape (up to frequency limit)
  Inference via F-test
Temporal basis functions

• Gamma Functions
  Bounded, asymmetrical (like BOLD)
  Set of different lags
  Inference via F-test

Two Gamma functions added
Temporal basis functions

- **Gamma Functions**
  - Bounded, asymmetrical (like BOLD)
  - Set of different lags
  - Inference via F-test

- **“Informed” Basis Set**
  - Best guess of canonical BOLD response
  - Variability captured by Taylor expansion
  - “Magnitude” inferences via t-test…?
Temporal basis functions

“Informed” Basis Set (Friston et al. 1998)

Canonical HRF (2 gamma functions)
Temporal basis functions

“Informed” Basis Set
(Friston et al. 1998)

Canonical HRF (2 gamma functions)

plus Multivariate Taylor expansion in:
- time (Temporal Derivative)
Temporal basis functions

“Informed” Basis Set (Friston et al. 1998)

Canonical HRF (2 gamma functions)

plus Multivariate Taylor expansion in:

- time (Temporal Derivative)
- width (Dispersion Derivative)
Design Matrix

3 regressors used to model each condition

The three basis functions are:

1. Canonical HRF
2. Derivatives with respect to time
3. Derivatives with respect to dispersion
Temporal basis functions

- “Informed” Basis Set

  “Magnitude” inferences via t-test on canonical parameters (providing canonical is a reasonable fit)
  “Latency” inferences via tests on ratio of derivative : canonical parameters
Which temporal basis set?

Example: rapid motor response to faces, *Henson et al, 2001*

- Canonical
- + Temporal
- + Dispersion
- + FIR

...canonical + temporal + dispersion derivatives appear sufficient
...may not be for more complex trials (eg stimulus-delay-response)
...but then such trials better modelled with separate neural components (ie activity no longer delta function) + constrained HRF (Zarahn, 1999)
Comparison of the fitted response

Haemodynamic response in a single voxel.

Left: Estimation using the simple model

Right: More flexible model with basis functions
Summary

SPM uses basis functions to model the hemodynamic response using a single basis function or a set of functions.

The most common choice is the `Canonical HRF' (Default in SPM)

By adding the time and dispersion derivatives one can account for variability in the signal change over voxels
Part II:
Correlated regressors
parametric/non-parametric
design
Multicollinearity

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_N x_{iN} + \varepsilon \]

Coefficients reflect an estimated change in \( y \) with every unit change in \( x_i \) while controlling for all other regressors.
Multicollinearity

\[ y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_N x_{iN} + \epsilon \]

\[ x_{i1} = \lambda_0 + \lambda x_{i2} + \nu \]

- low variance of \( \nu \)
- high variance of \( \nu \)

(e.g. age) (e.g. chronic disease duration)
Multicollinearity and estimability

OLS minimizes $e$ by

$$Xe = 0$$

with

$$e = Y - (X\beta_{\text{estim}})^{-1}$$

which gives

$$\beta_{\text{estim}} = (X^TX)^{-1}X^TY$$

cf covariance matrix

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high multicollinearity (i.e. variance of $\nu$ small)

$\Rightarrow$ inaccuracy of individual $\beta_{\text{estim}}$, high standard error

perfect multicollinearity (i.e. variance of $\nu = 0$)

$\Rightarrow det(X) = 0$

$\Rightarrow (X^TX)$ not invertible

$\Rightarrow \beta_{\text{estim}}$ not unique
Multicollinearity

(t- and [unidimensional] F-) testing of a single regressor (e.g. $R_1$) = testing for the component that is not explained by (is orthogonal to) the other/the reduced model (e.g. $R_2$)

$\Rightarrow$ multicollinearity is contrast specific

$\Rightarrow$ “conflating” correlated regressors by means of (multidimensional) F-contrasts permits assessing common contribution to variance

$(X_i \beta_{estim} = projection of Y_i onto X space)$
Multicollinearity

(relatively) little spread after projection onto

x-axis, y-axis or
f(x) = x

reflecting reduced efficiency for detecting dependencies of the observed data on the respective (combination of) regressors

(MRC CBU Cambridge, http://imaging.mrc-cbu.cam.ac.uk/imaging/DesignEfficiency)
Orthogonality matrix reflects the cosine of the angles between respective pairs of columns.

(SPM course Oct. 2010, Guillaume Flandin)
Orthogonalizing

leaves the parameter estimate of $R_1$ unchanged but alters the estimate of the $R_2$ parameter

assumes unambiguous causality between the orthogonalized predictor and the dependent variable by attributing the common variance to this one predictor only

hence rarely justified
Dealing with multicollinearity

- Avoid. (avoid dummy variables; when sequential scheme of predictors (stimulus – response) is inevitable: inject jittered delay (see B) or use a probabilistic $R_1$-$R_2$ sequence (see C))

- Obtain more data to decrease standard error of parameter estimates

- Use F-contrasts to assess common contribution to data variance

- Orthogonalizing might lead to self-fulfilling prophecies

(MRC CBU Cambridge, http://imaging.mrc-cbu.cam.ac.uk/imaging/DesignEfficiency)
Parametric vs. factorial design

Widely-used example
(Statistical Parametric Mapping, Friston et al. 2007)

Four button press forces
Parametric vs. factorial design

Which – when?

Limited prior knowledge, flexibility in contrasting beneficial ("screening"):

Large number of levels/continuous range:
Resources

- Slides from Methods for Dummies 2011
- Rik Henson Short SPM Course slides
- SPM 2012 Course
- SPM Manual and Data Set

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