

## Dynamic Causal Modelling (DCM)

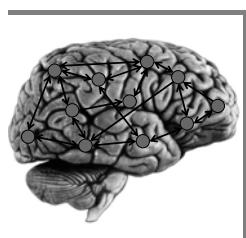
Presented by Uta Noppeney

With Thanks to and Slides from

Klaas Stephan

Will Penny

Karl Friston



Functional Imaging Lab  
Wellcome Dept. of Imaging Neuroscience  
Institute of Neurology  
University College London

## System analyses in functional neuroimaging

Analyses of regionally specific effects;  
which areas constitute a neuronal system?

Analyses of *inter-regional effects*:  
what are the interactions between the elements of a given neuronal system?

= the temporal correlation between spatially remote neurophysiological events

= the influence that the elements of a neuronal system exert over another  
**MODEL-dependent**

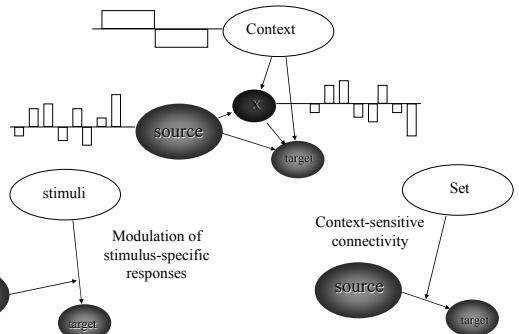
### • Functional Connectivity

Eigenimage analysis and PCA  
Nonlinear PCA  
ICA

### • Effective Connectivity

Psychophysiological Interactions  
MAR and State space Models  
Structure Equation Models  
Volterra Models  
Dynamic Causal Models

## Psychophysiological interactions



### • Functional Connectivity

Eigenimage analysis and PCA  
Nonlinear PCA  
ICA

### • Effective Connectivity

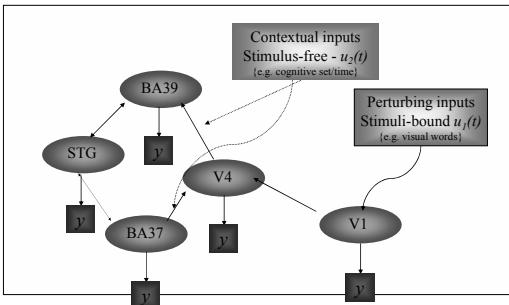
Psychophysiological Interactions  
MAR and State space Models  
Structure Equation Models  
Volterra Models  
Dynamic Causal Models

## Overview

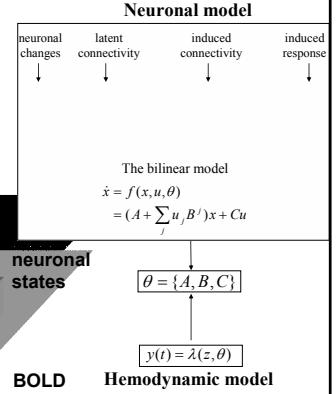
- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## The aim

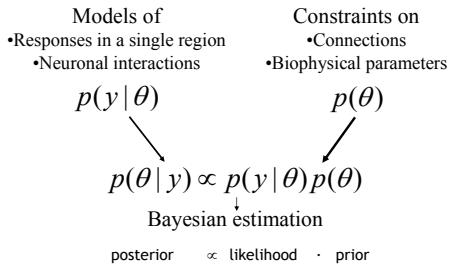
Functional integration and the modulation of specific pathways



## Conceptual overview



## Conceptual overview

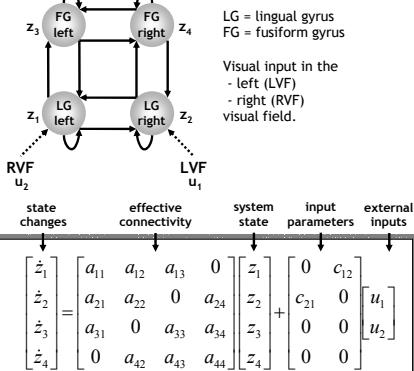


## DCM - Conceptual overview

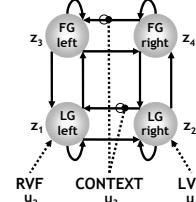
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## Overview

