

# M/EEG source analysis

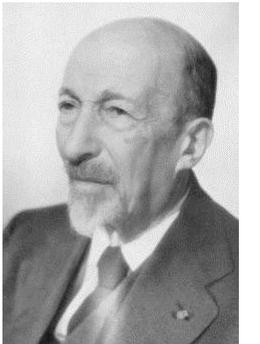


**Jérémie Mattout**

Lyon Neuroscience Research Center

*“Will it ever happen that mathematicians will know enough about the physiology of the brain, and neurophysiologists enough of mathematical discovery, for efficient cooperation to be possible”*

*Jacques Hadamard (mathematician, 1865-1963)*



# Introduction

## M/EEG source analysis

### Introduction

Ill-posed inverse problem

Why a Bayesian approach?

### Generative models

Sources

From sources to sensors

Sensors

### Bayesian inference

ECD model

Imaging models

Setting priors

Empirical Bayes

Comparing models

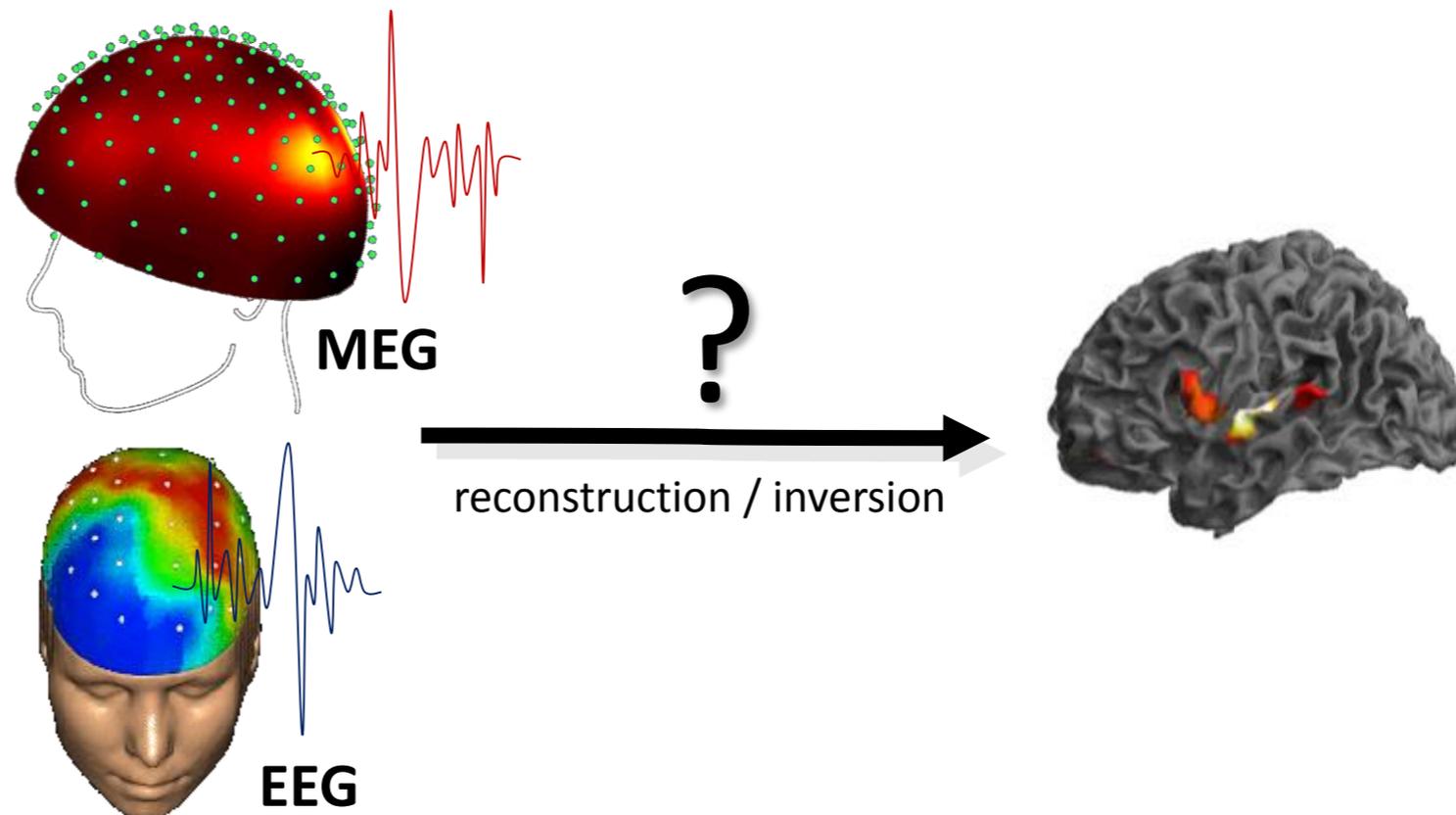
Group inference

EEG/MEG fusion

### Example

MMN study

Group multimodal inference



### Well-posed inverse problem:

- a solution exists ✓
- the solution is unique ✗
- the solution is stable ✗

*Jacques Hadamard* (mathematician, 1865-1963)



