

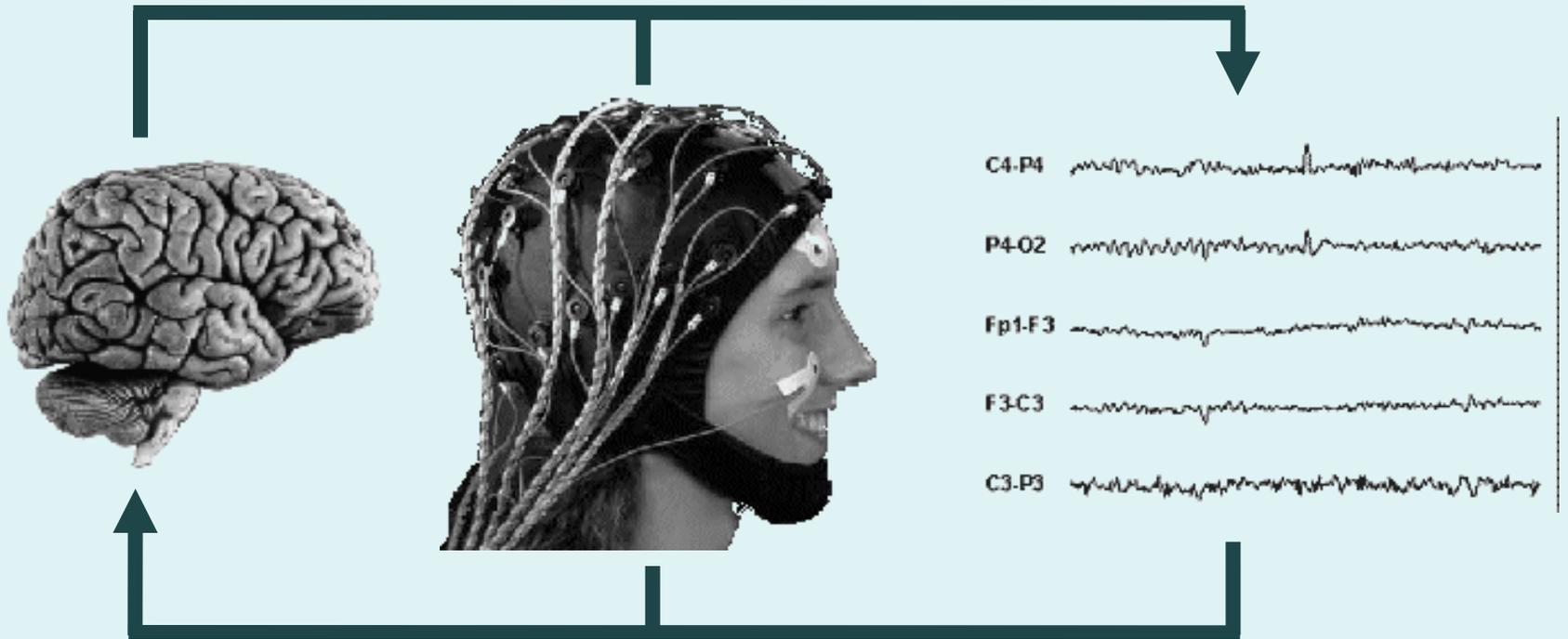
# **M/EEG source reconstruction: problems & solutions**

