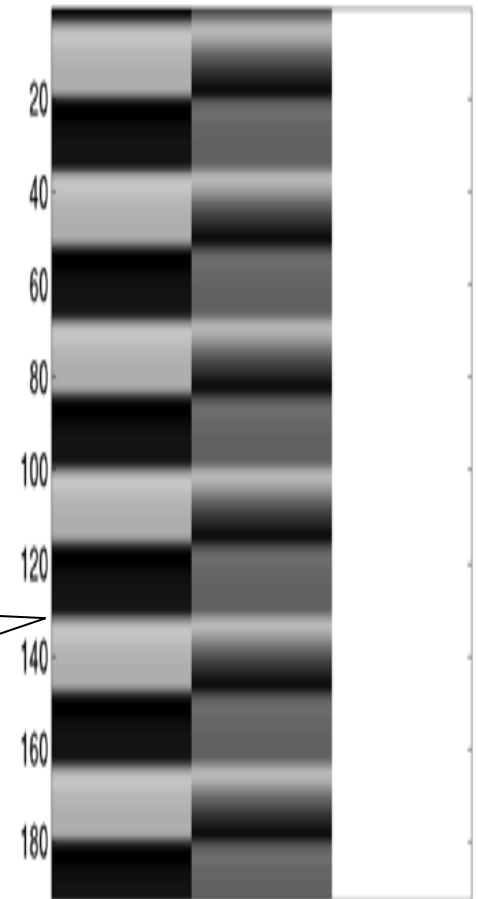
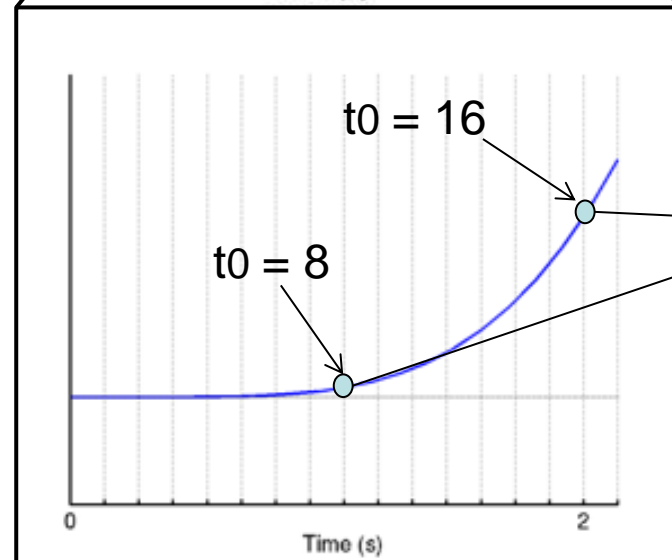
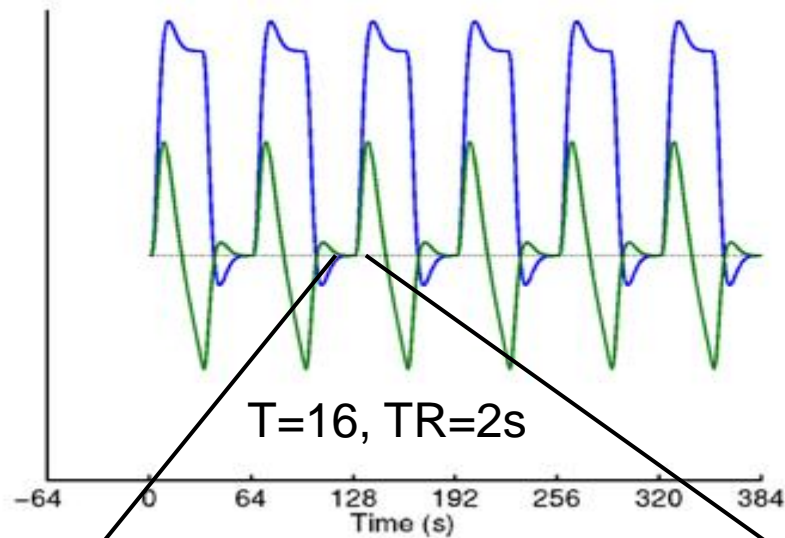
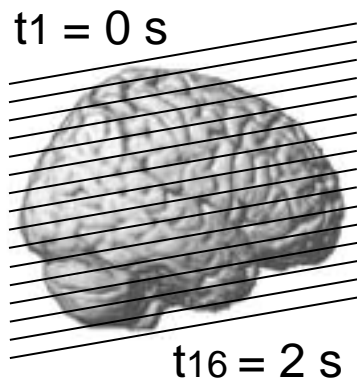


# Event-related fMRI

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University College London

# Slice Timing issue



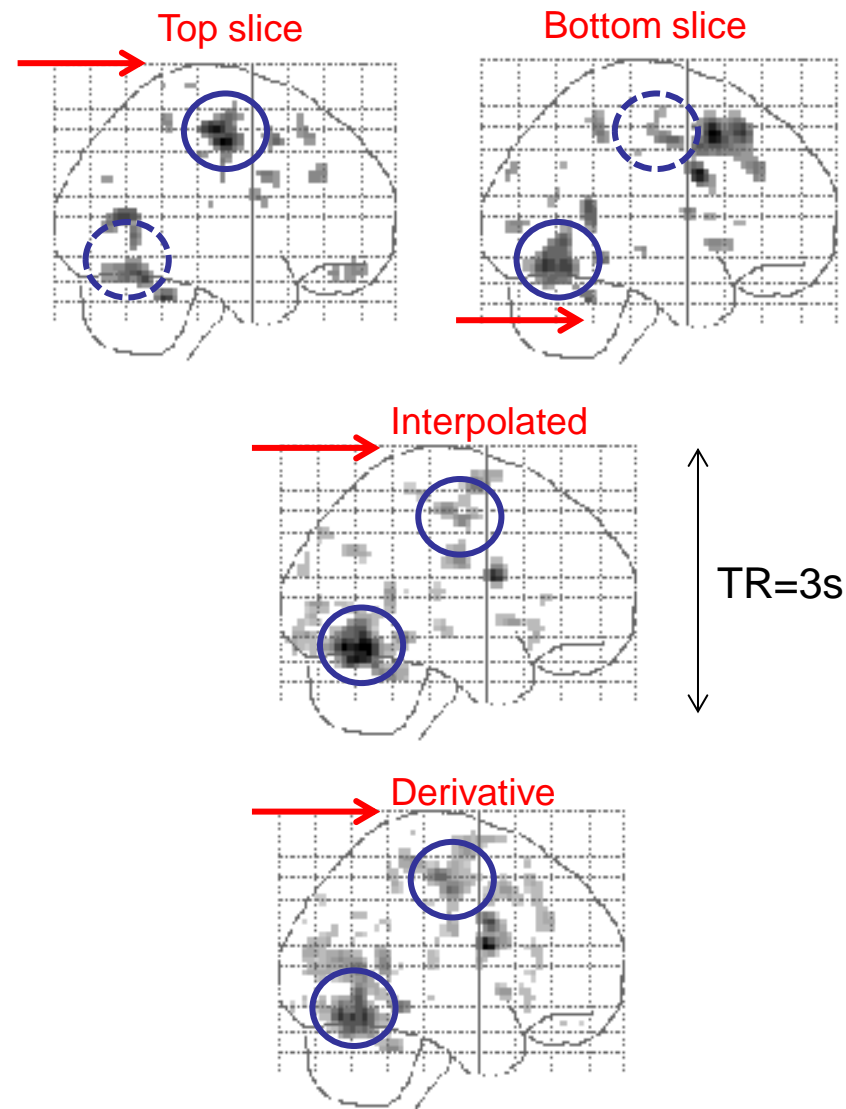
# Slice Timing issue

## “Slice-timing Problem”:

- Slices acquired at different times, yet model is the same for all slices
- different results (using canonical HRF) for different reference slices
- (slightly less problematic if middle slice is selected as reference, and with short TRs)

## Solutions:

1. Temporal interpolation of data  
“Slice timing correction”
2. More general basis set (e.g., with temporal derivatives)

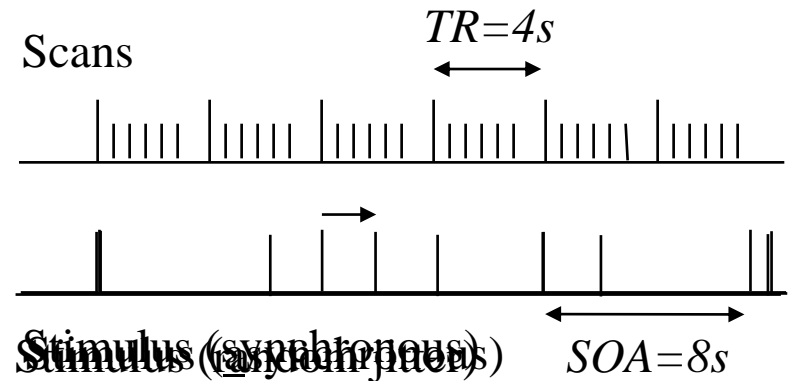


See Sladky et al, NeuroImage, 2012.

# Timing issues: Sampling

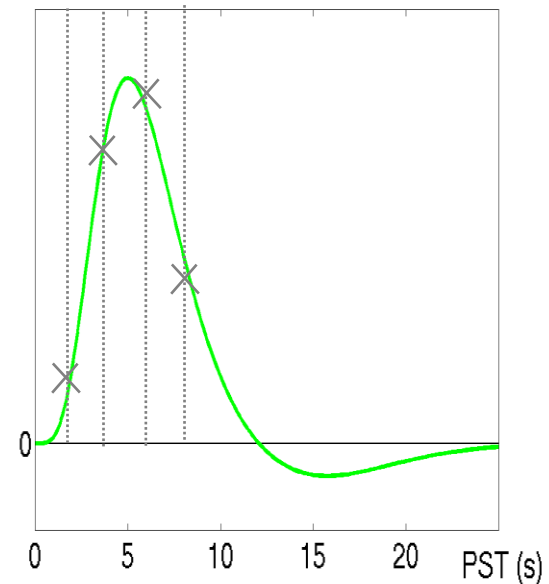
Typical TR for 48 slice EPI at 3mm spacing is  $\sim 4s$

Sampling at  $[0, 4, 8, 12 \dots]$  post-stimulus may miss peak signal.



Higher effective sampling by:

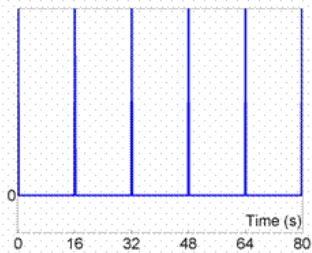
1. Asynchrony  
eg  $SOA = 1.5TR$
2. Random Jitter  
eg  $SOA = (2 \pm 0.5)TR$



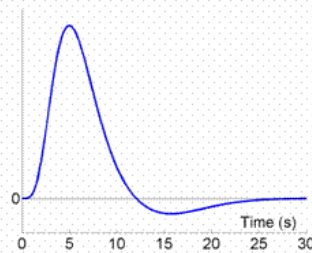
# Optimal SOA?

## 16s SOA

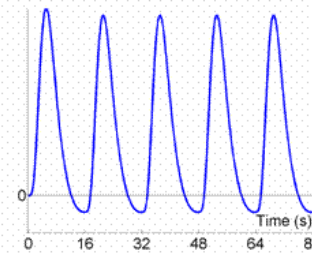
Stimulus (“Neural”)



IR



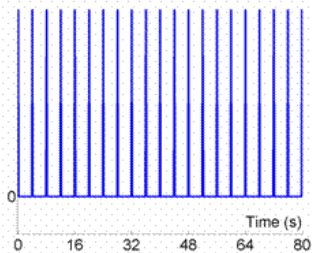
Predicted fMRI Data



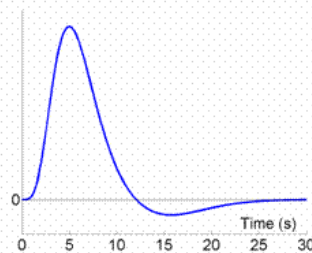
Not very efficient...

## 4s SOA

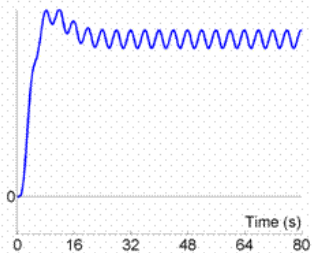
Stimulus (“Neural”)



IR



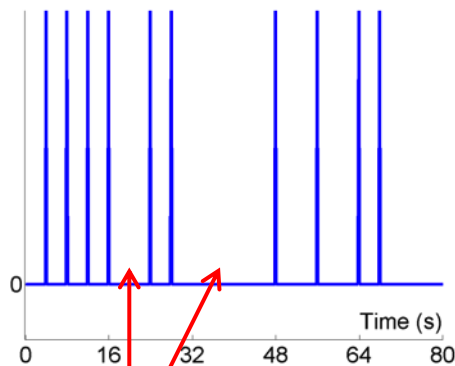
Predicted fMRI Data



Very inefficient...