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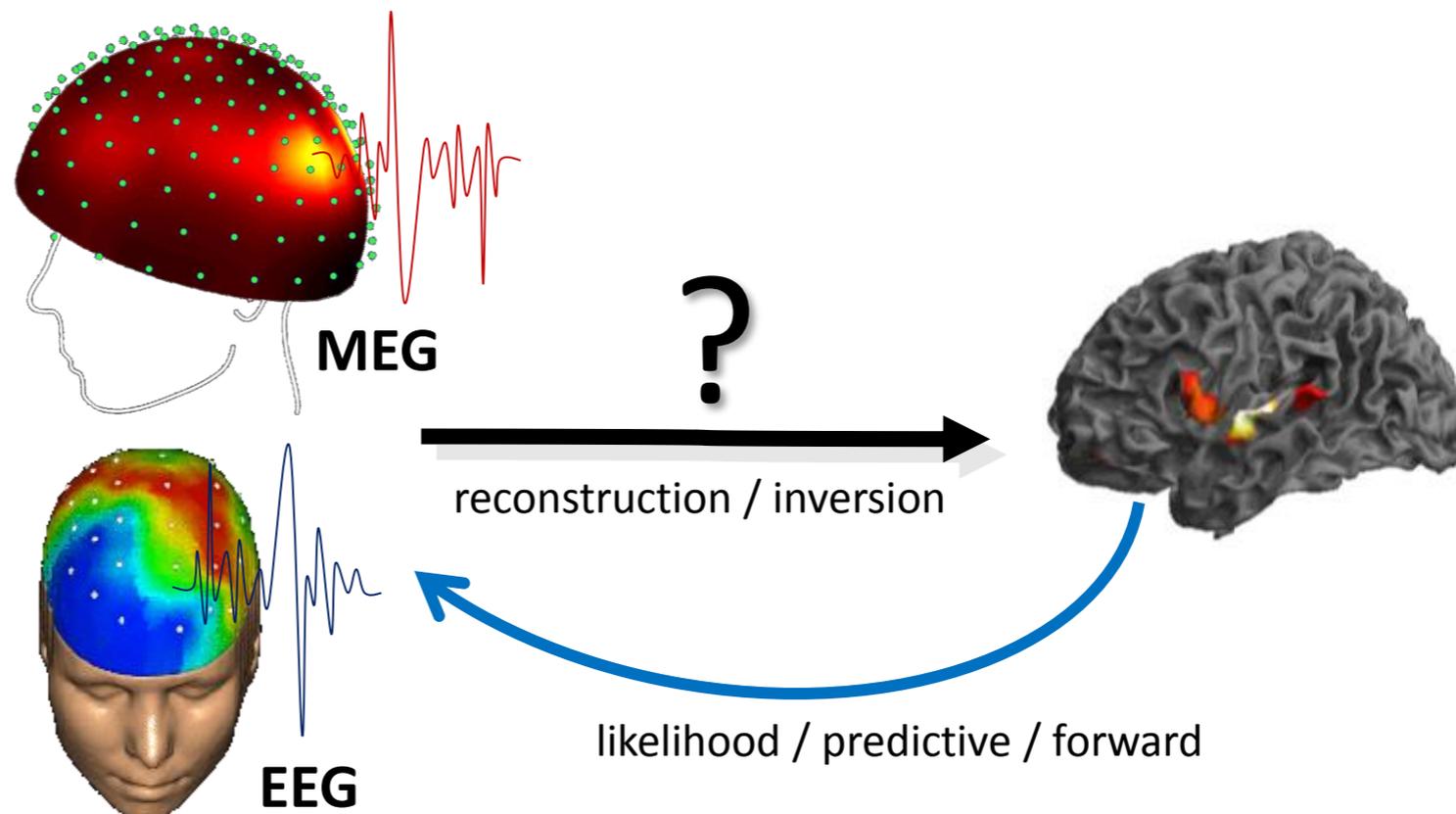