SPM-M/EEG course  
Lyon, April 2012

# Source localisation in M/EEG

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## Forward Problem

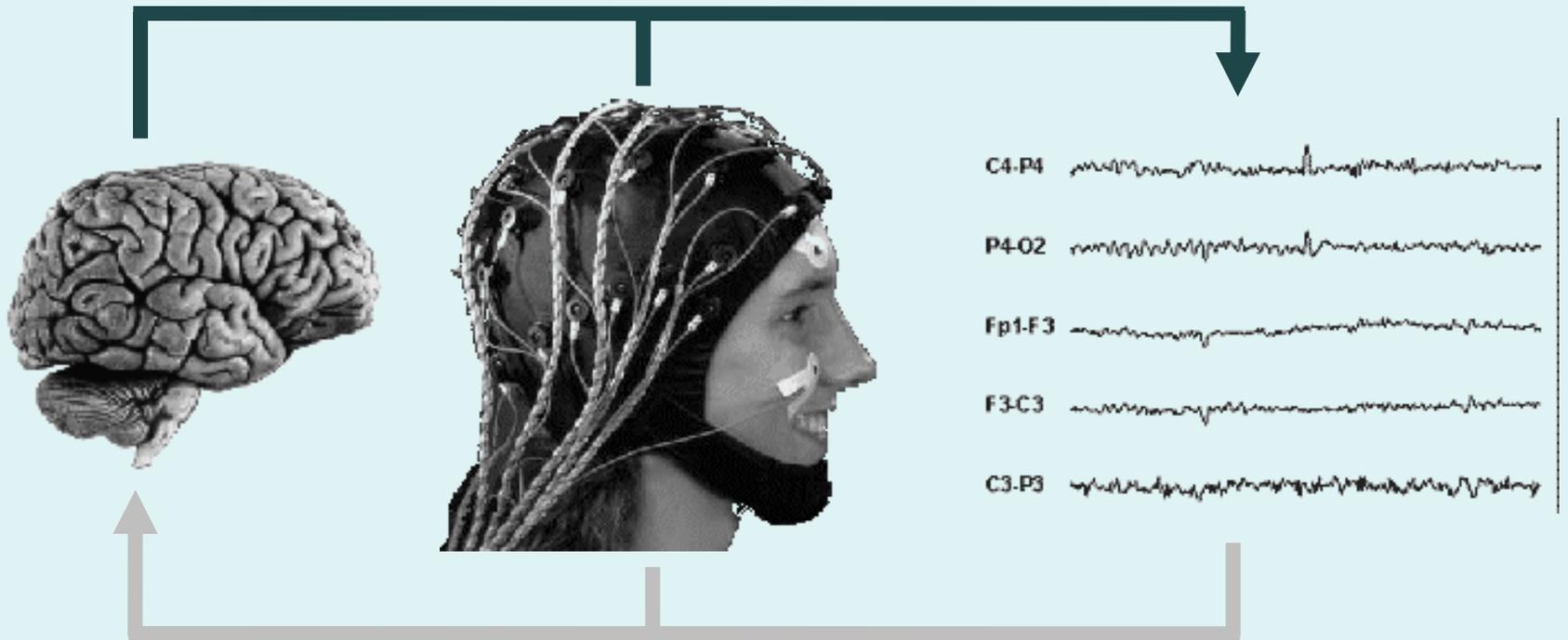


## Inverse Problem

# Source localisation in M/EEG

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## Forward Problem

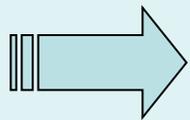
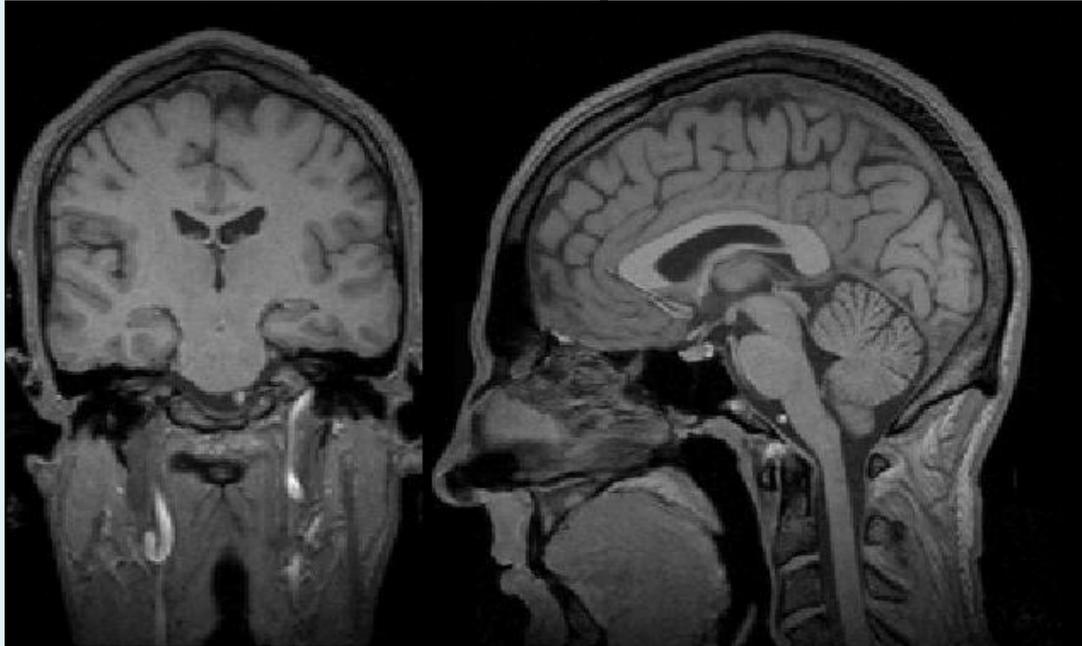


## Inverse Problem

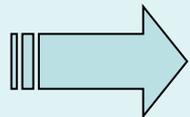
# Forward problem

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## Head anatomy



Head model : conductivity layout  
Source model : current dipoles



Solution by **Maxwell's equations**

# Maxwell's equations (1873)

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$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{j} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}$$

Ohm's law :

$$\vec{j} = \sigma \vec{E}$$

Continuity equation :

$$\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

# Solving the forward problem

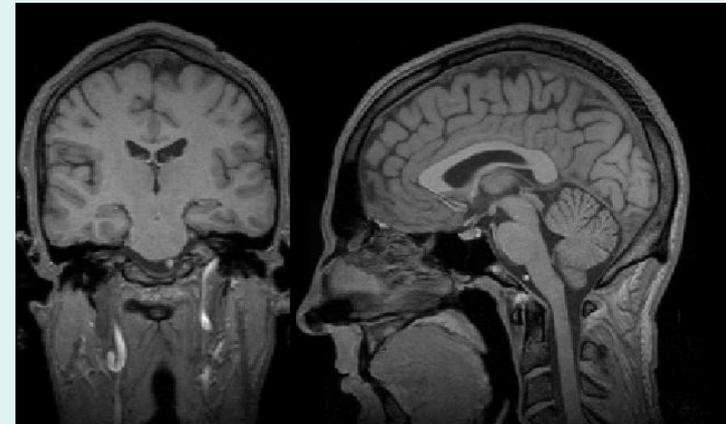
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From Maxwell's equations find:

$$M = f(\vec{j}, \vec{r})$$

where  $M$  are the measurements and  $f(.)$  depends on:

- signal recorded, EEG or MEG
- head model, i.e. conductivity layout adopted
- source location
- source orientation & amplitude