# Short randomised SOA

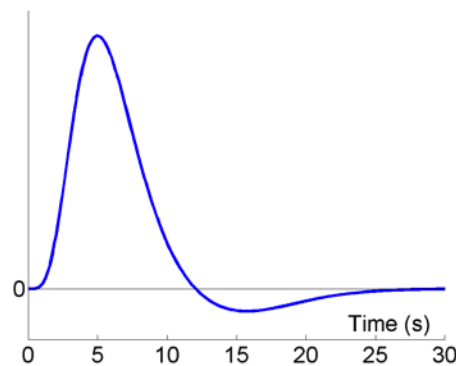
Stimulus (“Neural”)



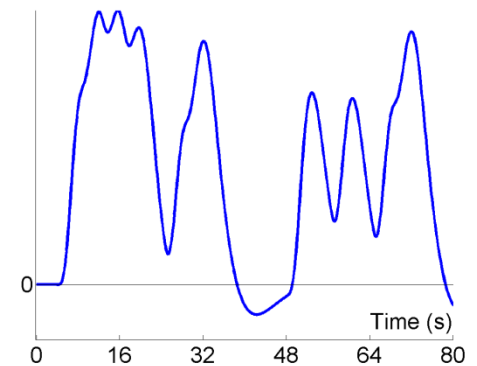
Null events



HRF



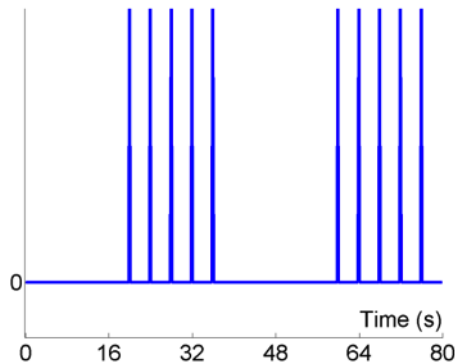
Predicted Data



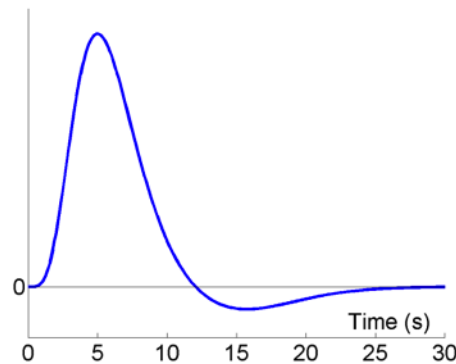
More efficient!

# Block design SOA

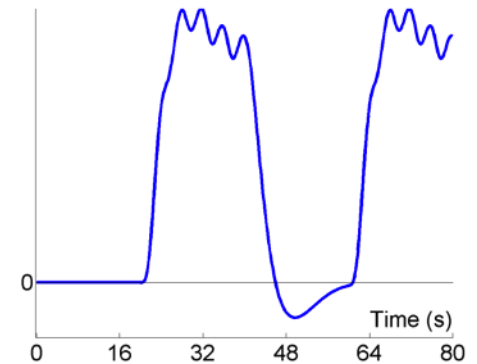
Stimulus (“Neural”)



HRF



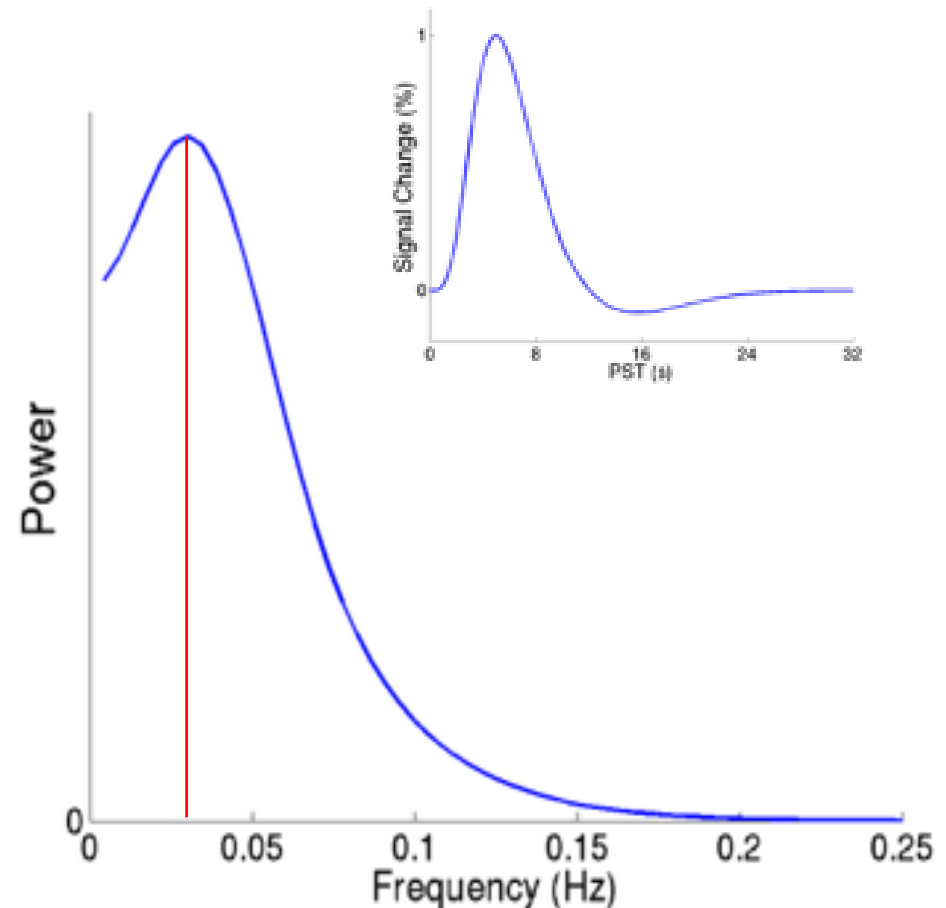
Predicted Data



Even more efficient!

# Design efficiency

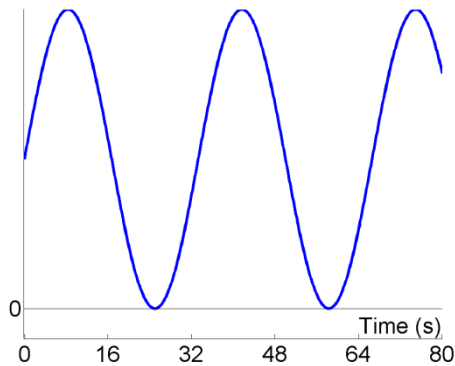
- ❑ HRF can be viewed as a filter.
- ❑ We want to maximise the signal passed by this filter.
- ❑ Dominant frequency of canonical HRF is  $\sim 0.03$  Hz.
- The most efficient design is a sinusoidal modulation of neuronal activity with period  $\sim 32$ s



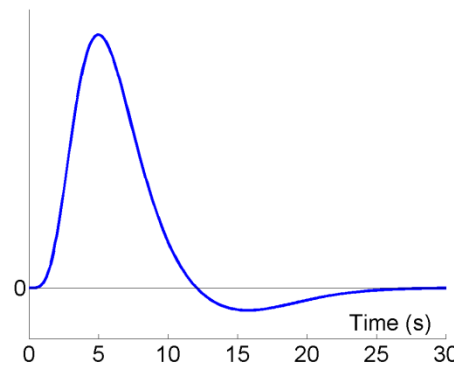


# Sinusoidal modulation, $f=1/32$

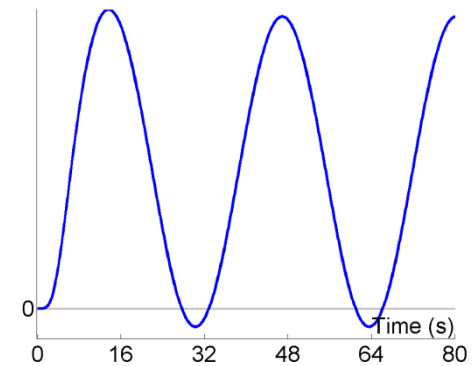
Stimulus ("Neural")



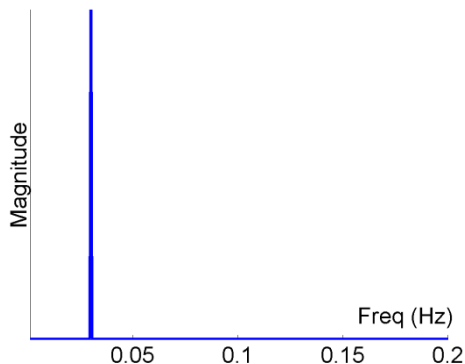
HRF



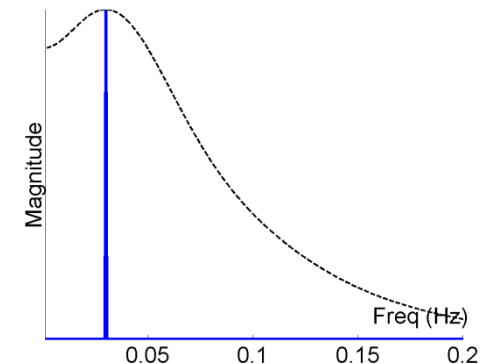
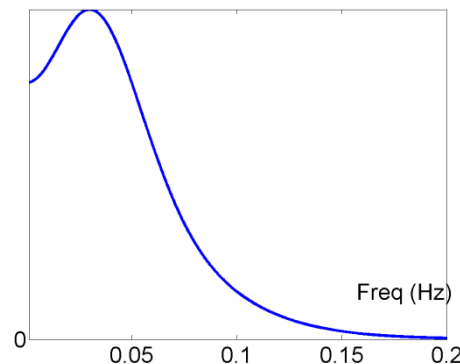
Predicted Data