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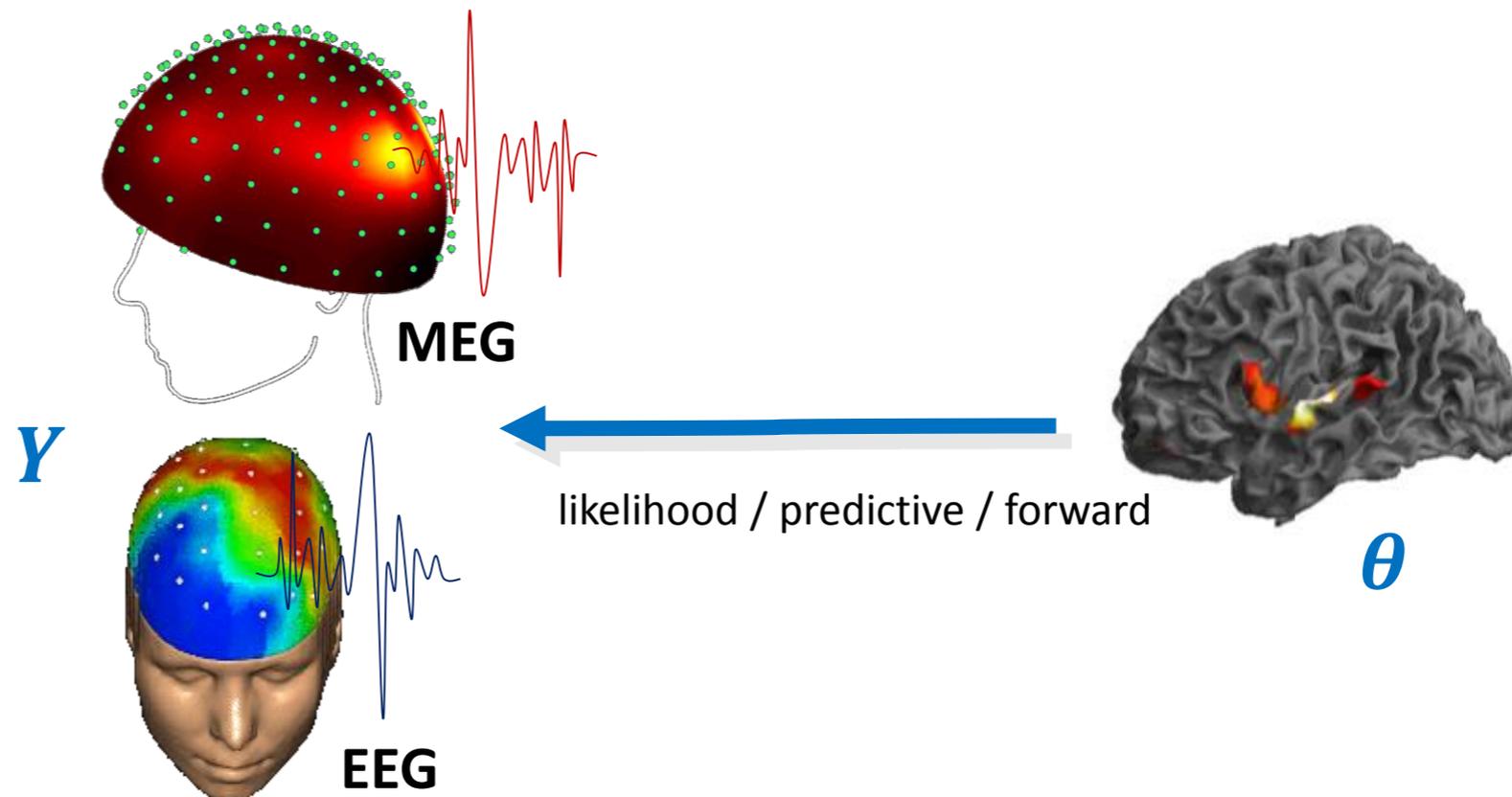
Group multimodal inference