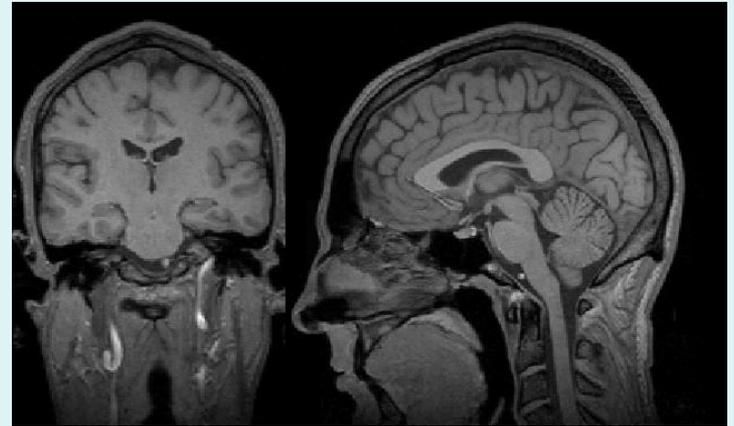


# Solving the forward problem

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From Maxwell's equations find:

$$M = f(\vec{j}, \vec{r})$$



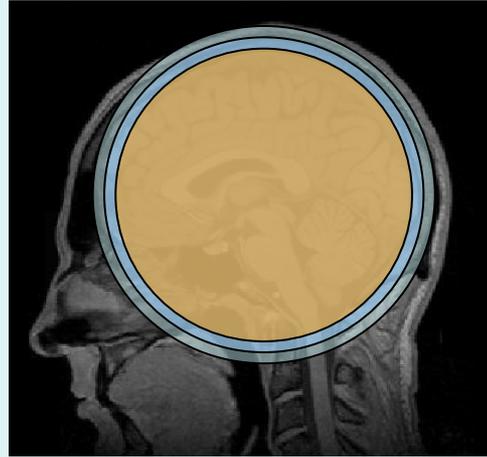
with  $f(.)$  as

- an analytical solution
  - highly symmetrical geometry, e.g. spheres, concentric spheres, etc.
  - homogeneous isotropic conductivity
- a numerical solution
  - more general (but still limited!) head model

# Analytical solution

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Example: 3 concentric spheres



Pro's:

- Simple model
- Exact mathematical solution
- Fast calculation

Con's:

- Human head is not spherical
- Conductivity is not homogeneous and isotropic.

# Numerical solution

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Usually “Boundary Element Method” (BEM) :

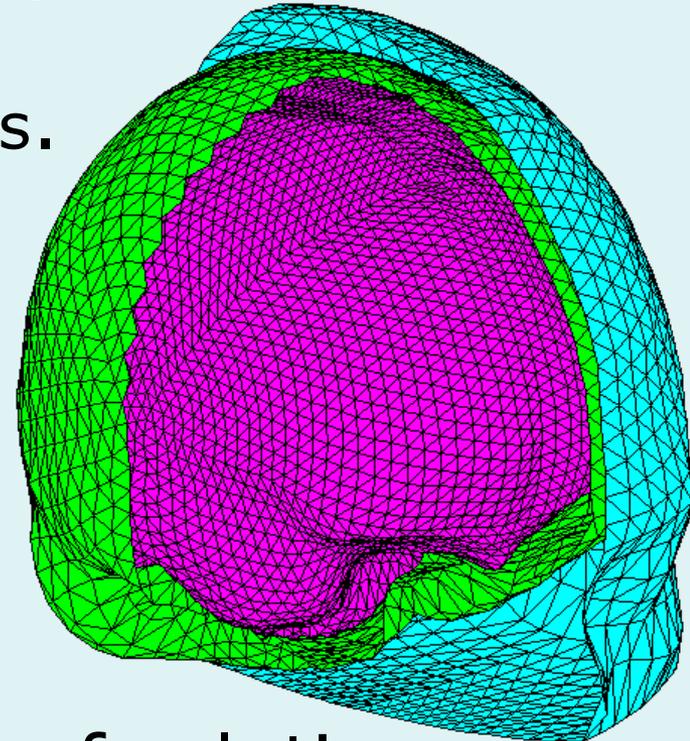
- Concentric sub-volumes of homogeneous and isotropic conductivity,
- Estimate values on the interfaces.

Pro's:

- More correct head shape modelling (not perfect though!)

Con's:

- Mathematical approximations of solution  
→ numerical errors
- Slow and intensive calculation



# Features of forward solution

Find forward solution (any):

$$M = f(\vec{j}, \vec{r})$$

with  $f(\cdot)$

- linear in  $\vec{j} = \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}^T$
- non-linear in  $\vec{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}^T$

If  $N$  sources with *known & fixed* location, then

$$M = f\left(\begin{bmatrix} \vec{r}_1 & \vec{r}_2 & \dots & \vec{r}_N \end{bmatrix}\right) \times \mathbf{J} = \mathbf{L} \times \mathbf{J}$$

# SPM solutions

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## Source space & head model

- Template Cortical Surface (TCS), in MNI space.
- Canonical Cortical Surface (CCS) = TCS warped to subject's anatomy
- Subject's Cortical Surface (SCS) = extracted from subject's own structural image (BrainVisa/FreeSurfer)