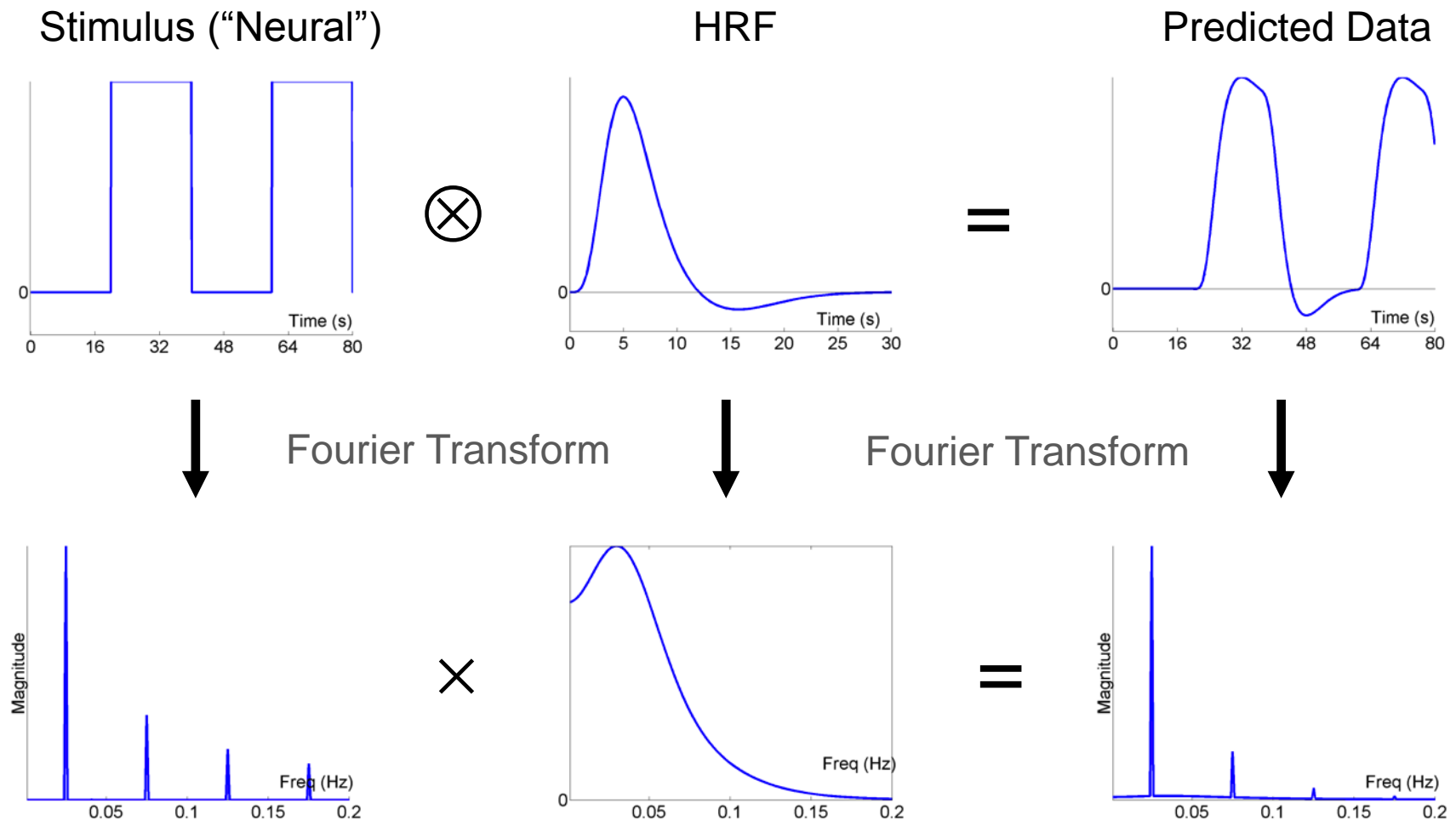
Fourier Transform



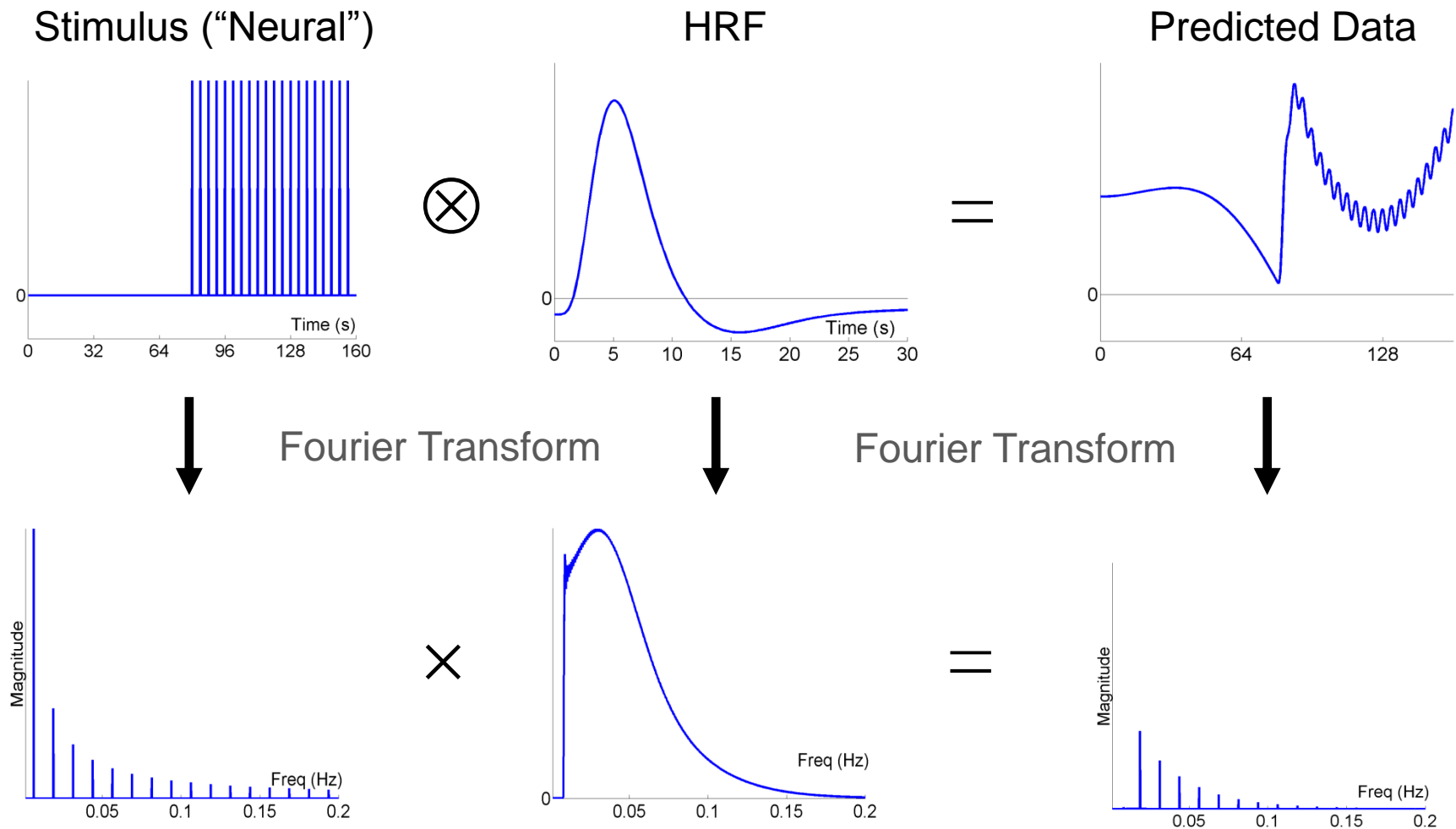
Fourier Transform



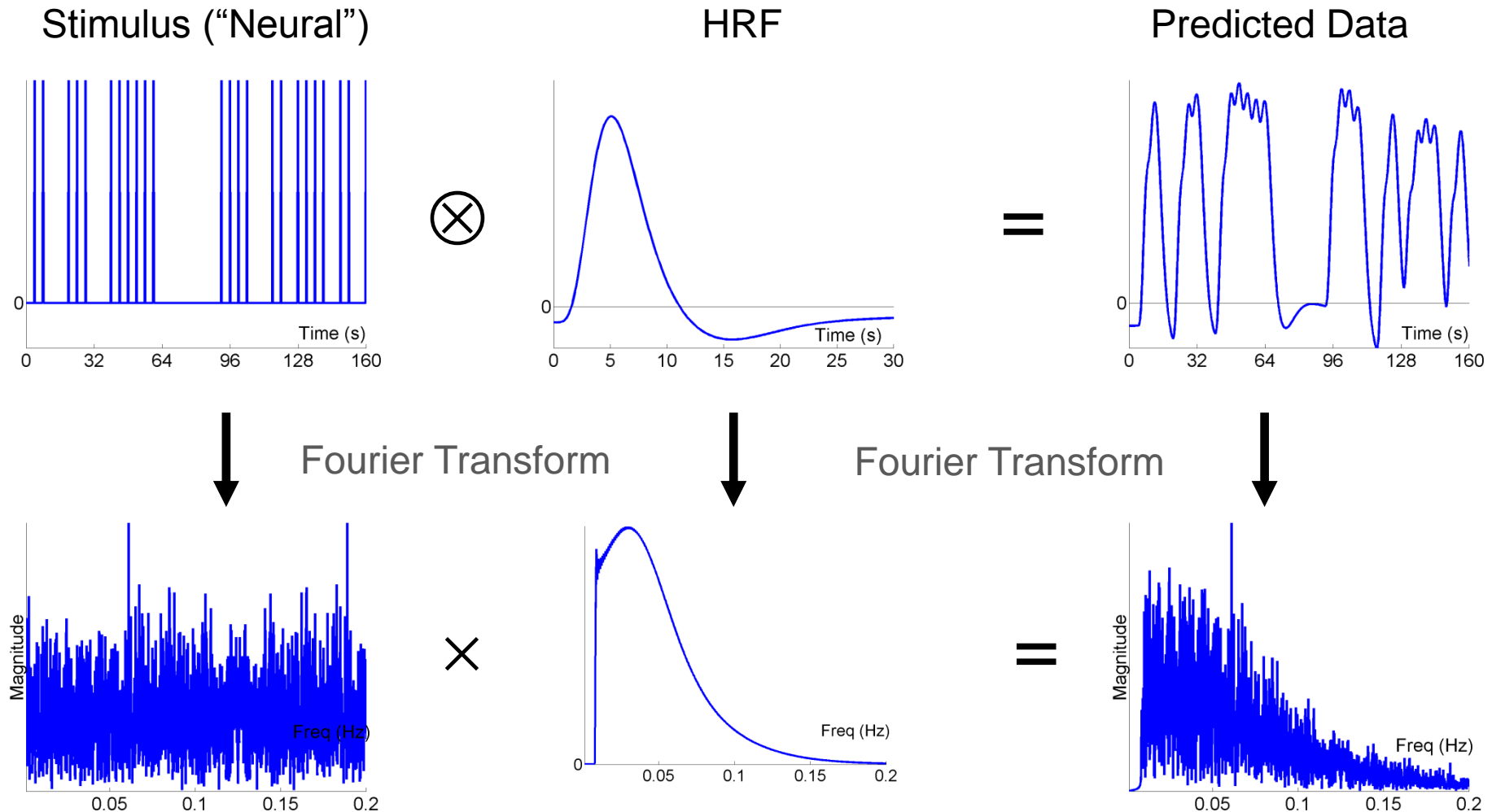
# Blocked: epoch = 20s



**Blocked: epoch = 80s, high-pass filter = 1/120s**



# Randomised Design, $\text{SOA}_{\min} = 4\text{s}$ , high pass filter = $1/120\text{s}$



Randomised design spreads power over frequencies

# Design efficiency

## Block designs:

- ❑ Generally efficient but often not appropriate.
- ❑ Optimal block length 16s with short SOA (beware of high-pass filter).

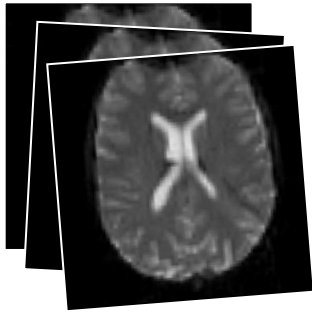
## Event-related designs:

- ❑ Efficiency depends on the contrast of interest
- ❑ With short SOAs 'null events' (jittered ITI) can optimise efficiency across multiple contrasts.
- ❑ Non-linear effects start to become problematic at  $\text{SOA} < 2\text{s}$

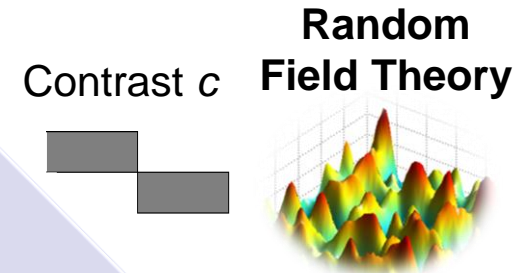
# Multiple testing *(random field theory)*

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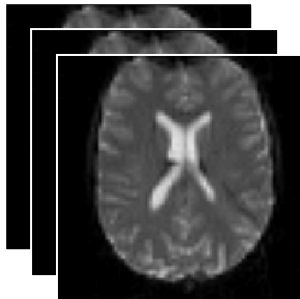
$$y = \begin{bmatrix} \text{checkered pattern} \end{bmatrix} \beta + \varepsilon$$



Pre-  
processings

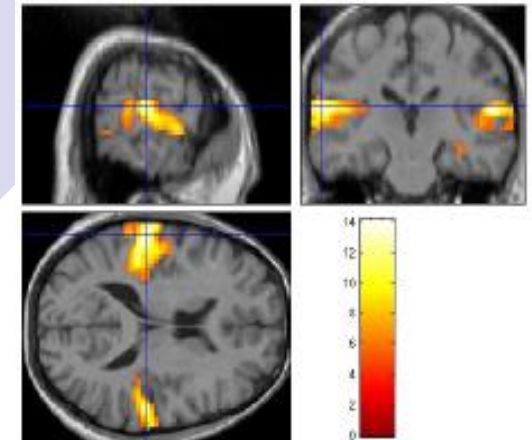
General  
Linear  
Model

Statistical  
Inference

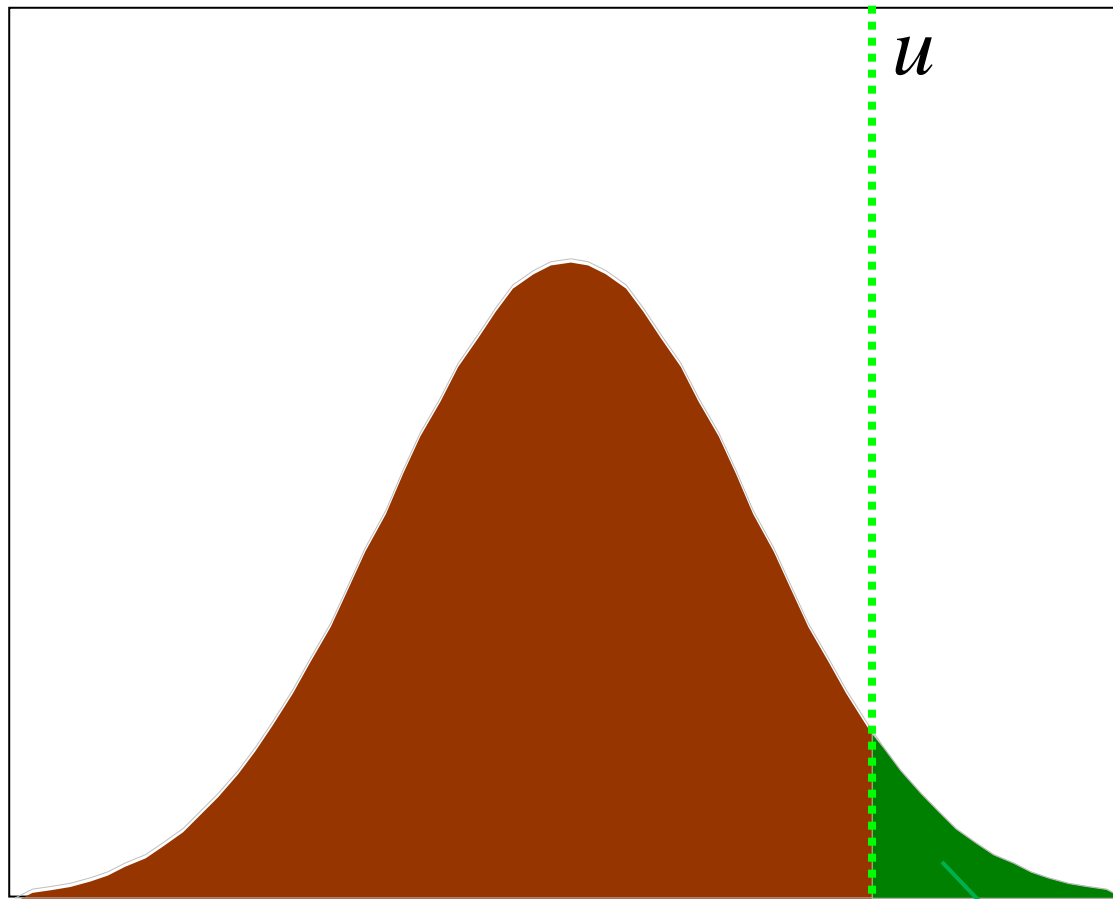


$$\hat{\beta} = (X^T X)^{-1} X^T y$$

$$\hat{\sigma}^2 = \frac{\hat{\varepsilon}^T \hat{\varepsilon}}{\text{rank}(X)}$$



# Inference at a single voxel



Null distribution of test statistic  $T$

Null Hypothesis  $H_0$ :  
zero activation

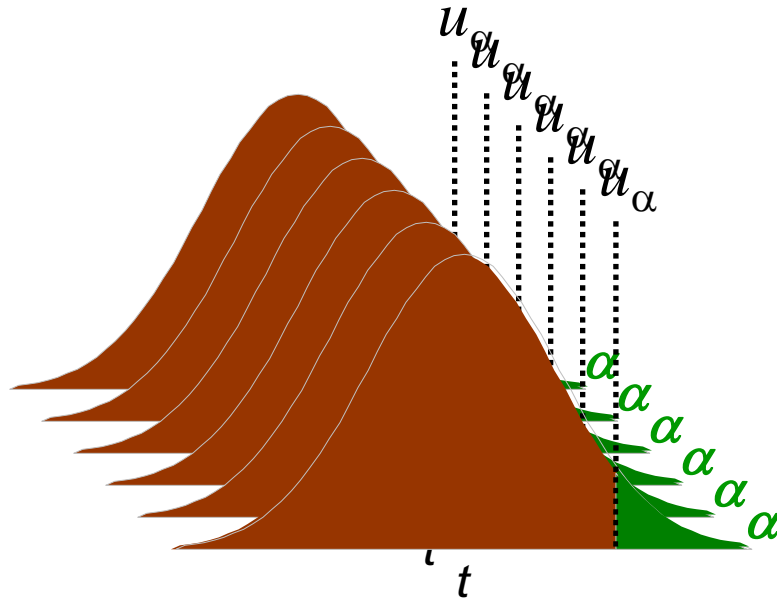
Decision rule (threshold)  $u$ :  
determines false positive  
rate  $\alpha$

$\Rightarrow$  Choose  $u$  to give acceptable  
 $\alpha$  under  $H_0$

$$\alpha = p(t > u | H_0)$$



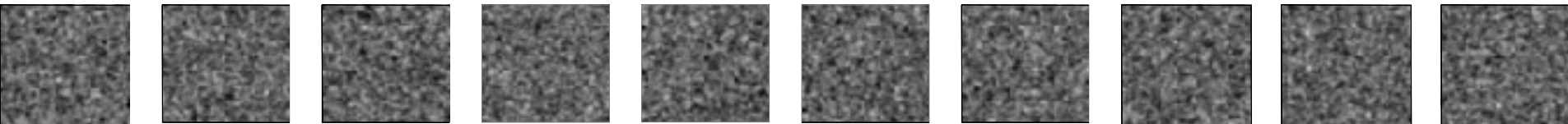
# Multiple tests



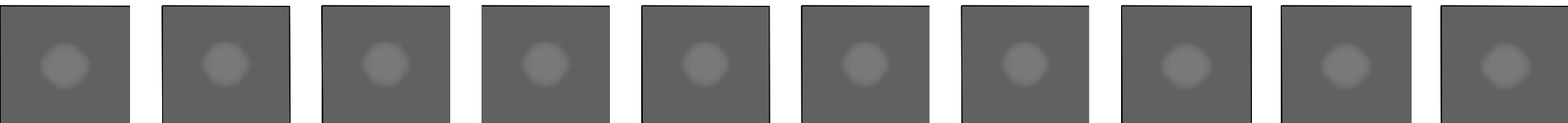
If we have 100,000 voxels,  
 $\alpha=0.05 \Rightarrow 5,000$  false positive voxels.