### Bayesian inference enables:

- to incorporate priors on the solution
- to account for uncertainty through probabilistic distributions
- to yield a unique and optimal solution given a likelihood model and priors over model parameters

# Generative models



A particular generative model is fully defined by:

- A **data** likelihood density function

$$p(Y|\theta)$$

- A prior distribution over **source** parameters  $\theta$

$$p(\theta)$$

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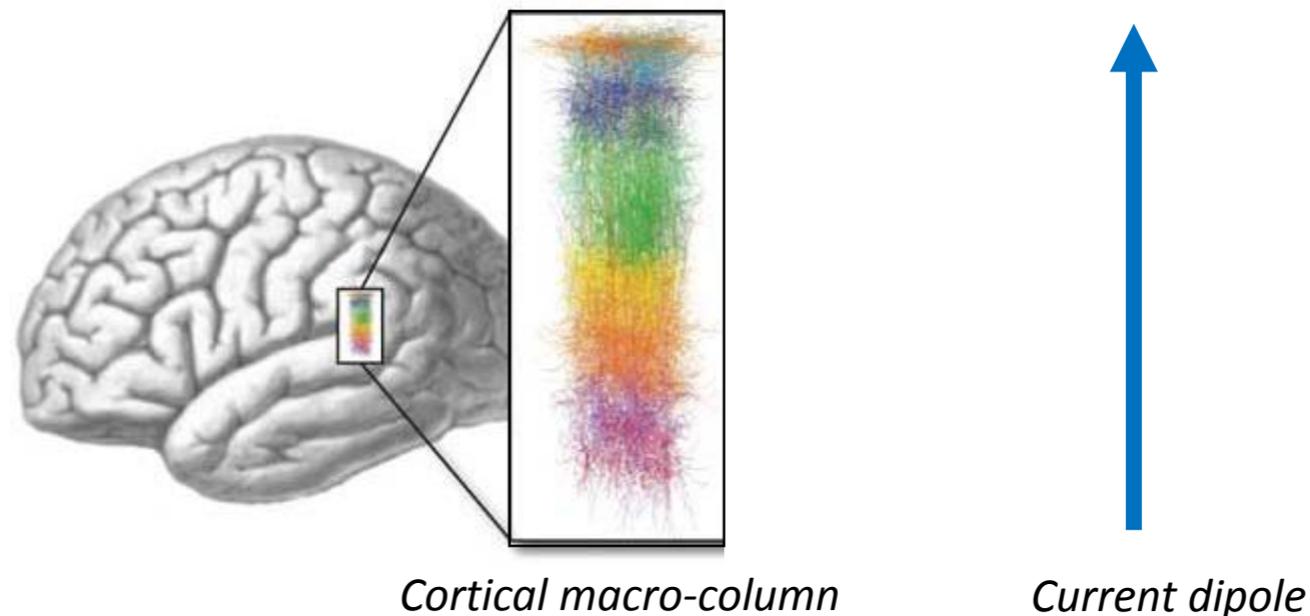
EEG/MEG fusion