# SPM solutions

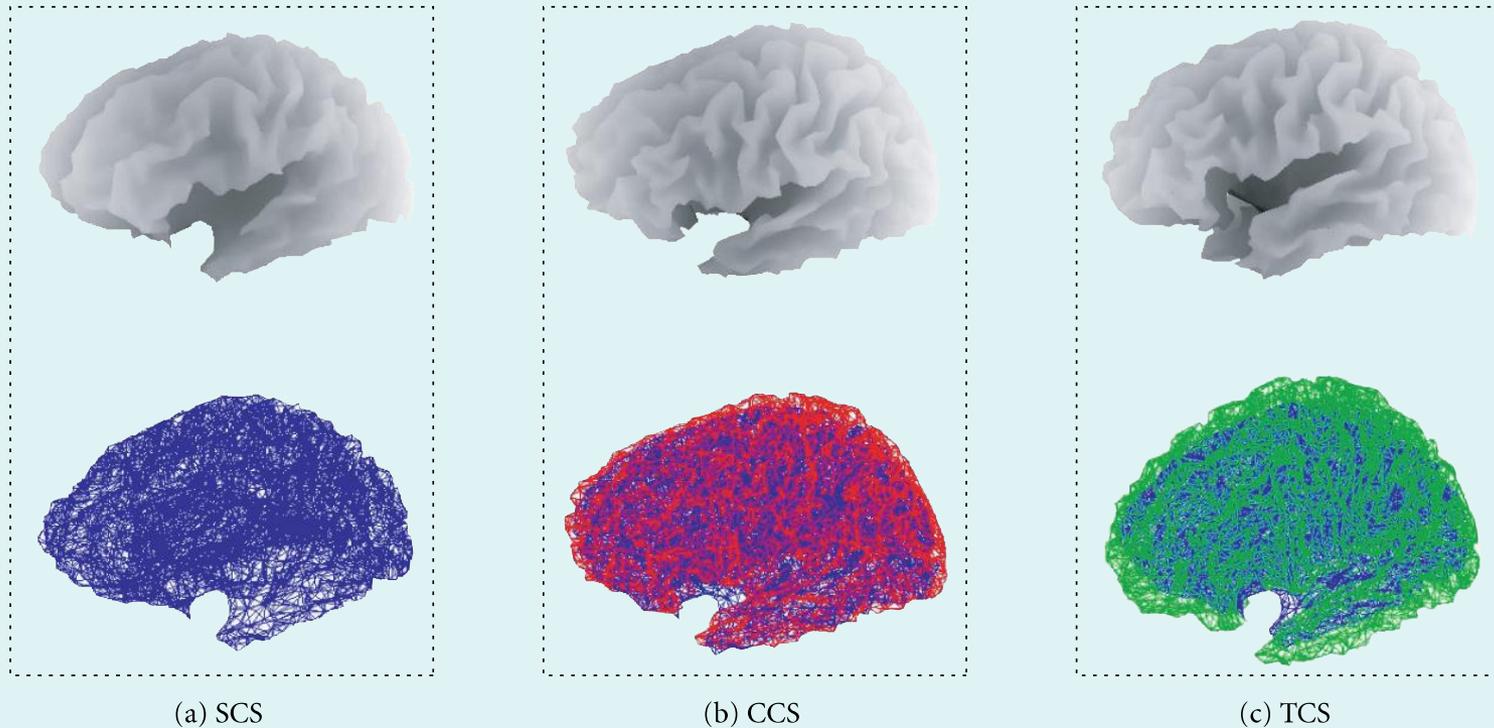


FIGURE 4: Surface rendering (upper row) and meshes (lower row) encoding the three cortical models: SCS (a), CCS (b), and TCS (c). CCS (red) and TCS (green) meshes are superimposed on the SCS mesh (blue).

# SPM solutions

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## Source space & head model

- Template Cortical Surface (TCS), in MNI space.
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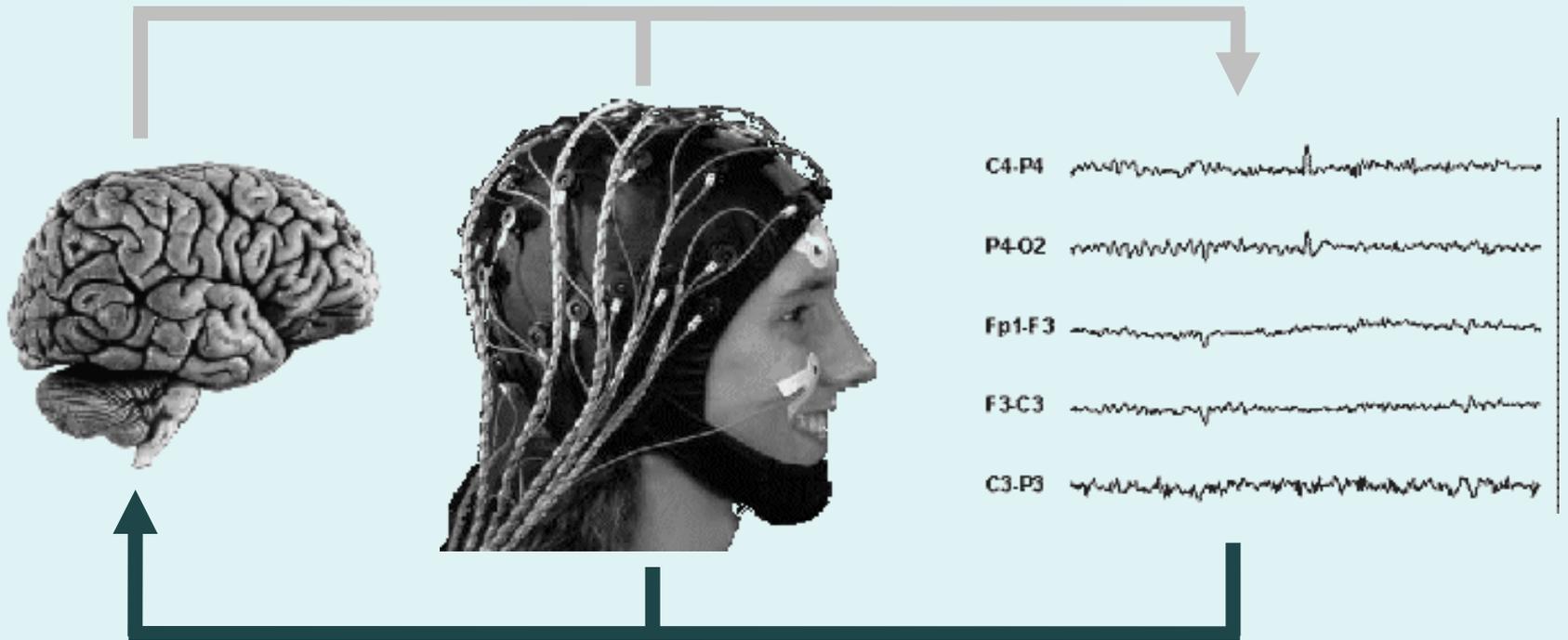
## Forward solutions:

- Single sphere
- Overlapping spheres
- Concentric spheres
- BEM
- ... (new things get added to SPM & FieldTrip)