### Example: linear dynamic system



### Extension: bilinear dynamic system

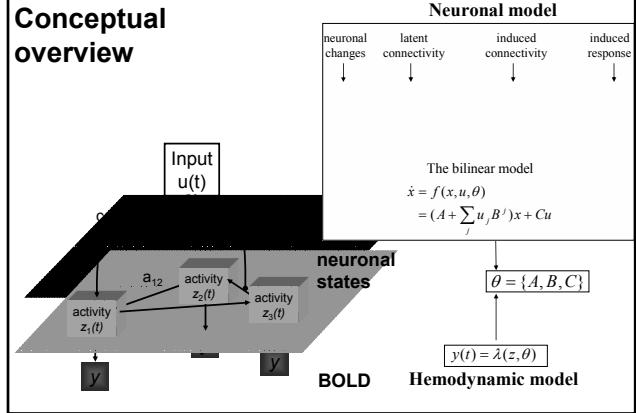


$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \left[ \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & 0 & a_{24} \\ a_{31} & 0 & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix} + u_3 \begin{bmatrix} 0 & b_{12}^3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{34}^3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 & c_{12} & 0 \\ c_{21} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

## Bilinear state equation in DCM

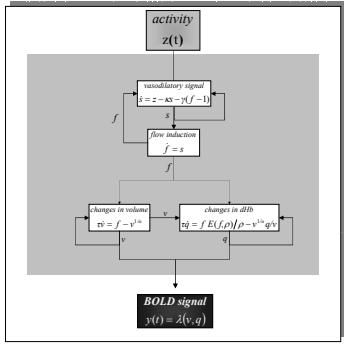
$$\begin{array}{ccccccc}
 \text{state} & \text{intrinsic} & \text{modulation of} & \text{system} & \text{direct} & \text{m external} \\
 \text{changes} & \text{connectivity} & \text{connectivity} & \text{state} & \text{inputs} & \text{inputs} \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 \left[ \begin{array}{c} \dot{z}_1 \\ \vdots \\ \dot{z}_n \end{array} \right] = & \left[ \begin{array}{ccc} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{array} \right] & \left[ \begin{array}{ccc} b_{11}' & \cdots & b_{1n}' \\ \vdots & \ddots & \vdots \\ b_{n1}' & \cdots & b_{nn}' \end{array} \right] & \left[ \begin{array}{c} z_1 \\ \vdots \\ z_n \end{array} \right] + & \left[ \begin{array}{ccc} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{array} \right] & u_1 \\ 
 \dot{z} = (A + \sum_{j=1}^m u_j B^j) z + C u \longrightarrow \theta^n = \{A, B^1 \dots B^m, C\}
 \end{array}$$

## Conceptual overview



## The hemodynamic “Balloon” model

- 5 hemodynamic parameters:  
 $\theta^h = \{\kappa, \gamma, \tau, \alpha, \rho\}$   
 ↓  
 important for model fitting, but of no interest for statistical inference
- Empirically determined prior distributions.
- Computed separately for each area (like the neural parameters).

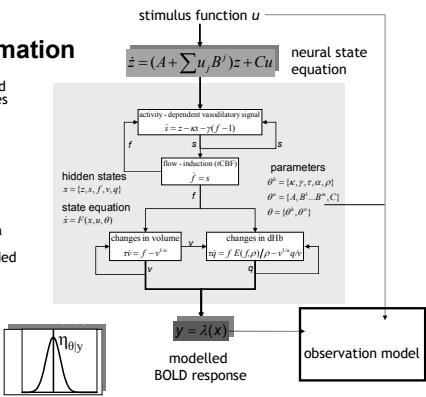


## Overview

- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example 1: attention to visual motion

## Overview: parameter estimation

- Combining the neural and hemodynamic states gives the complete forward model.
- An observation model includes measurement error  $\epsilon$  and confounds  $X$  (e.g. drift).
- Bayesian parameter estimation by means of a Levenberg–Marquardt gradient ascent, embedded into an EM algorithm.
- Result: Gaussian a posteriori parameter distributions, characterised by mean  $\eta_{\theta|y}$  and covariance  $C_{\theta|y}$ .