### Example

MMN study

Group multimodal inference

- Observable from scalp:  
the synchronous and additive activities of numerous neighbouring neurons



- $\theta$  :
- Dipole location (x, y, z)
  - Dipole orientation ( $O_x$ ,  $O_y$ ,  $O_z$ )
  - Dipole strength

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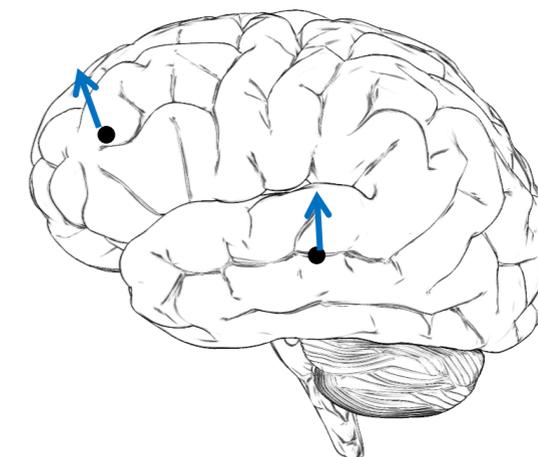
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MMN study

Group multimodal inference

- **Equivalent Current Dipole (ECD)**
  - Only a few activated sources
  - Each source corresponds to a fairly large brain area
  - Each source activity is modelled by one current dipole

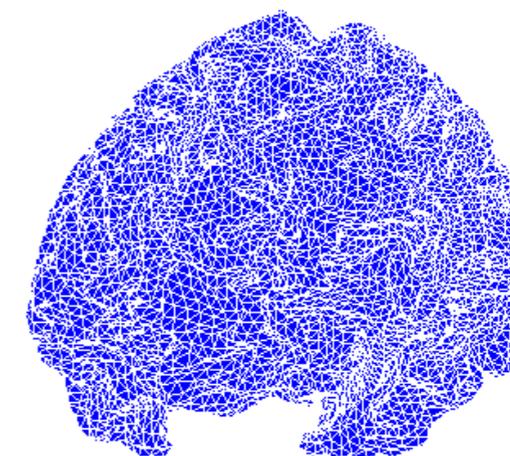
→ Only a few parameters  $\theta$  to be estimated  
(location, orientation and strength)



*e.g. early response to auditory stimulus*

- **Distributed or imaging approach**
  - The whole brain/cortex may be active
  - The source space is discretized using a grid over the whole brain (voxels) or a cortical mesh (nodes)
  - Each voxel or node is the location of a dipolar source
  - Each dipole models the activity of a small brain region

→ Many parameters  $\theta$  to be estimated  
(strength only)