This is clearly undesirable; to correct for this we can define a null hypothesis for a collection of tests.

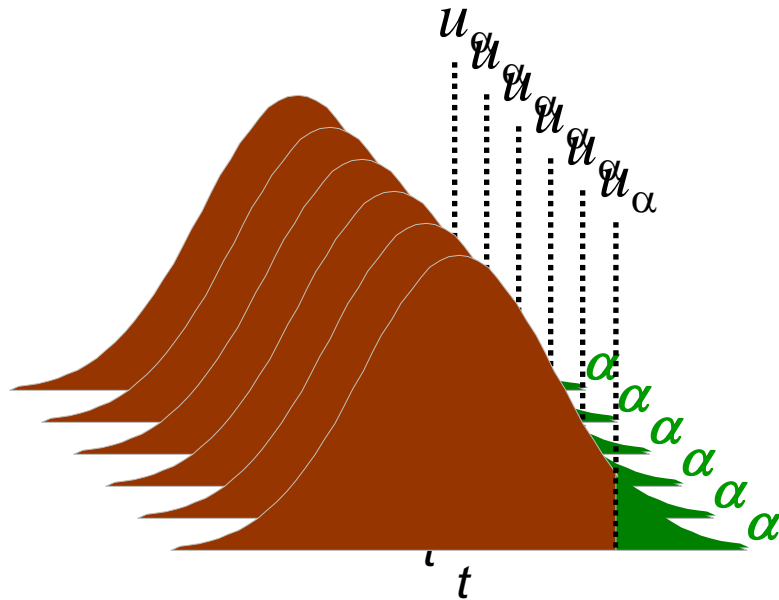
Noise



Signal

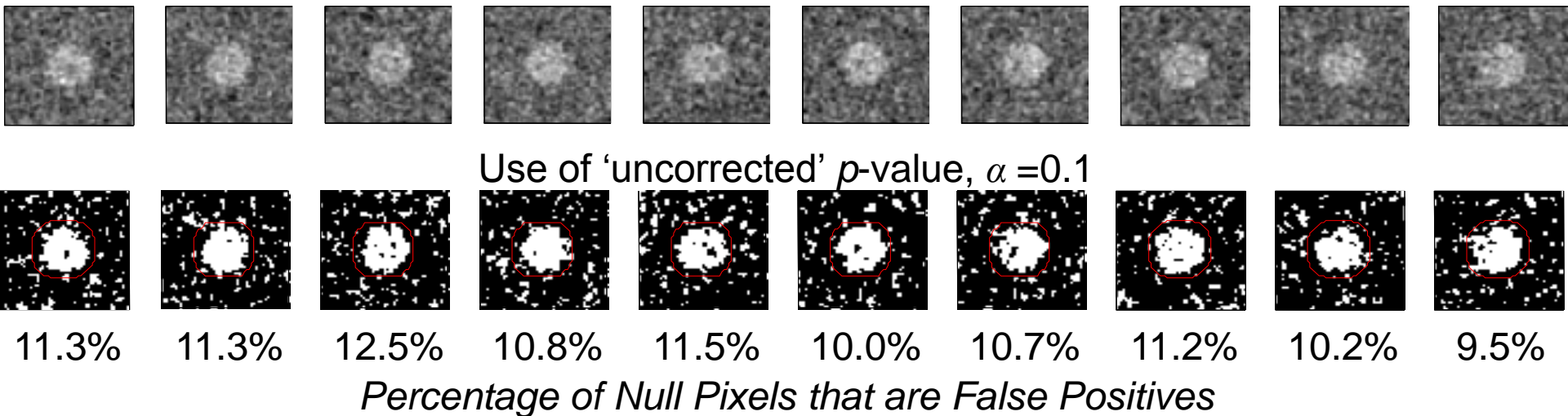


# Multiple tests



If we have 100,000 voxels,  
 $\alpha=0.05 \Rightarrow 5,000$  false positive voxels.

This is clearly undesirable; to correct for this we can define a null hypothesis for a collection of tests.



# Family-Wise Null Hypothesis

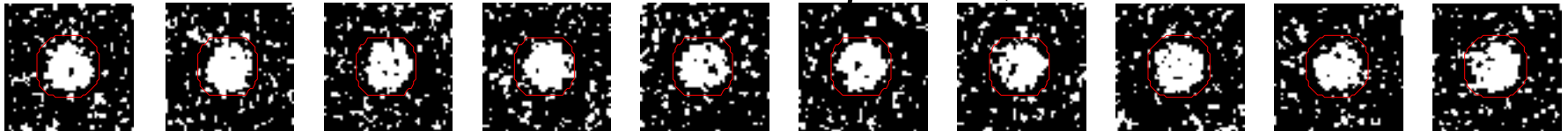
***Family-Wise Null Hypothesis:***  
*Activation is zero everywhere*

If we reject a voxel null hypothesis at *any* voxel,  
we reject the family-wise Null hypothesis

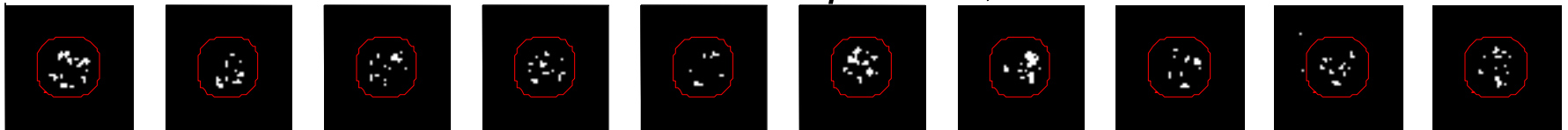
A FP ***anywhere*** in the image gives a **Family Wise Error** (FWE)

Family-Wise Error rate (FWER) = 'corrected'  $p$ -value

Use of 'uncorrected'  $p$ -value,  $\alpha = 0.1$



Use of 'corrected'  $p$ -value,  $\alpha = 0.1$



FWE

# Bonferroni correction

The Family-Wise Error rate (FWER),  $\alpha_{FWE}$ , for a family of  $N$  tests follows the inequality:

$$\alpha_{FWE} \leq N\alpha$$

where  $\alpha$  is the test-wise error rate.

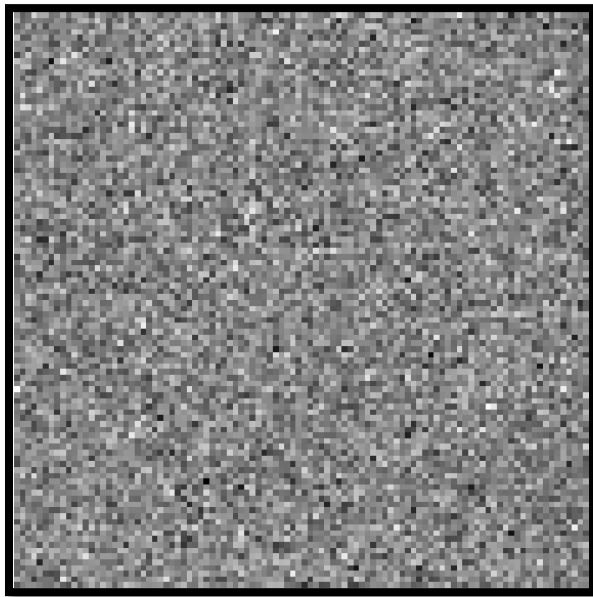
Therefore, to ensure a particular FWER choose:

$$\alpha = \frac{\alpha_{FWE}}{N}$$

This correction does not require the tests to be independent but becomes very stringent if dependence.

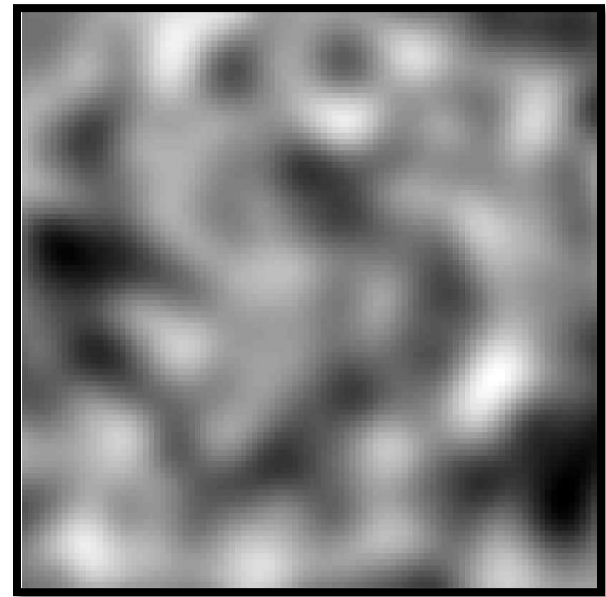
# Spatial correlations

100 x 100 independent tests



Discrete data

Spatially correlated tests (FWHM=10)



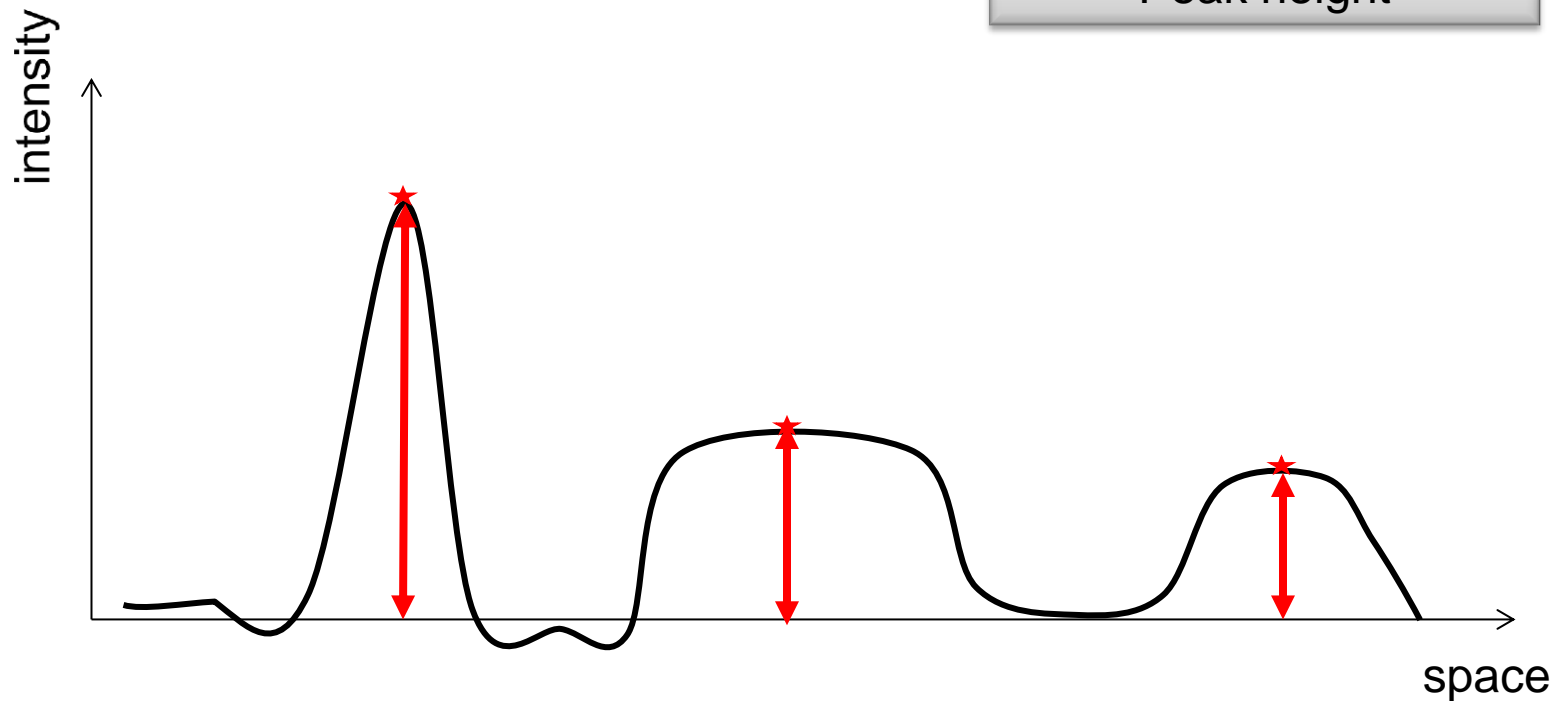
Spatially extended data

Bonferroni is too conservative for spatially correlated data.