## Overview: parameter estimation

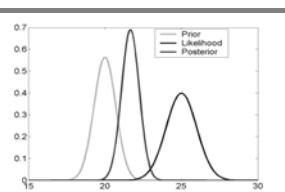
- Models of
- Responses in a single region
  - Neuronal interactions
- Constraints on
- Connections
  - Biophysical parameters
- $$p(y|\theta)$$
- $$p(\theta|y) \propto p(y|\theta)p(\theta)$$
- $$p(\theta|y) \propto p(y|\theta)p(\theta)$$
- posterior  $\propto$  likelihood · prior
- Bayesian estimation

## Priors in DCM

### Bayes Theorem

$$p(\theta | y) \propto p(y | \theta) \cdot p(\theta)$$

posterior  $\propto$  likelihood  $\cdot$  prior



- needed for Bayesian estimation, embody constraints on parameter estimation
- express our prior knowledge or "belief" about parameters of the model
- hemodynamic parameters: empirical priors
- temporal scaling: principled prior
- coupling parameters: shrinkage priors

## Priors in DCM

### Principled priors:

- System stability: In the absence of input, the neuronal states must return to a stable mode

- Constraints on prior variance of intrinsic connections (A): Probability <0.001 of obtaining a non-negative Lyapunov exponent (largest real eigenvalue of the intrinsic coupling matrix)

- Self-inhibition: Priors on the decay rate constant  $\sigma$  ( $\eta=1$ ,  $C_{\sigma}=0.105$ ): these allow for neural transients with a half life in the range of 300 ms to 2 seconds

- Shrinkage priors** for coupling parameters ( $\eta=0$ )  $\rightarrow$  conservative estimates!

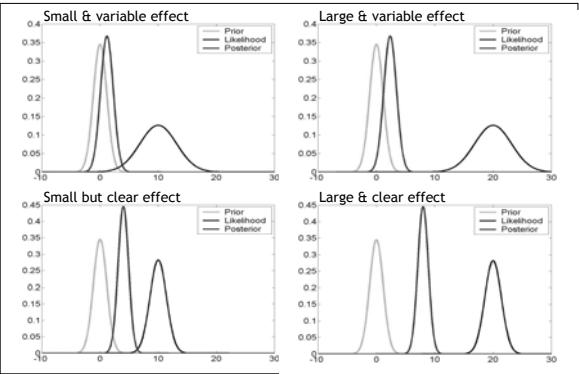
$$\begin{bmatrix} \sigma \\ a_{ij} \\ b_{ij} \\ c_{ij} \\ \theta^k \end{bmatrix} = \begin{bmatrix} \sigma \\ a_{ij} \\ b_{ij} \\ c_{ij} \\ \theta^k \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \eta_\theta & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.105 & \dots & 0 \\ C_4 & \vdots & C_s \\ \vdots & \ddots & 1 \\ 0 & \dots & C_{11} \end{bmatrix}$$

### Temporal scaling:

Identical in all areas by factorising A and B with  $\sigma$  (a single rate constant for all regions): all connection strengths are relative to the self-connections.

$$A \rightarrow \sigma A = \sigma \begin{bmatrix} -1 & a_{12} & \dots \\ a_{21} & -1 & \dots \\ \vdots & \ddots & \ddots \end{bmatrix}$$

## Shrinkage Priors



## Bayesian estimation: univariate Gaussian case

### Normal densities

$$p(\theta) = N(\theta; \eta_p, \sigma_p^2)$$

$$p(y | \theta) = N(y; \theta x, \sigma_e^2)$$

$$p(\theta | y) = N(\theta; \eta_{\theta|y}, \sigma_{\theta|y}^2)$$

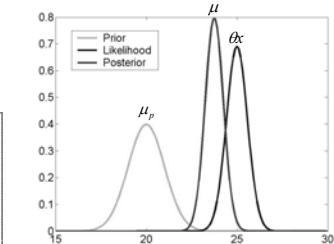
$$\frac{1}{\sigma_{\theta|y}^2} = \frac{x^2}{\sigma_e^2} + \frac{1}{\sigma_p^2}$$

$$\eta_{\theta|y} = \sigma_{\theta|y}^2 \left( \frac{x}{\sigma_e} y + \frac{1}{\sigma_e^2} \eta_p \right)$$