*e.g. MRI-derived cortical mesh*

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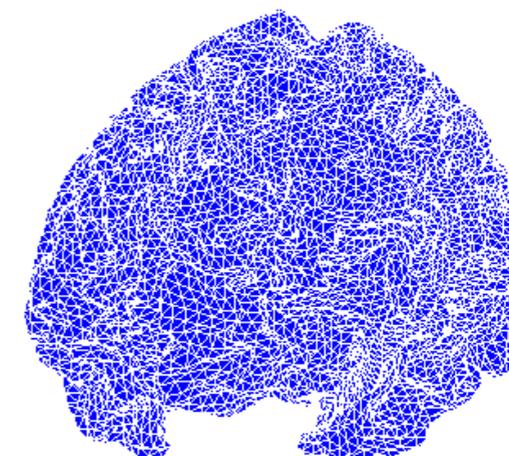
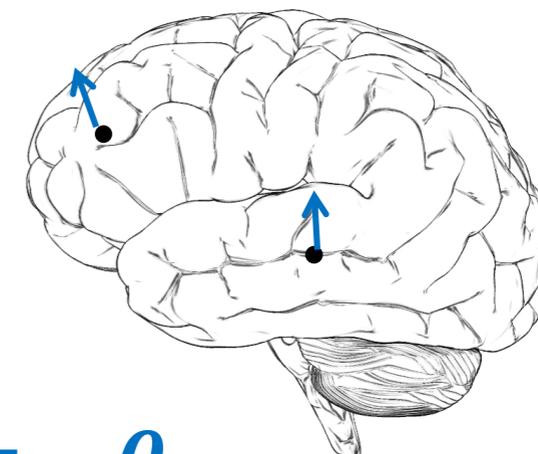
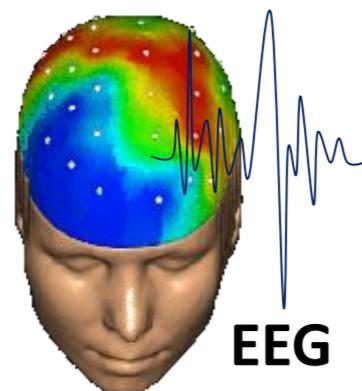
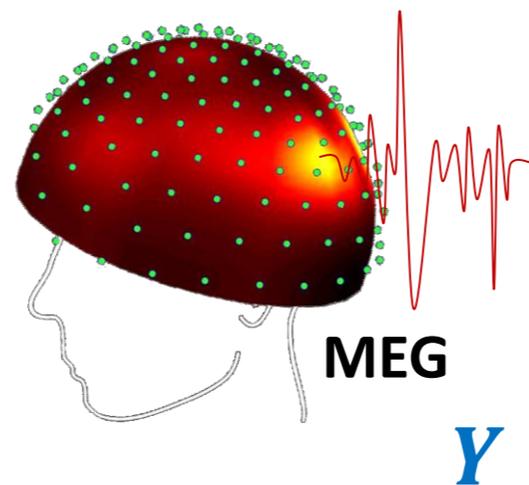
MMN study  
Group multimodal inference



James Clerk Maxwell  
(1831 - 1879)

# From sources to sensors

- Predicting the sensor data  $Y$  from known source parameters  $\theta$ :
  - requires solving the Maxwell's equations in a quasi-static regime
  - amounts to solving a well-posed forward problem
  - involves approximations



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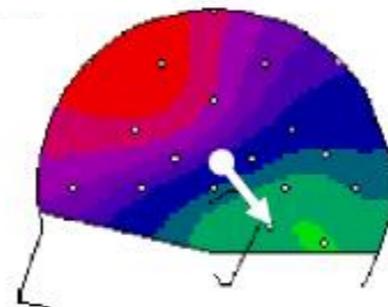
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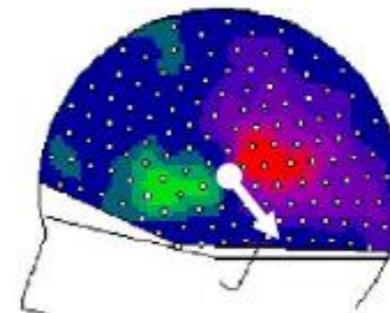
Group multimodal inference

# From sources to sensors

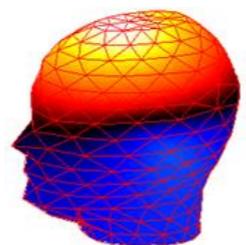
EEG



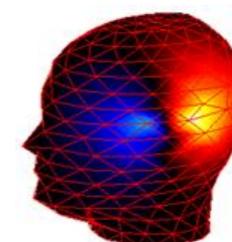
MEG



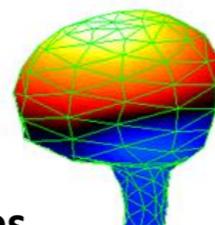
- EEG is sensitive to both radial and tangential sources



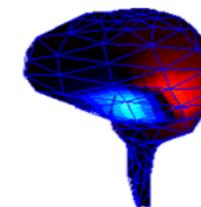
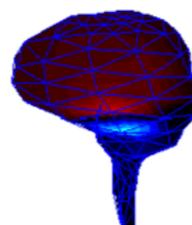
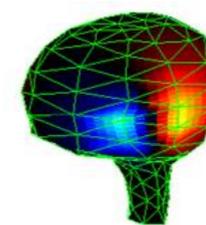
- MEG is barely sensitive to radial sources