# Topological inference

Peak level inference

**Topological feature:**  
Peak height

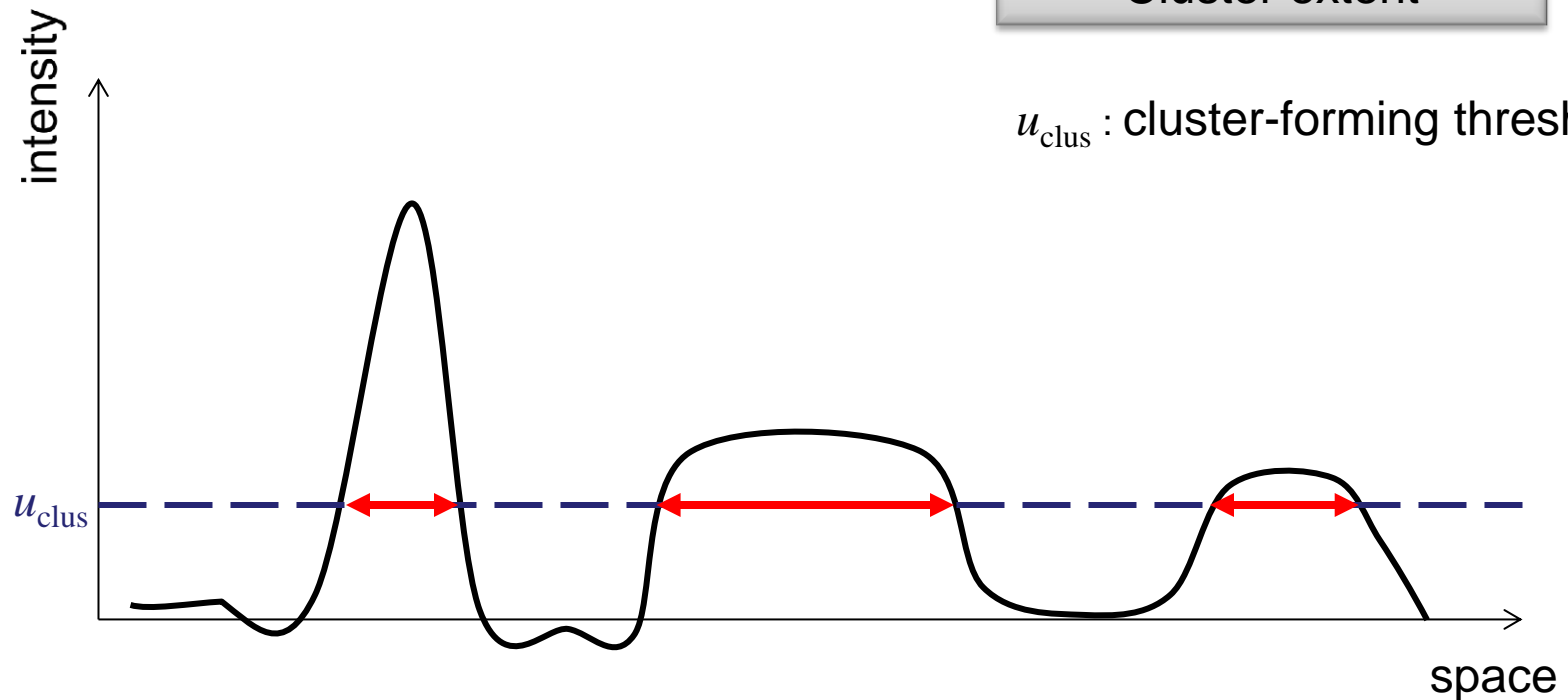


# Topological inference

## Cluster level inference

**Topological feature:**  
Cluster extent

$u_{\text{clus}}$  : cluster-forming threshold

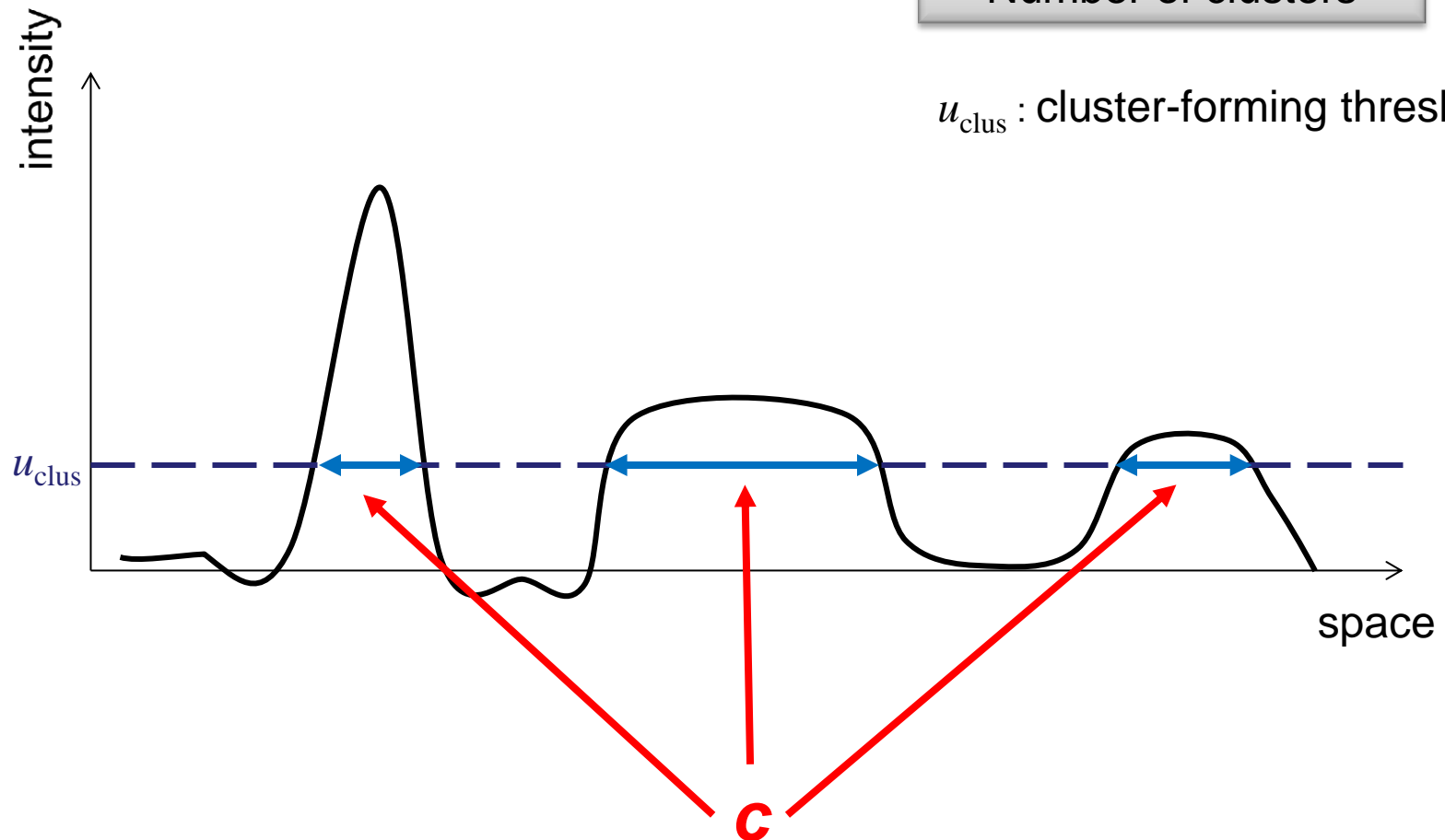


# Topological inference

Set level inference

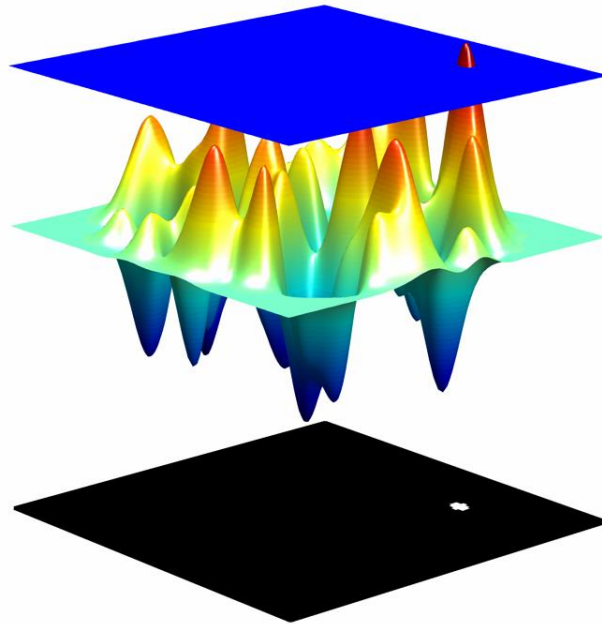
**Topological feature:**  
Number of clusters

$u_{\text{clus}}$  : cluster-forming threshold





# RFT and Euler Characteristic



$$\begin{aligned}
 FWER &= p(FWE) \\
 &\approx E[\chi_u] \\
 &\propto \lambda(\Omega) |\Lambda|^{1/2} u \exp(-u^2/2) / (2\pi)^{3/2}
 \end{aligned}$$

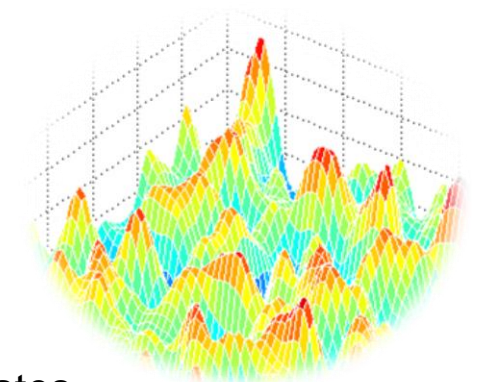
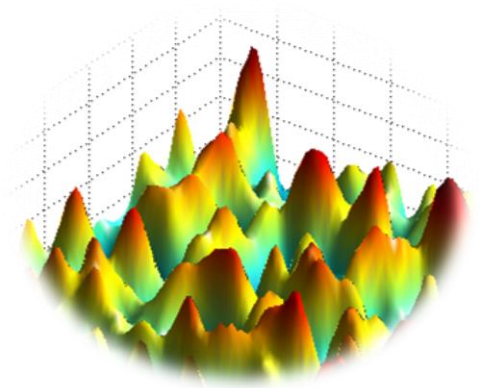
Search volume

Roughness  
(1/smoothness)

Threshold

# Random Field Theory

- ❑ The statistic image is assumed to be a good lattice representation of an underlying continuous stationary random field.  
Typically,  $\text{FWHM} > 3$  voxels  
(combination of intrinsic and extrinsic smoothing)
- ❑ Smoothness of the data is unknown and estimated:  
very precise estimate by pooling over voxels  $\Rightarrow$  stationarity assumptions (esp. relevant for cluster size results).
- ❑ *A priori* hypothesis about where an activation should be, reduce search volume  $\Rightarrow$  Small Volume Correction:
  - mask defined by (probabilistic) anatomical atlases
  - mask defined by separate "functional localisers"
  - mask defined by orthogonal contrasts
  - (spherical) search volume around previously reported coordinates

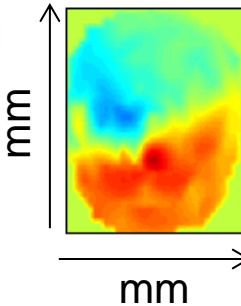
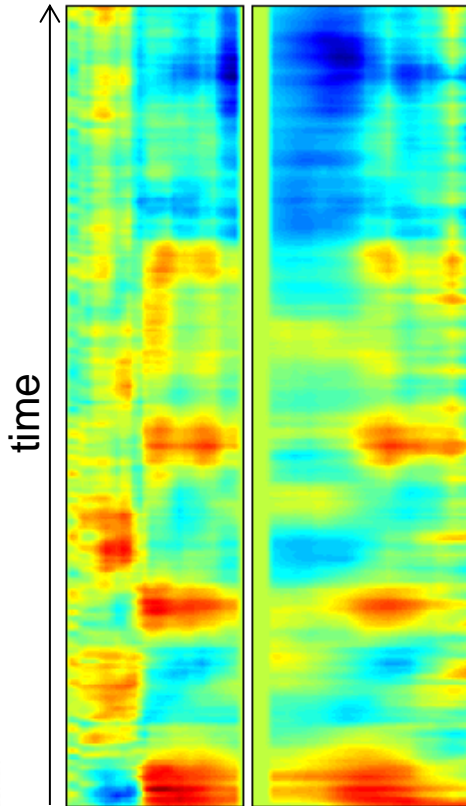
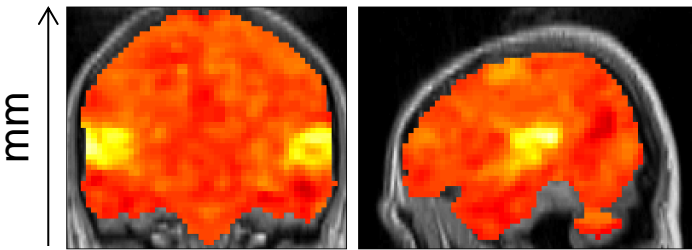


# Conclusion

- ❑ There is a ***multiple testing problem*** and *corrections* have to be applied on  $p$ -values (for the volume of interest only (see Small Volume Correction)).
- ❑ Inference is made about ***topological features*** (peak height, spatial extent, number of clusters).  
Use results from the ***Random Field Theory***.
- ❑ **Control of *FWER*** (probability of a false positive anywhere in the image): very specific, not so sensitive.
- ❑ **Control of *FDR*** (expected proportion of false positives amongst those features declared positive (the *discoveries*)): less specific, more sensitive.