Relative precision weighting

Univariate linear model

$$y = \theta x + e$$



## Bayesian estimation: multivariate Gaussian case

### Normal densities

$$p(\theta) = N(\theta; \eta_p, C_p)$$

$$p(y | \theta) = N(y; X\theta, C_e)$$

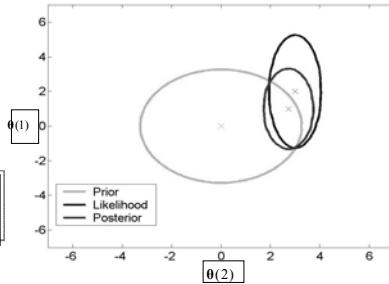
$$p(\theta | y) = N(\theta; \eta_{\theta|y}, C_{\theta|y})$$

$$C_{\theta|y}^{-1} = X^T C_e^{-1} X + C_p^{-1}$$

$$\eta_{\theta|y} = C_{\theta|y} (X^T C_e^{-1} y + C_p^{-1} \eta_p)$$

General Linear Model

$$y = X\theta + e$$



One step if  $C_e$  is known.

## Bayesian estimation: nonlinear case

Local linearization by 1<sup>st</sup> order Taylor:

$$y = h(\theta) + e$$

$$h(\theta) = h(\eta_{\theta|y}^i) + \frac{\partial h(\eta_{\theta|y}^i)}{\partial \theta} (\theta - \eta_{\theta|y}^i)$$

$$\mathbf{J} = \frac{\partial h(\eta_{\theta|y}^i)}{\partial \theta}$$

$$\Delta \theta = \theta - \eta_{\theta|y}^i$$

$$\mathbf{r}' = \mathbf{y} - h(\eta_{\theta|y}^i)$$

$$= \mathbf{J} \Delta \theta + e$$



### Current estimates

$$\eta_{\theta|y}^i, C_{\theta|y}^i$$

$$p(\theta) = N(\theta; \eta_p, C_p)$$

$$p(\Delta \theta) = N(\Delta \theta; \eta_p - \eta_{\theta|y}^i, C_p)$$

### Likelihood

$$p(y | \theta) = N(y; h(\theta), C_e)$$

$$p(\mathbf{r} | \Delta \theta) = N(\mathbf{r}; \mathbf{J} \Delta \theta, C_e)$$

### Gradient ascent (Fisher scoring) with priors

$$(C_{\theta|y}^{i+1})^{-1} = \mathbf{J}^T C_e^{-1} \mathbf{J} + C_p^{-1}$$

$$\eta_{\theta|y}^{i+1} = \eta_{\theta|y}^i + C_{\theta|y}^{i+1} (\mathbf{J}^T C_e^{-1} \mathbf{r} + C_p^{-1} (\eta_p - \eta_{\theta|y}^i))$$

## EM and gradient ascent

- Bayesian parameter estimation by means of expectation maximisation (EM)

- **E-step:**  
gradient ascent (Fisher scoring & Levenberg-Marquardt regularisation) to compute

- (i) the conditional mean  $\eta_{\theta|y}$  (= expansion point of gradient ascent),
- (ii) the conditional covariance  $C_{\theta|y}$

- **M-step:**  
Estimation of hyperparameters  $\lambda_i$  for error covariance components  $Q_i$

$$C_e = \sum \lambda_i Q_i$$

- Note: Gaussian assumptions about the posterior (Laplace approximation)

## Parameter estimation: output in command window (new)

```

E-Step: 1 F: -1.514001e+003 (dp:8.299907e-002)
E-Step: 2 F: -1.200724e+003 dp: 9.638851e-001
E-Step: 3 F: -1.115951e+003 dp: 2.703493e-001
E-Step: 4 F: -1.077757e+003 dp: 2.002973e-002
E-Step: 5 F: -1.075699e+003 dp: 4.219233e-003
E-Step: 6 F: -1.075663e+003 dp: 1.030322e-003
E-Step: 7 F: -1.075661e+003 dp: 3.595806e-004
E-Step: 8 F: -1.075661e+003 dp: 2.273264e-006