# Source localisation in M/EEG

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Forward Problem



Inverse Problem



# Useful priors for cinema audiences

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- Things further from the camera appear smaller
- People are about the same size
- Planes are much bigger than people

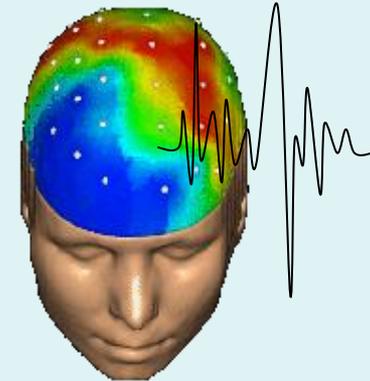
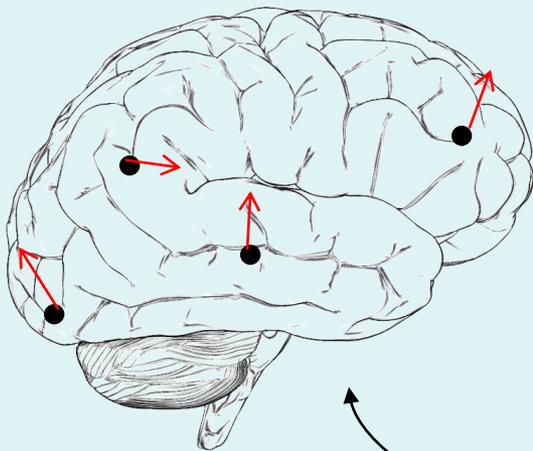
# M/EEG inverse problem

## Probabilistic framework

forward computation

Likelihood & Prior

$$p(Y|q, M) \quad p(q|M)$$



inverse computation

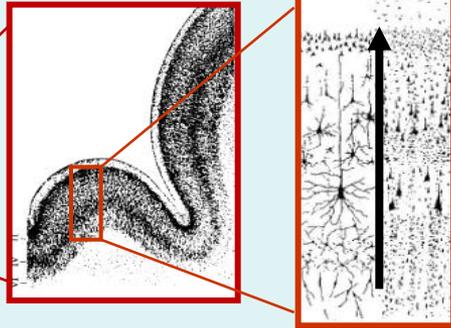
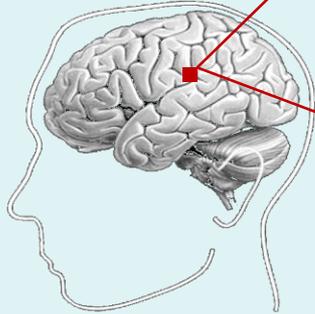
Posterior & Evidence

$$p(q|Y, M) \quad p(Y|M)$$



# Distributed or imaging model

Likelihood



$$Y = LJ + \varepsilon \quad P(Y|\theta, M) = N(LJ, \Sigma)$$

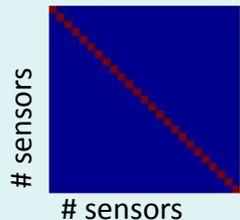
Parameters  $\theta$ :  $(J, \Sigma)$

Hypothesis  $M$ : distributed (linear) model, gain matrix  $L$ , Gaussian distributions

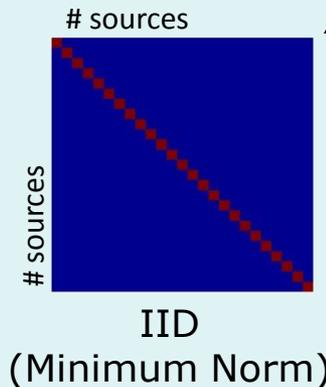
Priors

*Sensor level:*

$$\Sigma = \sigma^2 I$$



*Source level:*  $P(J) = N(0, \Delta)$

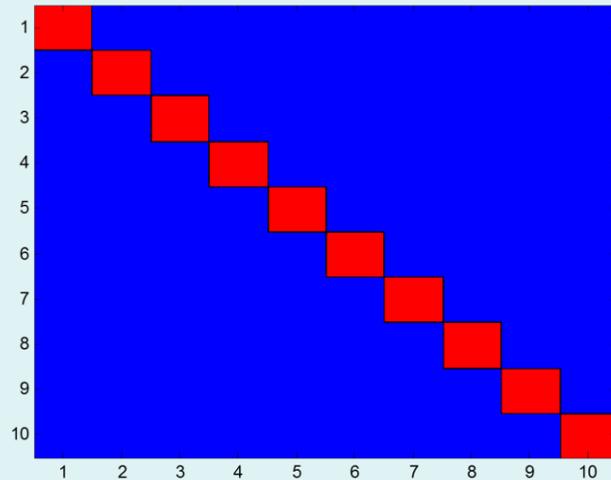




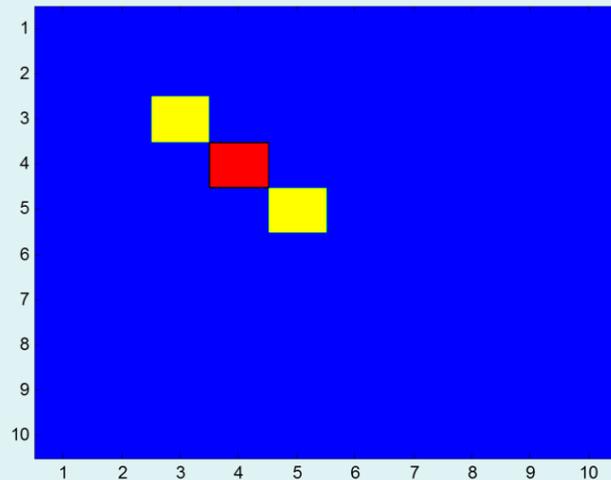
# Minimum norm solution

= "allow all sources to be active, but keep energy to a minimum"