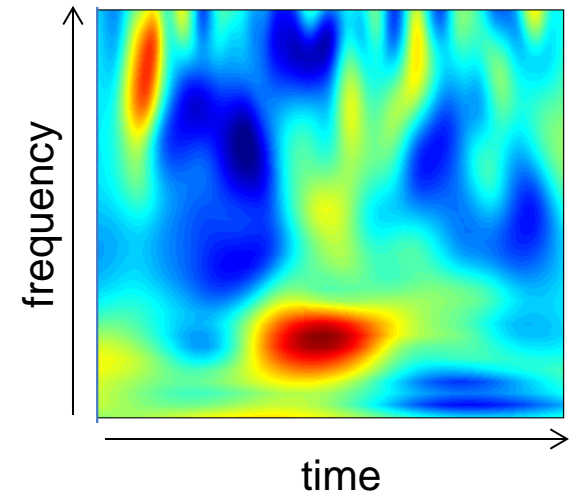
# Statistical Parametric Maps

fMRI, VBM,  
M/EEG source reconstruction

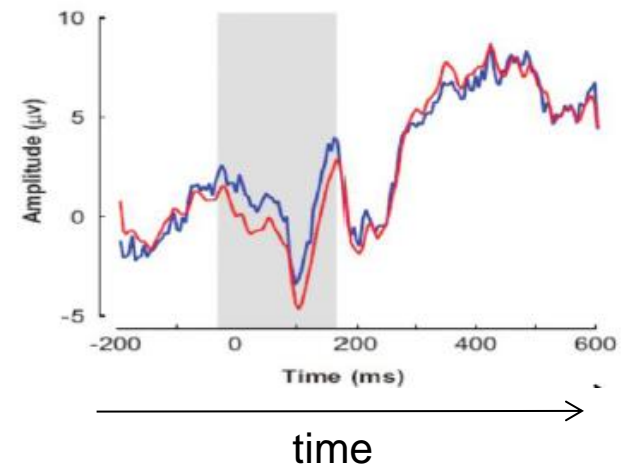


M/EEG  
2D+t  
scalp-time

M/EEG 2D time-frequency



M/EEG 1D channel-time

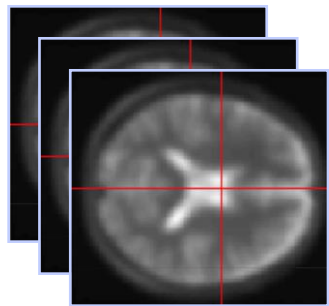


# Group Analyses

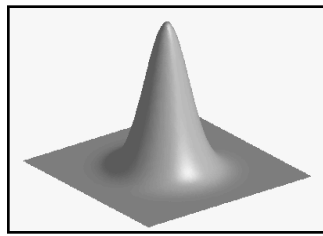
*Guillaume Flandin*

Wellcome Trust Centre for Neuroimaging  
University College London

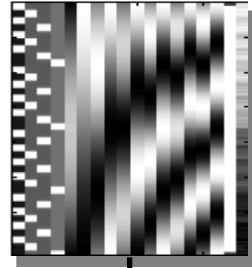
Image time-series



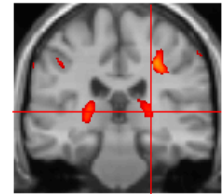
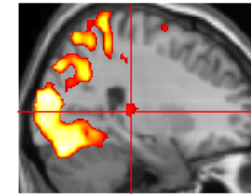
Spatial filter



Design matrix



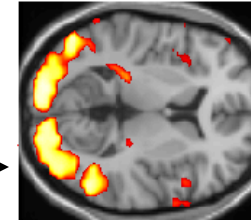
Statistical Parametric Map



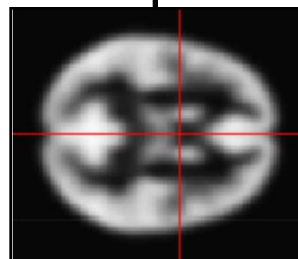
Realignment

Smoothing

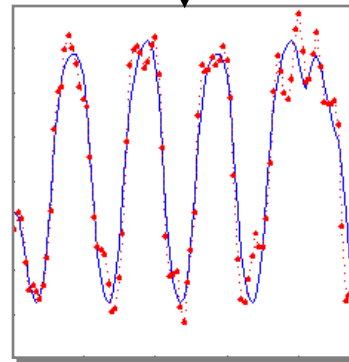
General Linear Model



Normalisation



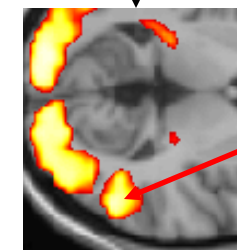
Anatomical  
reference



Parameter estimates

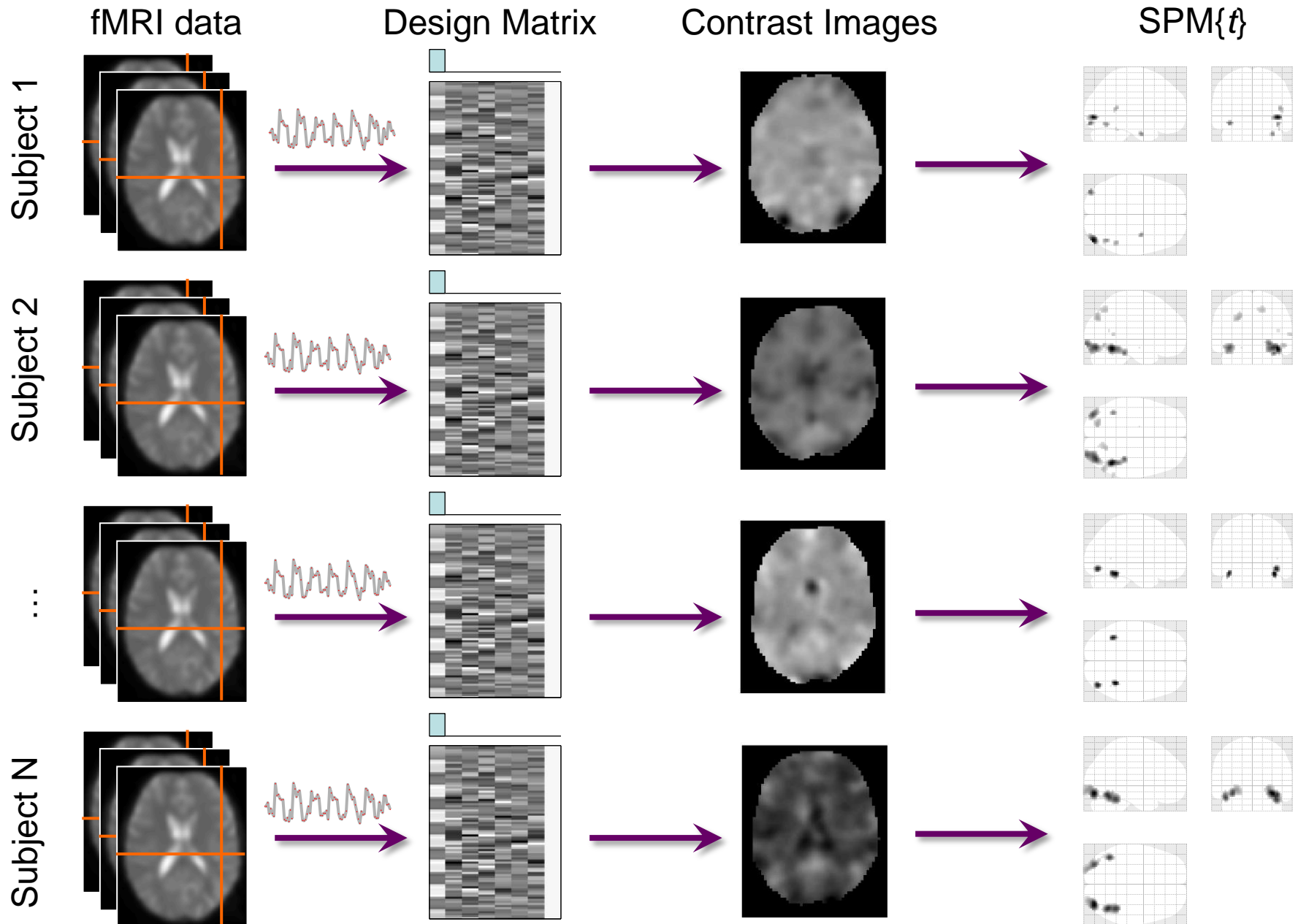
Statistical  
Inference

RFT

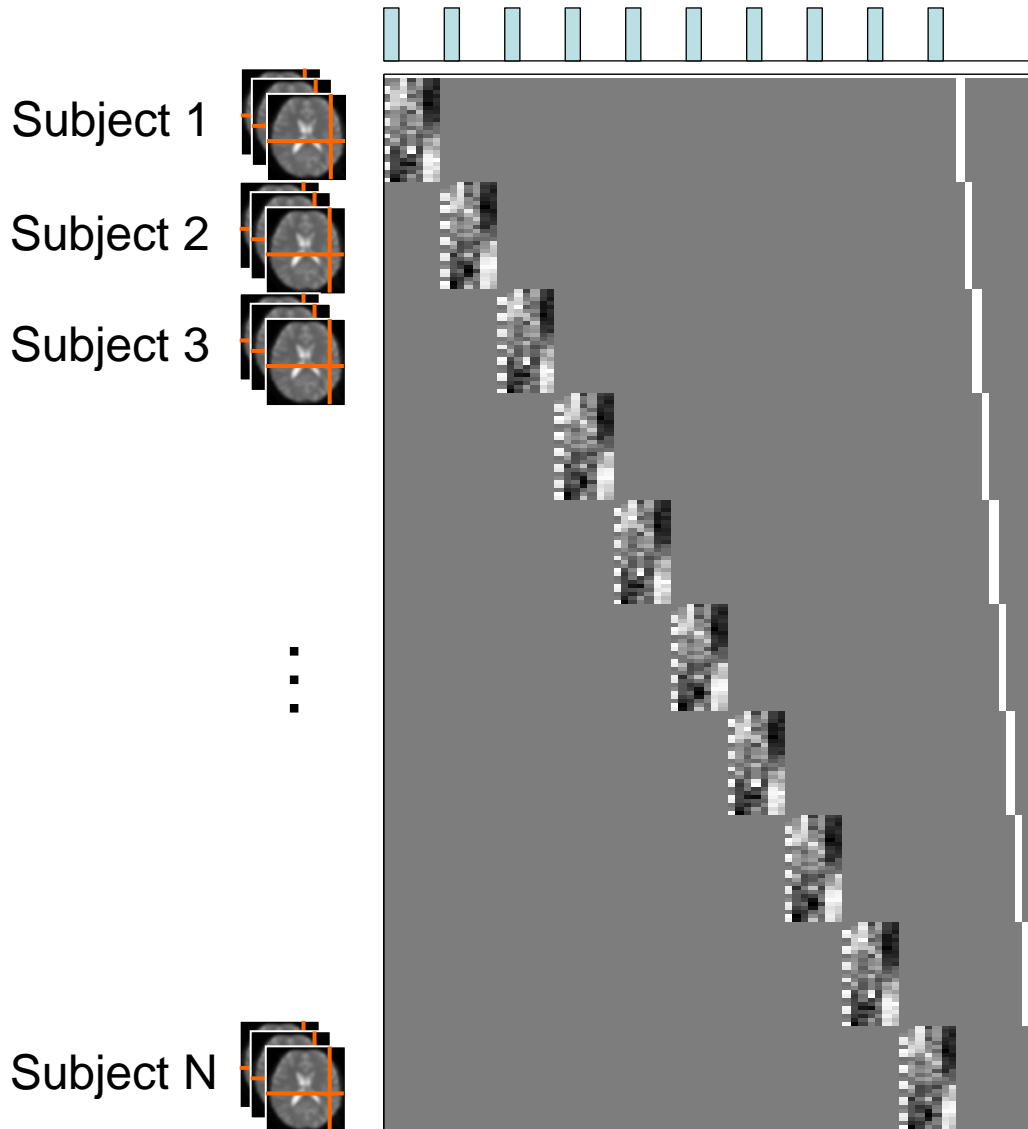


$p < 0.05$

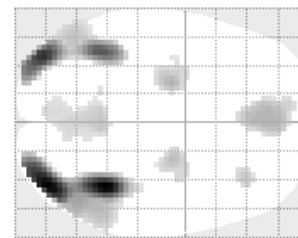
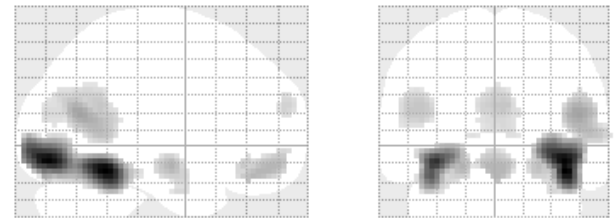
# GLM: repeat over subjects