```

$$dp = \|\Delta\theta\|_2$$

Change of the norm of the parameter vector (= magnitude of update)

objective function

$$F = \frac{1}{2} \left( -(y - h(\theta))^T C_e^{-1} (y - h(\theta)) - (\theta_{\theta|y} - \theta_p)^T C_p^{-1} (\theta_{\theta|y} - \theta_p) - \log |C_e| - \log |C_p| + \log |C_{\theta|y}| \right)$$

## Parameter estimation in DCM

- Combining the neural and hemodynamic states gives the complete forward model:

$$x = \{z, s, f, v, q\}$$

$$\theta = \theta^n + \theta^h$$

$$\dot{x} = f(x, u, \theta)$$

$$y = \lambda(x) = h(u, \theta)$$

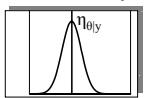
- The observation model includes measurement error  $\varepsilon$  and confounds  $X$  (e.g. drift):

$$y = h(u, \theta) + X\beta + \varepsilon$$

- Bayesian parameter estimation under Gaussian assumptions by means of EM and gradient ascent.

$$y - h(u, \eta_{\theta|y}) \rightarrow \min$$

- Result:**  
Gaussian *a posteriori* parameter distributions with mean  $\eta_{\theta|y}$  and covariance  $C_{\theta|y}$ .



## Overview

- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## DCM parameters = rate constants

Generic solution to the ODEs in DCM:

$$\frac{dz}{dt} = az \quad \rightarrow \quad z(t) = \exp(at) + c$$

Decay function:

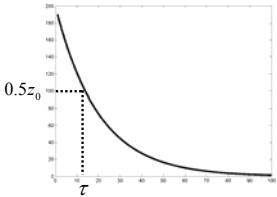
$$z(t) = z_0 \exp(-at)$$

Half-life  $\tau$ :

$$z(\tau) = 0.5z_0$$

$$= z_0 \exp(-a\tau)$$

$$\rightarrow a = \ln 2 / \tau$$



## Interpretation of DCM parameters

- Dynamic model (differential equations)  
→ neural parameters correspond to rate constants (inverse of time constants → Hz!)  
→ speed at which effects take place

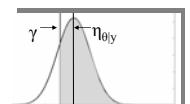
- Identical temporal scaling in all areas by factorising A and B with σ:  
all connection strengths are relative to the self-connections.

- Each parameter is characterised by the mean ( $\eta_{\theta|y}$ ) and covariance of its *a posteriori* distribution. Its mean can be compared statistically against a chosen threshold  $\gamma$ .

$$\theta^n = \{A, B, C, \sigma\}$$

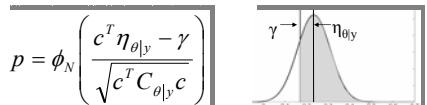
$$p = \ln 2 / \tau_p$$

$$A \rightarrow \sigma A = \sigma \begin{bmatrix} -1 & a_{12} & \dots \\ a_{21} & -1 & \dots \\ \vdots & \ddots & \ddots \end{bmatrix}$$



## Inference about DCM parameters: single-subject analysis

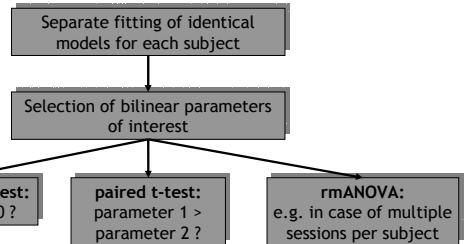
- Bayesian parameter estimation in DCM: Gaussian assumptions about the *a posteriori* distributions of the parameters
- Use of the cumulative normal distribution to test the probability by which a certain parameter (or contrast of parameters  $c^T \eta_{\theta|y}$ ) is above a chosen threshold  $\gamma$ :



- $\gamma$  can be chosen as a function of the expected half life of the neural process, e.g.  $\gamma = \ln 2 / \tau$