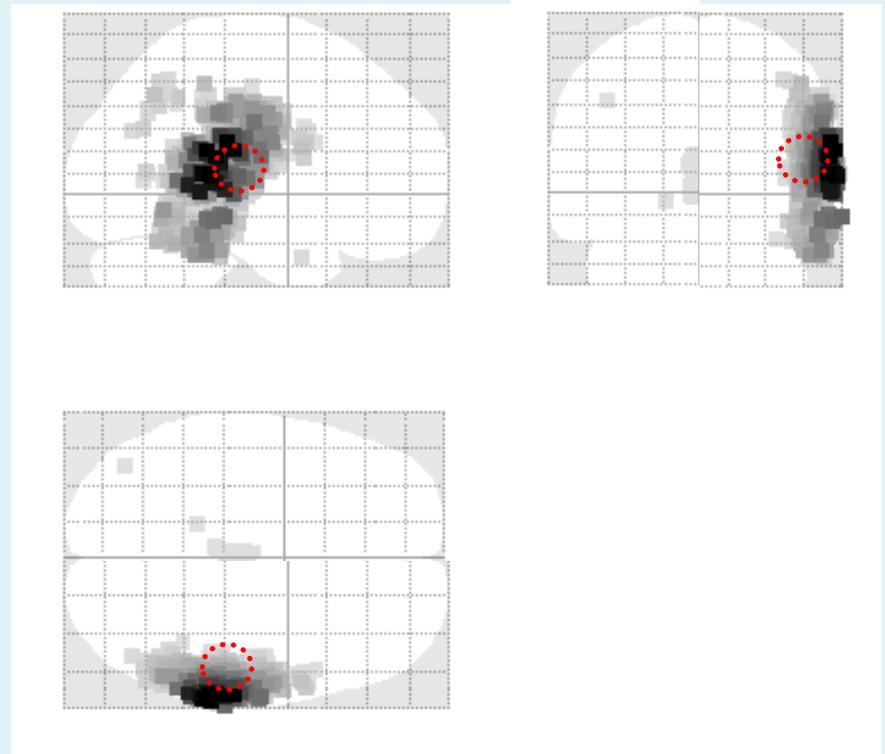
Prior



True  
(focal  
source)

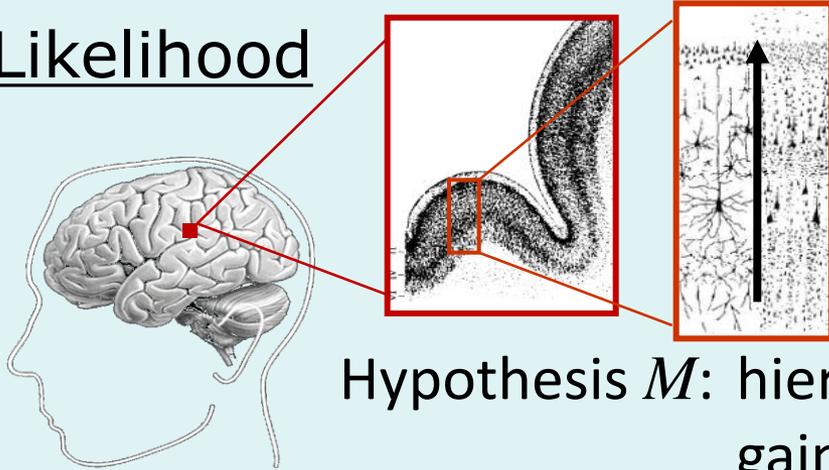


Solution



# Incorporating multiple constraint

## Likelihood



$$Y = LJ + \varepsilon \quad P(Y|\theta, M) = N(LJ, \Sigma)$$

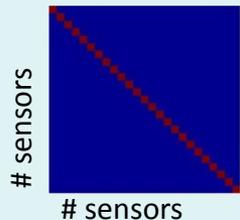
Parameters  $\theta: (J, \sigma, \lambda)$

Hypothesis  $M$ : hierarchical model, Gaussian distributions, gain matrix  $L$ , variance components  $Q^i$