- EEG is sensitive to conductivities



- MEG is barely sensitive to conductivities



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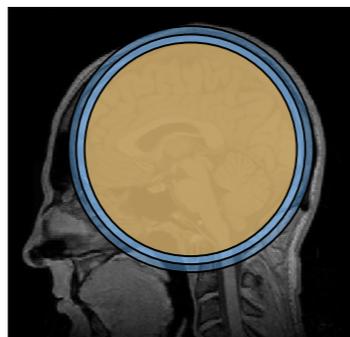
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## Example

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Group multimodal inference

# From sources to sensors

## Simple Head Model



*Concentric spheres*

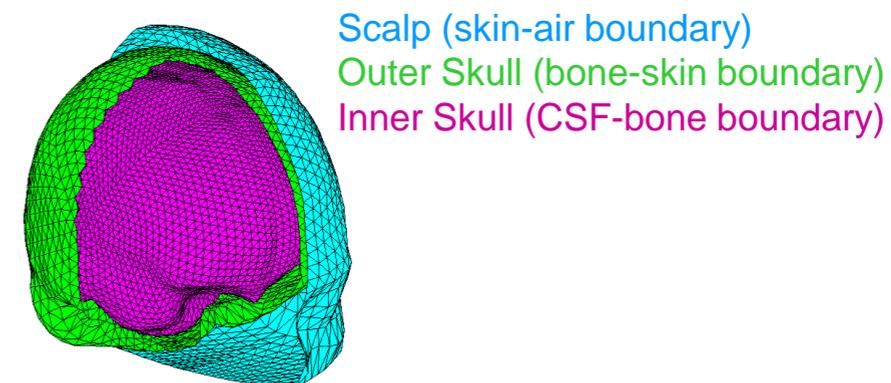
### Pros :