## Inference about DCM parameters: group analysis

- In analogy to “random effects” analyses in SPM, 2<sup>nd</sup> level analyses can be applied to DCM parameters:

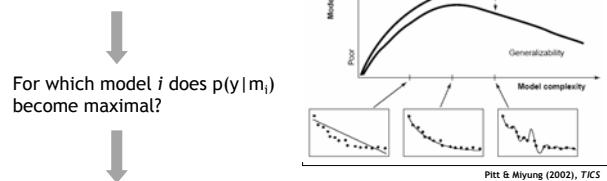


## Overview

- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## Model comparison and selection

Given competing hypotheses on structure & functional mechanisms of a system, which model is the best?



For which model  $i$  does  $p(y|m_i)$  become maximal?

Which model represents the best balance between model fit and model complexity?

## Bayesian Model Selection

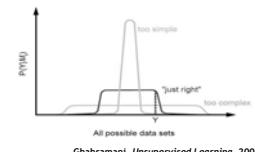
Bayes theorem:

$$p(\theta|y, m) = \frac{p(y|\theta, m)p(\theta|m)}{p(y|m)}$$

Model evidence:

$$p(y|m) = \int p(y|\theta, m) \cdot p(\theta|m) d\theta$$

Occam's Razor:



## Bayesian Model Selection

Model evidence:

$$p(y|m) = \int p(y|\theta, m) \cdot p(\theta|m) d\theta$$

Laplace approximation:

$$\begin{aligned} F &= \text{accuracy}(m) - \text{complexity}(m) \\ &= -\frac{1}{2} \log |\mathbf{C}_e| - \frac{1}{2} (\mathbf{y} - \mathbf{h}(\mathbf{\theta}))^\top \mathbf{C}_e^{-1} (\mathbf{y} - \mathbf{h}(\mathbf{\theta})) \\ &\quad - \frac{1}{2} (\mathbf{\theta}_{\theta|y} - \mathbf{\theta}_p)^\top \mathbf{C}_p^{-1} (\mathbf{\theta}_{\theta|y} - \mathbf{\theta}_p) - \frac{1}{2} \log |\mathbf{C}_p| + \frac{1}{2} \log |\mathbf{C}_{\theta|y}| \end{aligned}$$

The log model evidence can be represented as:

$$\log p(y|m) = \text{accuracy}(m) - \text{complexity}(m)$$

## Approximations to model evidence

Bayesian information criterion (BIC):

$$BIC(y | m) = \text{accuracy}(m) - \frac{p}{2} \log N_s$$

Akaike information criterion (AIC):

$$AIC(y | m) = \text{accuracy}(m) - p$$

Bayes factor:

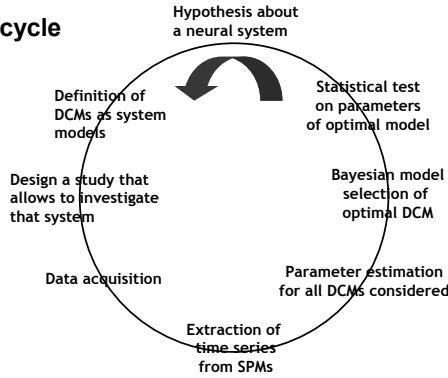
$$B_y = \frac{p(y | m = i)}{p(y | m = j)}$$

Penny et al., 2004, *NeuroImage*

## Overview

- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## The DCM cycle

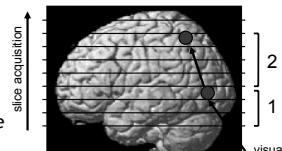


## Planning a DCM-compatible study

- **Suitable experimental design:**
  - preferably multi-factorial (e.g. 2 x 2)
  - e.g. one factor that varies the driving (sensory) input
  - and one factor that varies the contextual input
- **Hypothesis and model:**
  - define specific *a priori* hypothesis
  - Which alternative models?
  - which parameters are relevant to test this hypothesis?
- **TR:**
  - as short as possible (optimal: < 2 s)