## Priors

*Sensor level:*

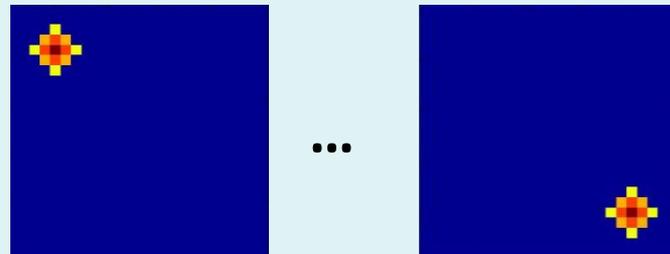
$$\Sigma = \sigma^2 I$$



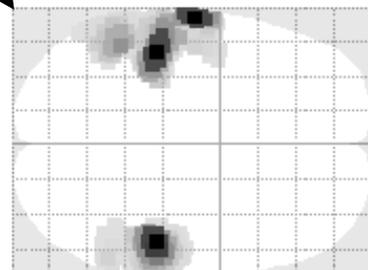
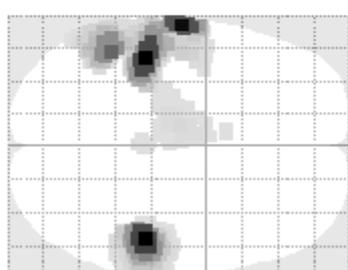
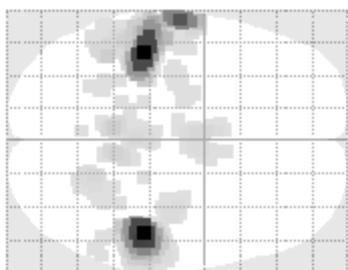
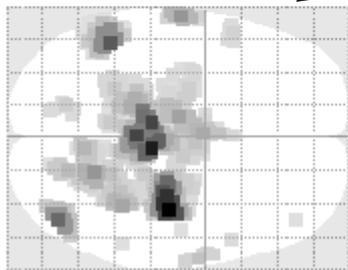
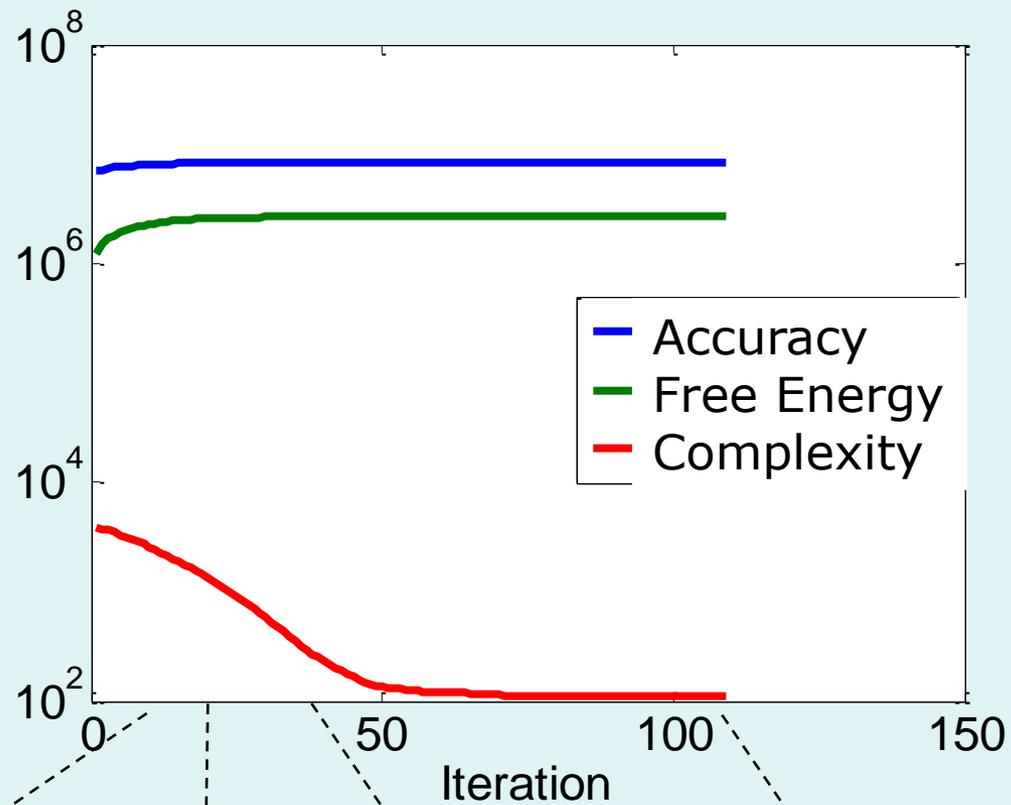
*Source level:*  $P(J) = N(0, \Delta)$

$$\Delta = \lambda_1 Q^1 + \dots + \lambda_k Q^k$$

$$\log l = N(a, b)$$

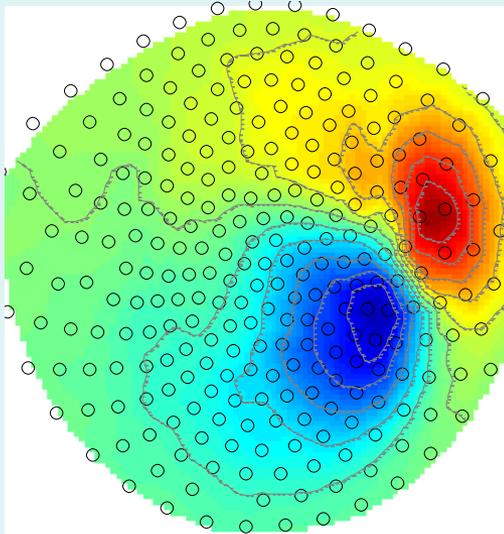


For example: Multiple Sparse Priors (MSP)