# Fixed effects analysis (FFX)



Modelling all subjects at once



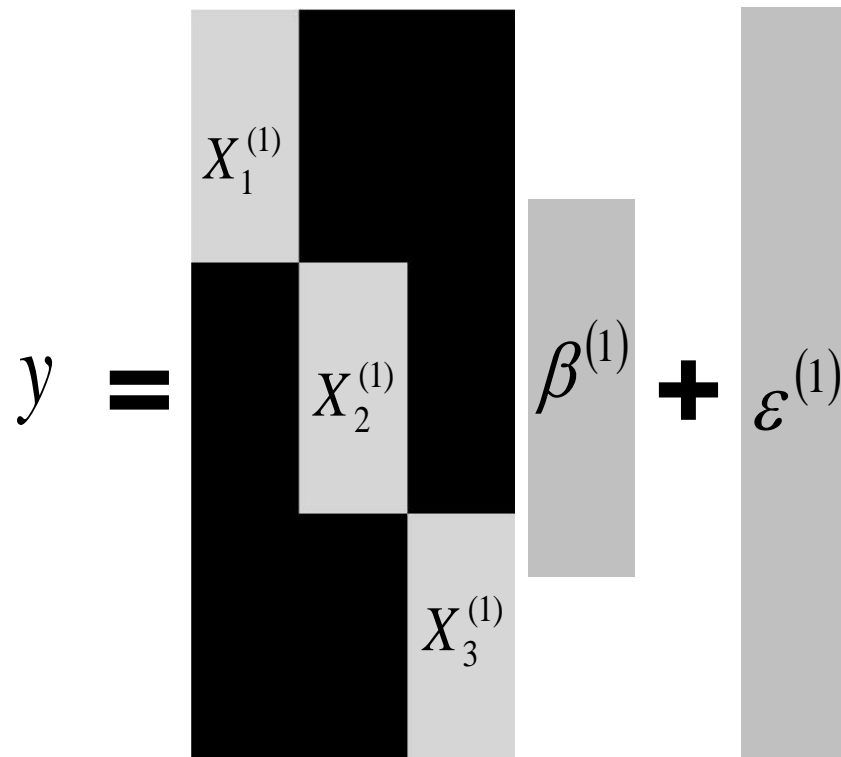
variance over subjects at each voxel



# Fixed effects analysis (FFX)

$$y = X^{(1)} \beta^{(1)} + \varepsilon^{(1)}$$

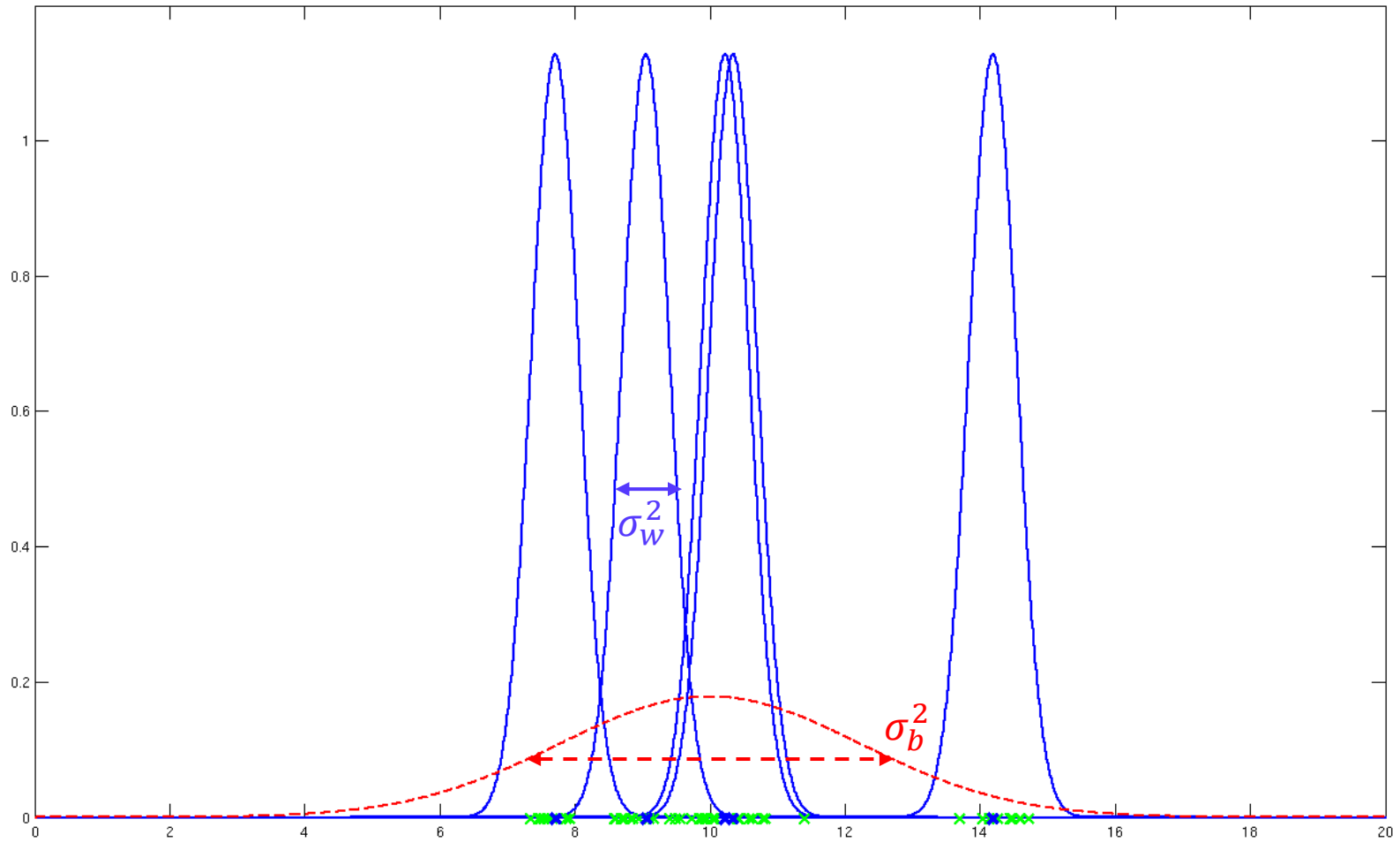
Modelling all subjects at once



The diagram illustrates the equation  $y = X^{(1)} \beta^{(1)} + \varepsilon^{(1)}$  using a block matrix representation. On the left, the vector  $y$  is represented by a tall black rectangle. To its right is an equals sign. Next is the design matrix  $X^{(1)}$ , depicted as a 3x3 grid of squares. The top-left, middle-right, and bottom-right squares are light gray and labeled  $X_1^{(1)}$ ,  $X_2^{(1)}$ , and  $X_3^{(1)}$  respectively. The other five squares are black. To the right of the grid is a tall gray rectangle representing the parameter vector  $\beta^{(1)}$ . This is followed by a plus sign and another tall gray rectangle representing the error vector  $\varepsilon^{(1)}$ .

- ✓ Simple model
- ✓ Lots of degrees of freedom
- ✗ Large amount of data
- ✗ Assumes common variance over subjects at each voxel

# Random effects



Probability model underlying random effects analysis

# Fixed vs random effects

With **Fixed Effects Analysis (FFX)** we compare the group effect to the *within-subject variability*. It is not an inference about the population from which the subjects were drawn.

With **Random Effects Analysis (RFX)** we compare the group effect to the *between-subject variability*. It is an inference about the population from which the subjects were drawn. If you had a new subject from that population, you could be confident they would also show the effect.

# Fixed vs random effects

- ❑ Fixed isn't "wrong", just usually isn't of interest.

- ❑ Summary:

- **Fixed effects inference:**

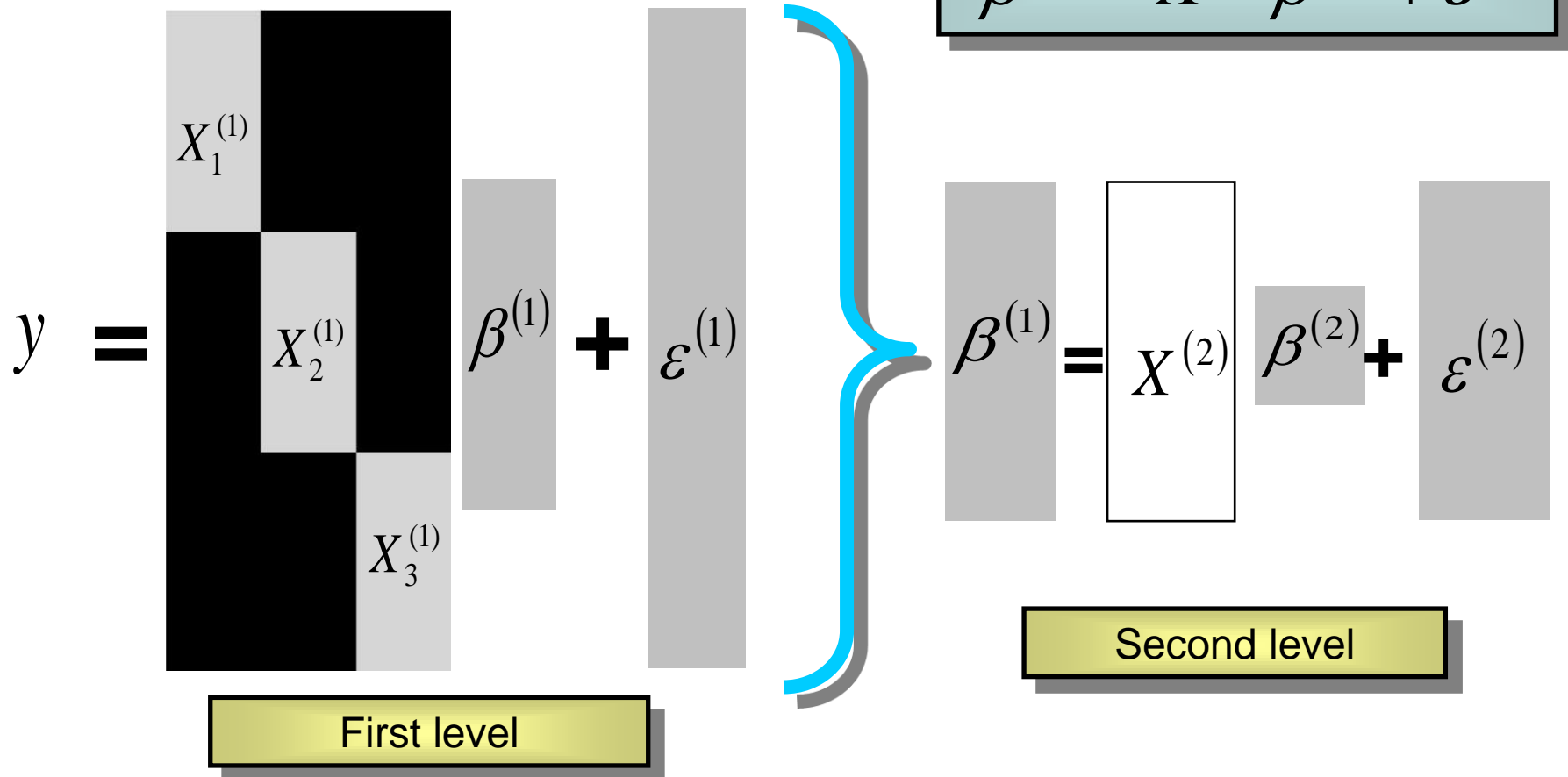
- "I can see this effect in this cohort"*

- **Random effects inference:**

- "If I were to sample a new cohort from the same population I would get the same result"*

# Hierarchical models

Example: Two level model



# Summary Statistics RFX Approach

## First level

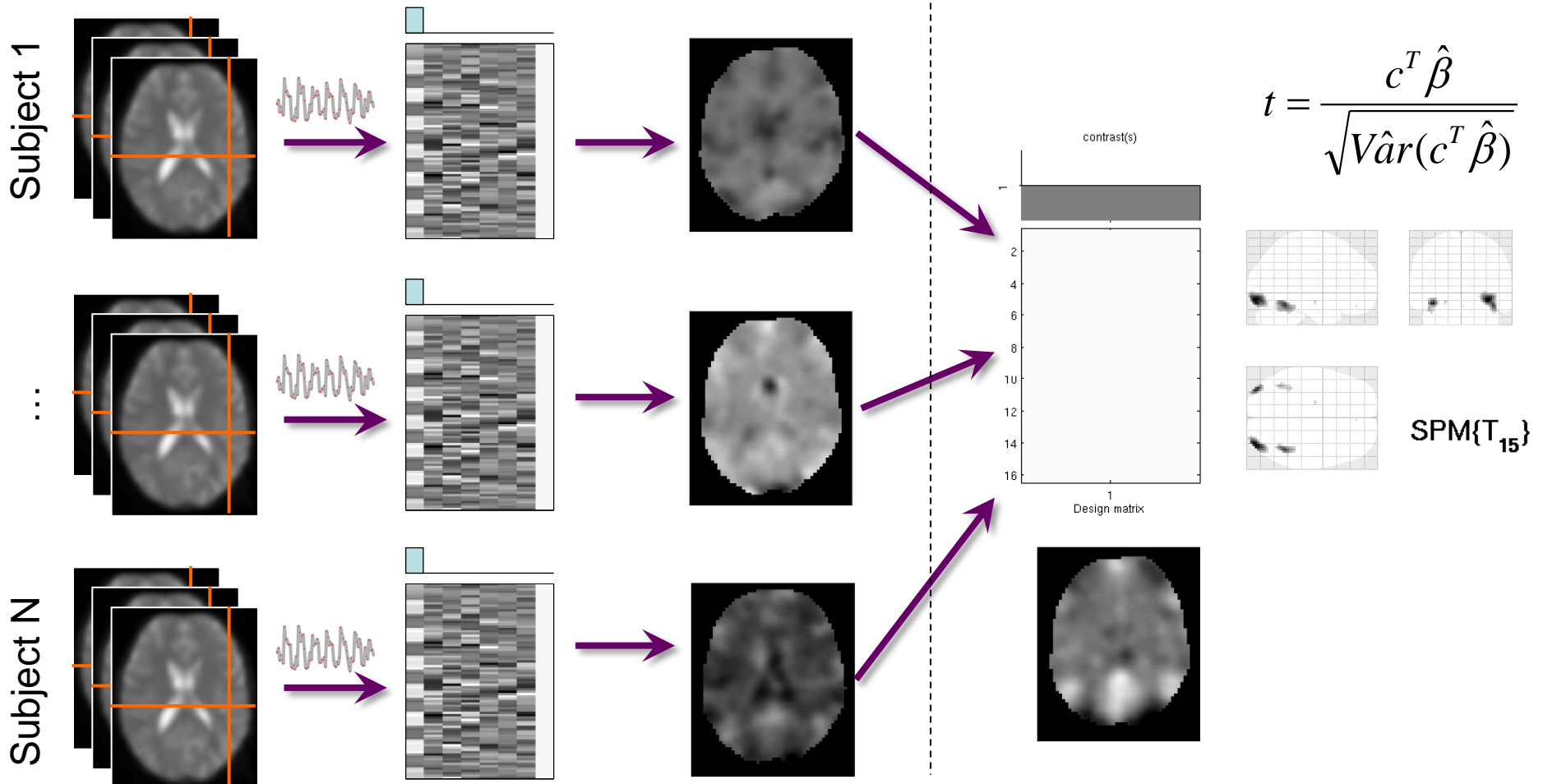
## Second level

fMRI data

Design Matrix

Contrast Images

One-sample t-test @ second level



$$t = \frac{c^T \hat{\beta}}{\sqrt{\text{Var}(c^T \hat{\beta})}}$$

## Assumptions

- ❑ The summary statistics approach is exact if for each session/subject:
  - Within-subjects variances the same
  - First level design the same (e.g. number of trials)
- ❑ Other cases: summary statistics approach is robust against typical violations.