## Timing problems at long TRs

- **Two potential timing problems in DCM:**
  1. wrong timing of inputs
  2. temporal shift between regional time series because of multi-slice acquisition
- DCM is robust against timing errors up to approx.  $\pm 1$  s
  - compensatory changes of  $\sigma$  and  $\theta$
- **Possible corrections:**
  - restriction of the model to neighbouring regions
  - in both cases: adjust temporal reference bin in SPM defaults (defaults.stats.fmri.t0)



## Practical steps of a DCM study - I

1. **Conventional SPM analysis (subject-specific)**
  - DCMs are fitted separately for each session
    - consider concatenation of sessions or adequate 2<sup>nd</sup> level analysis
2. **Extraction of time series**, e.g. via VOI tool in SPM
  - cave: anatomical & functional standardisation important for group analyses!

## Practical steps of a DCM study - II

3. Possibly definition of a new design matrix, if the "normal" design matrix does not represent the inputs appropriately.  
 • NB: DCM only reads timing information of each input from the design matrix, no parameter estimation necessary.

### 4. Definition of model

- via DCM-GUI or directly in MATLAB



## Practical steps of a DCM study - III

### 5. DCM parameter estimation

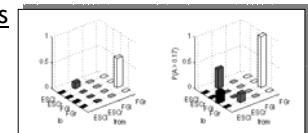
- cave: models with many regions & scans can crash MATLAB!

### 6. Model comparison and selection:

- Which of all models considered is the optimal one?  
 → Bayesian model selection tool

### 7. Testing the hypothesis

Statistical test on the relevant parameters of the optimal model



## Overview

- DCM - Conceptual overview
- Neural and hemodynamic levels in DCM
- Parameter estimation
  - Priors in DCM
  - Bayesian parameter estimation in non-linear systems
- Interpretation of parameters
- Bayesian model selection
- Practical steps of a DCM study
- Example: attention to visual motion

## Attention to motion in the visual system

Stimuli 250 radially moving dots at 4.7 degrees/s

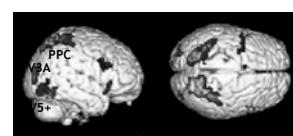
Pre-Scanning

5 x 30s trials with 5 speed changes (reducing to 1%)  
 Task - detect change in radial velocity



Scanning (no speed changes)

6 normal subjects, 4 x 100 scan sessions;  
 each session comprising 10 scans of 4 different  
 conditions



F A F N F A F N S .....

F - fixation point only

A - motion stimuli with attention (detect changes)

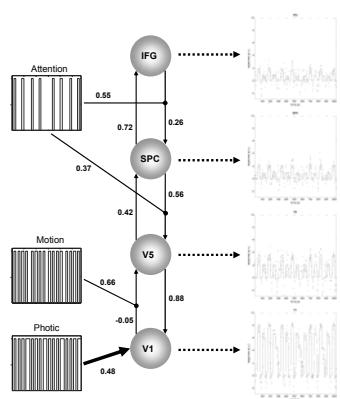
N - motion stimuli without attention

S - no motion

Büchel & Friston 1997, Cereb. Cortex  
 Büchel et al. 1998, Brain

## A simple DCM of the visual system

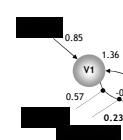
- Visual inputs drive V1, activity then spreads to hierarchically arranged visual areas.
- Motion modulates the strength of the V1→V5 forward connection.
- The intrinsic connection V1→V5 is insignificant in the absence of motion ( $a_{21}=-0.05$ ).
- Attention increases the backward-connections IFG→SPC and SPC→V5.



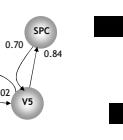
Re-analysis of data from  
 Friston et al., NeuroImage 2003

## Comparison of three simple models

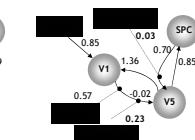
**Model 1:**  
 attentional modulation of V1→V5



**Model 2:**  
 attentional modulation of SPC→V5



**Model 3:**  
 attentional modulation of V1→V5 and SPC→V5



Bayesian model selection:

Model 1 better than model 2,  
 model 1 and model 3 equal

→ Decision for model 1:  
 in this experiment, attention primarily modulates V1→V5