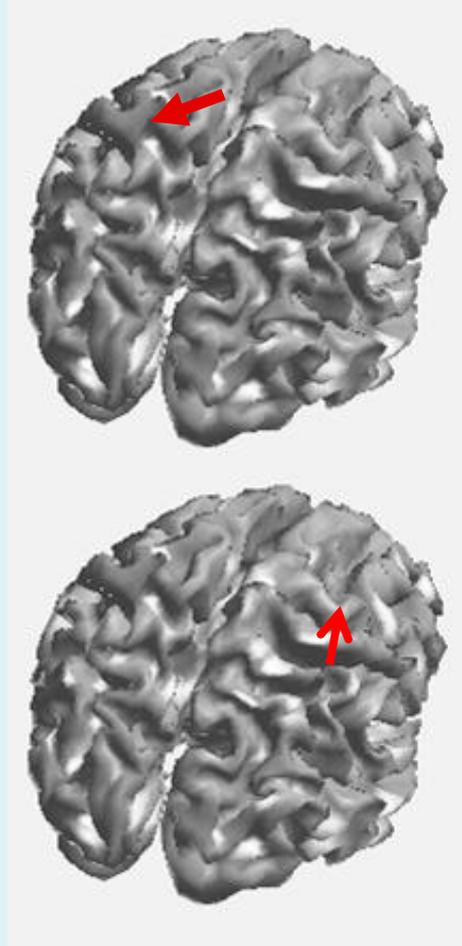


# Dipole fitting

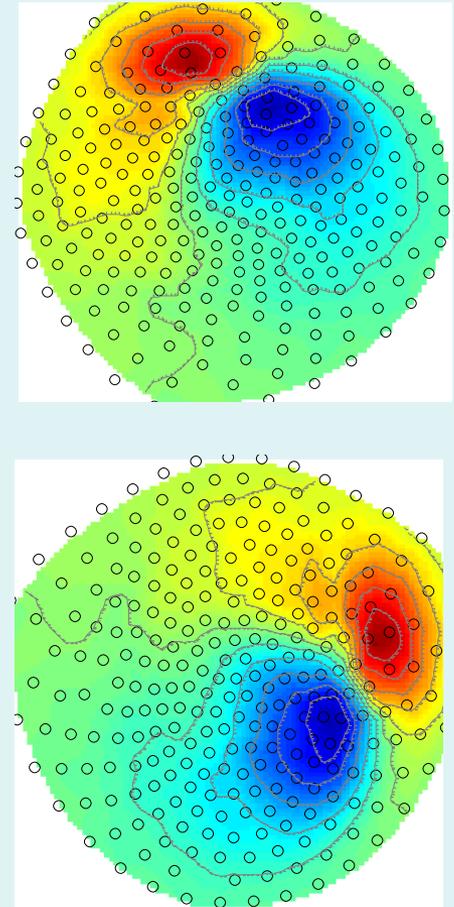
Measured data



Estimated position



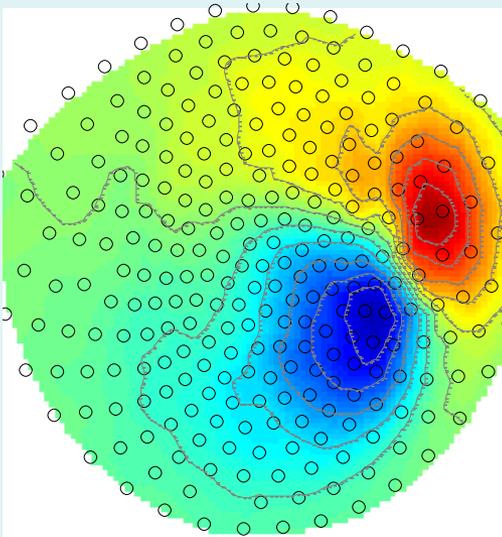
Estimated data



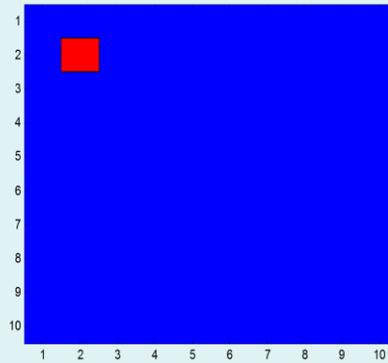
Constraint: *very few dipoles!*

# Dipole fitting

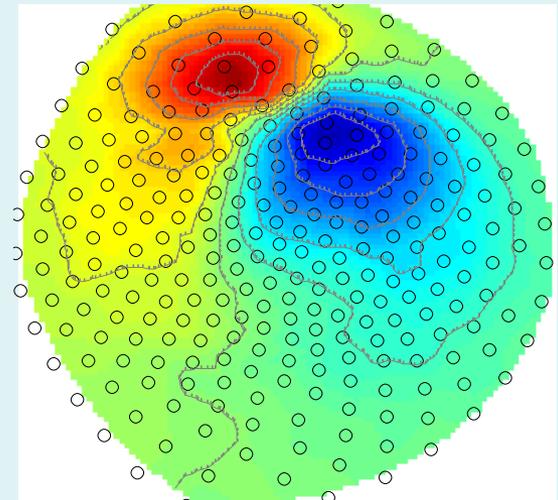
Measured data



Prior source covariance

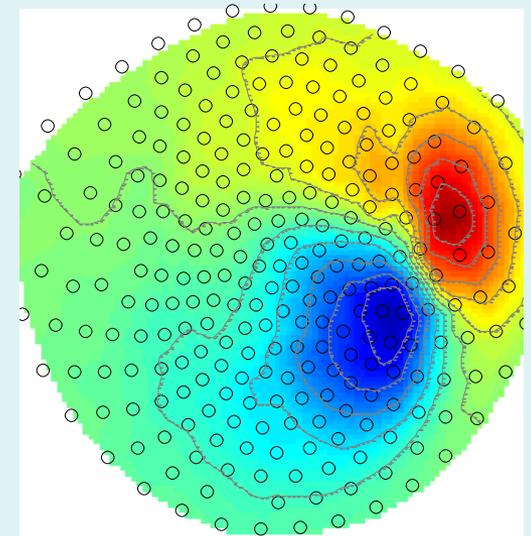
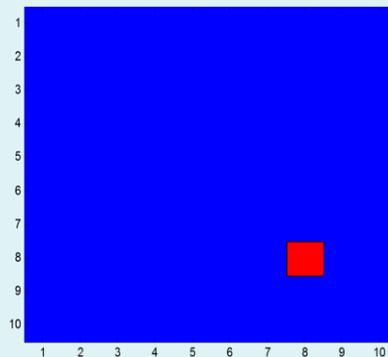
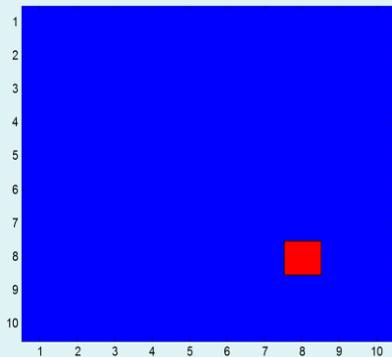


Estimated data



?

True source covariance



# Conclusion

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- Solving the Forward Problem is not exciting but necessary...  
...MEG or EEG? individual sMRI available?  
sensor location available?
- M/EEG inverse problem can be solved...  
...If you provide some *prior* knowledge!
- All prior knowledge encapsulated in a covariance matrices (sensors & sources)
- Can test between models and priors (a.k.a. constraints) in a Bayesian framework.

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# Thank you for your attention

And many thanks to Gareth and Jérémie for the borrowed slides.