*Mixed-effects and fMRI studies.* Friston et al., NeuroImage, 2005.

*Statistical Parametric Mapping: The Analysis of Functional Brain Images.* Elsevier, 2007.

*Simple group fMRI modeling and inference.* Mumford & Nichols. NeuroImage, 2009.

# ANOVA & non-sphericity

## ❑ One effect per subject:

- Summary statistics approach
- One-sample t-test at the second level

## ❑ More than one effect per subject or multiple groups:

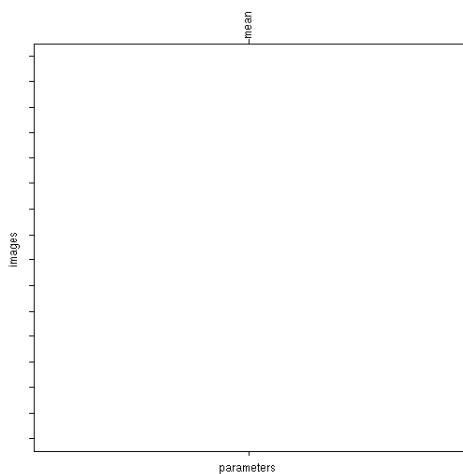
- Non-sphericity modelling
- Covariance components and ReML



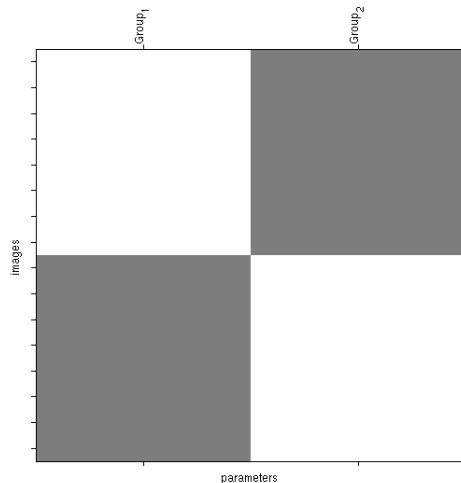
# Summary

- ❑ Group Inference usually proceeds with **RFX analysis**, not FFX. Group effects are compared to between rather than within subject variability.
- ❑ **Hierarchical models** provide a gold-standard for RFX analysis but are computationally intensive.
- ❑ **Summary statistics** approach is a robust method for RFX group analysis.
- ❑ Can also use '**ANOVA**' or '**ANOVA within subject**' at second level for inference about multiple experimental conditions or multiple groups.

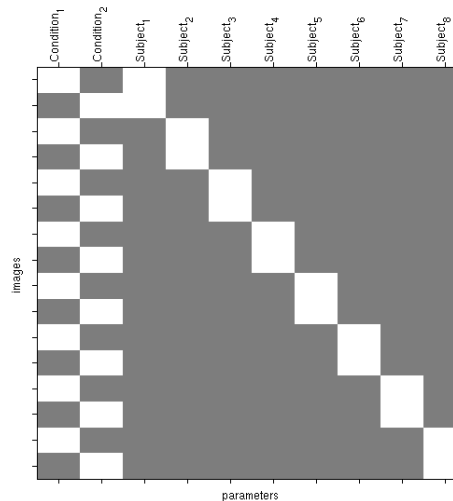
## One-sample t-test



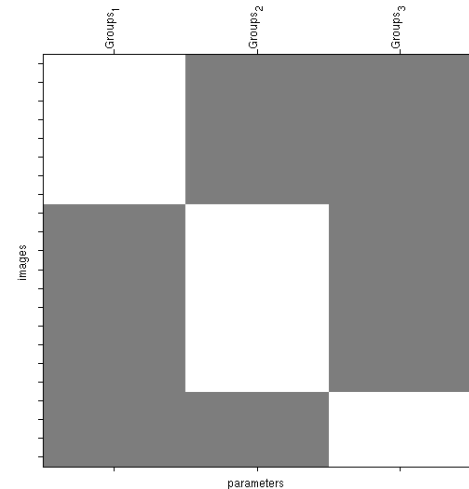
## Two-sample t-test



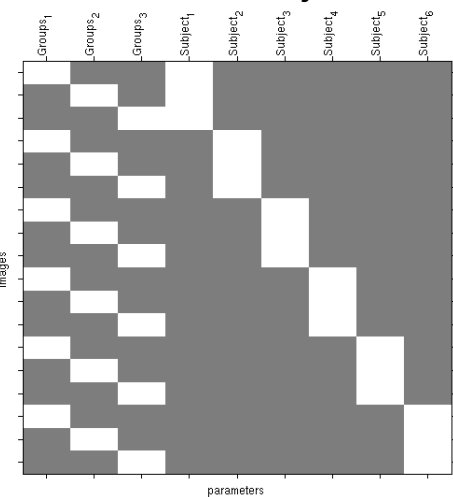
## Paired t-test



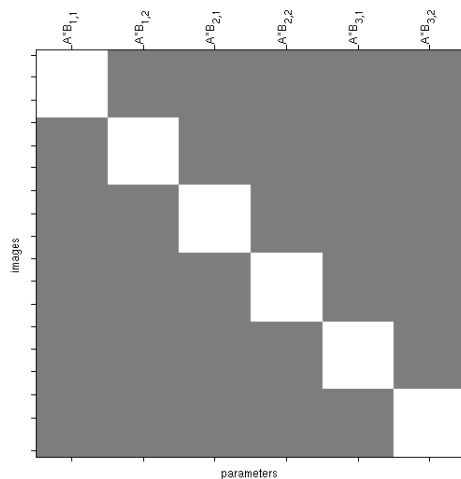
## One-way ANOVA



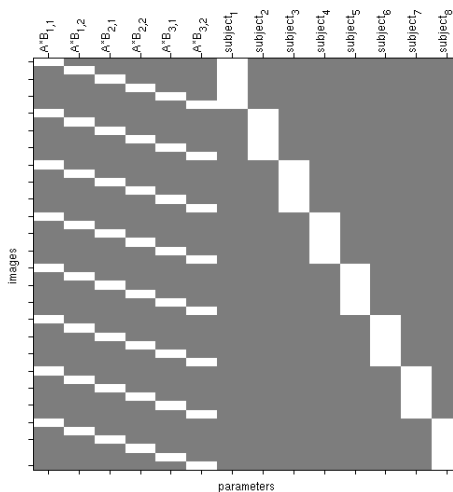
## One-way ANOVA within-subject



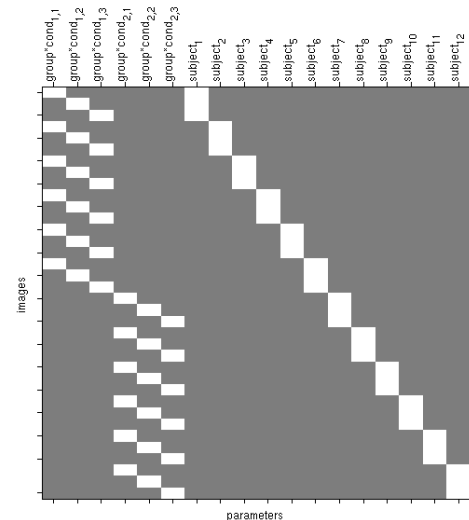
## Full Factorial



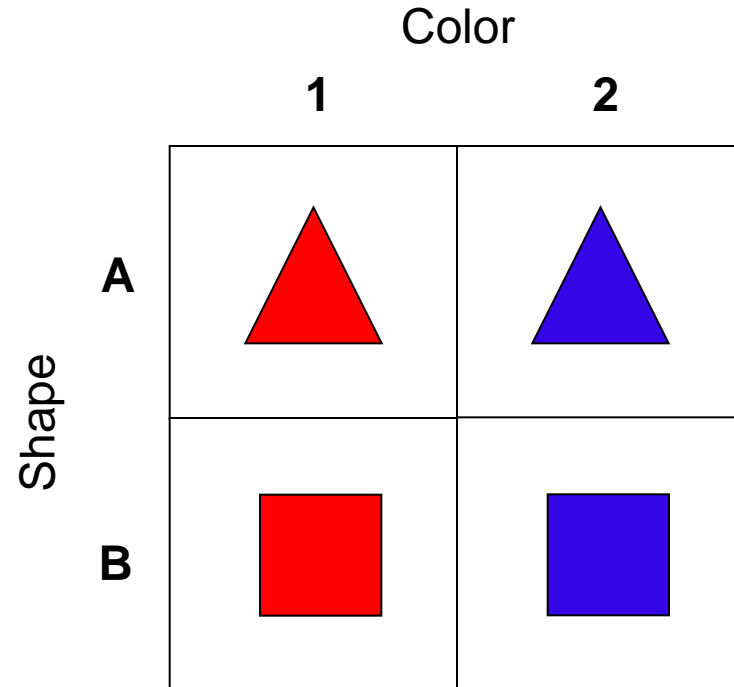
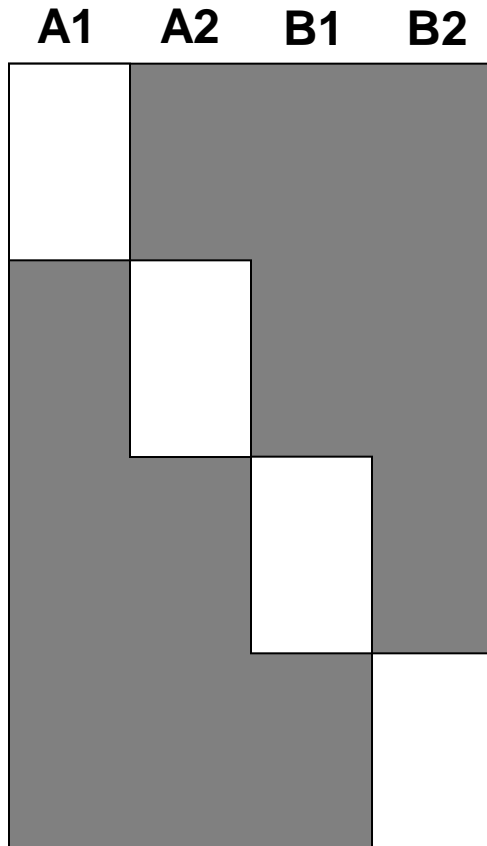
## Flexible Factorial



## Flexible Factorial



# 2x2 factorial design



**Main effect of Shape:**

$$(A1+A2) - (B1+B2) : 1 \ 1 \ -1 \ -1$$

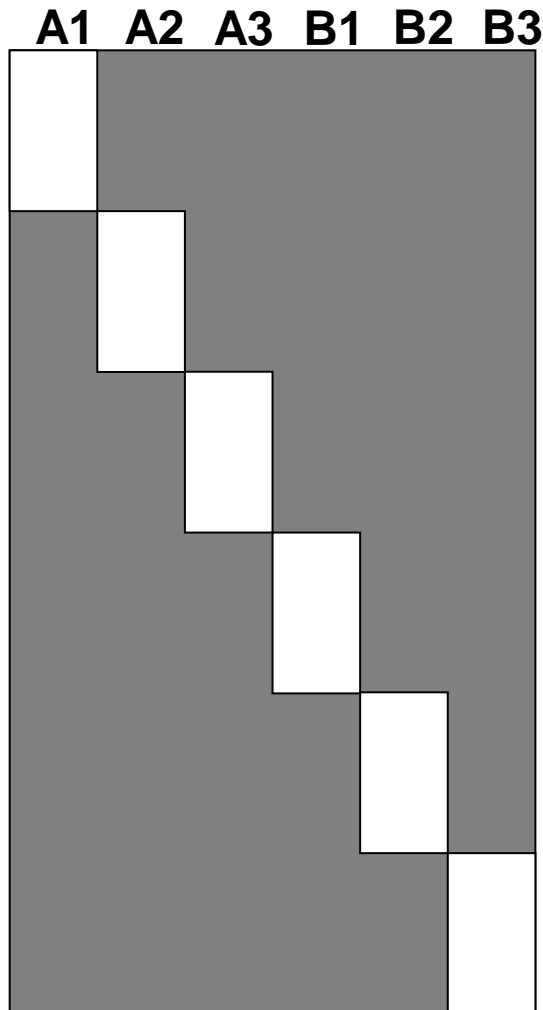
**Main effect of Color:**

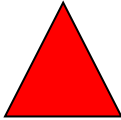
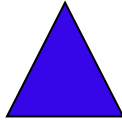
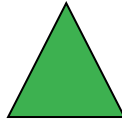
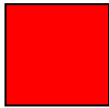
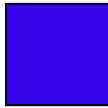

$$(A1+B1) - (A2+B2) : 1 \ -1 \ 1 \ -1$$

**Interaction Shape x Color:**

$$(A1-B1) - (A2-B2) : 1 \ -1 \ -1 \ 1$$

# 2x3 factorial design



		Color		
		1	2	3
Shape	A			
	B			

**Main effect of Shape:**

$$(A1+A2+A3) - (B1+B2+B3) : 1 \ 1 \ 1 \ -1 \ -1 \ -1$$

**Main effect of Color:**

$$(A1+B1) - (A2+B2) : 1 \ -1 \ 0 \ 1 \ -1 \ 0$$

$$(A2+B2) - (A3+B3) : 0 \ 1 \ -1 \ 0 \ 1 \ -1$$

$$(A1+B1) - (A3+B3) : 1 \ 0 \ -1 \ 1 \ 0 \ -1$$

**Interaction Shape x Color:**

$$(A1-B1) - (A2-B2) : 1 \ -1 \ 0 \ -1 \ 1 \ 0$$

$$(A2-B2) - (A3-B3) : 0 \ 1 \ -1 \ 0 \ -1 \ 1$$

$$(A1-B1) - (A3-B3) : 1 \ 0 \ -1 \ -1 \ 0 \ 1$$