**Fast analytic solution**

### Cons :

**Heads are not spherical**

## Realistic Head Model



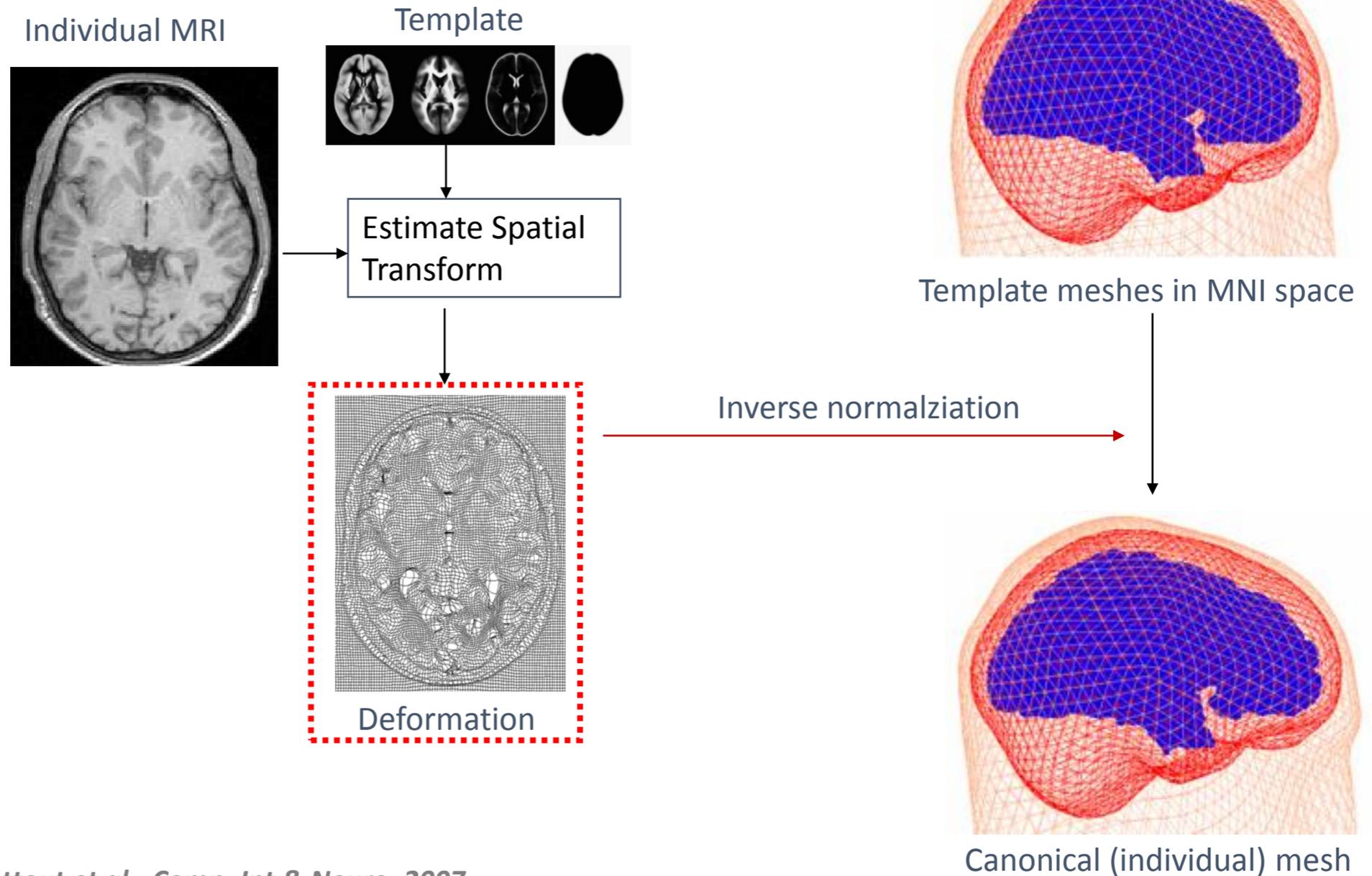
*Boundary element method (BEM)*

**Realistic geometry**

**Slow approximate numerical solutions**

# From sources to sensors

- Automated extraction of individual meshes



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- Data features  $Y$  to be fitted/explained
  - Evoked response
  - Induced response
  - Steady-state response

- Accounting for noise  $\varepsilon$  in the data

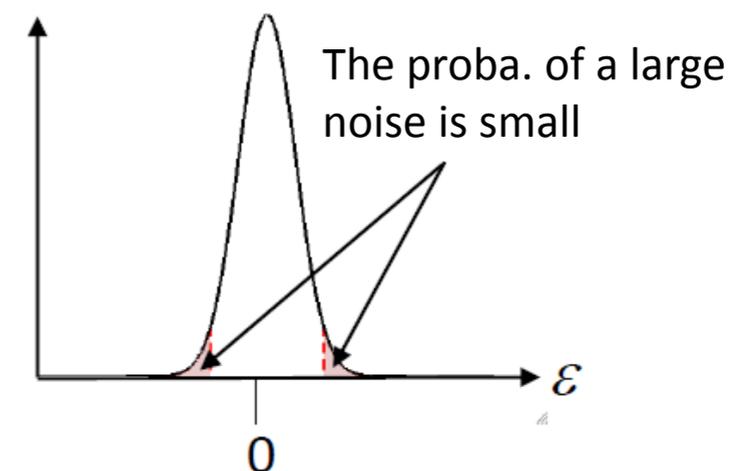
$$Y = g(\theta) + \varepsilon$$

Gaussian noise

$$p(\varepsilon) = N(\mathbf{0}, C_\varepsilon)$$

Data likelihood

$$p(Y|\theta) = N(g(\theta), C_\varepsilon)$$



# Bayesian inference

$$P(\theta|Y, M) = \frac{P(Y|\theta, M)P(\theta|M)}{P(Y|M)}$$

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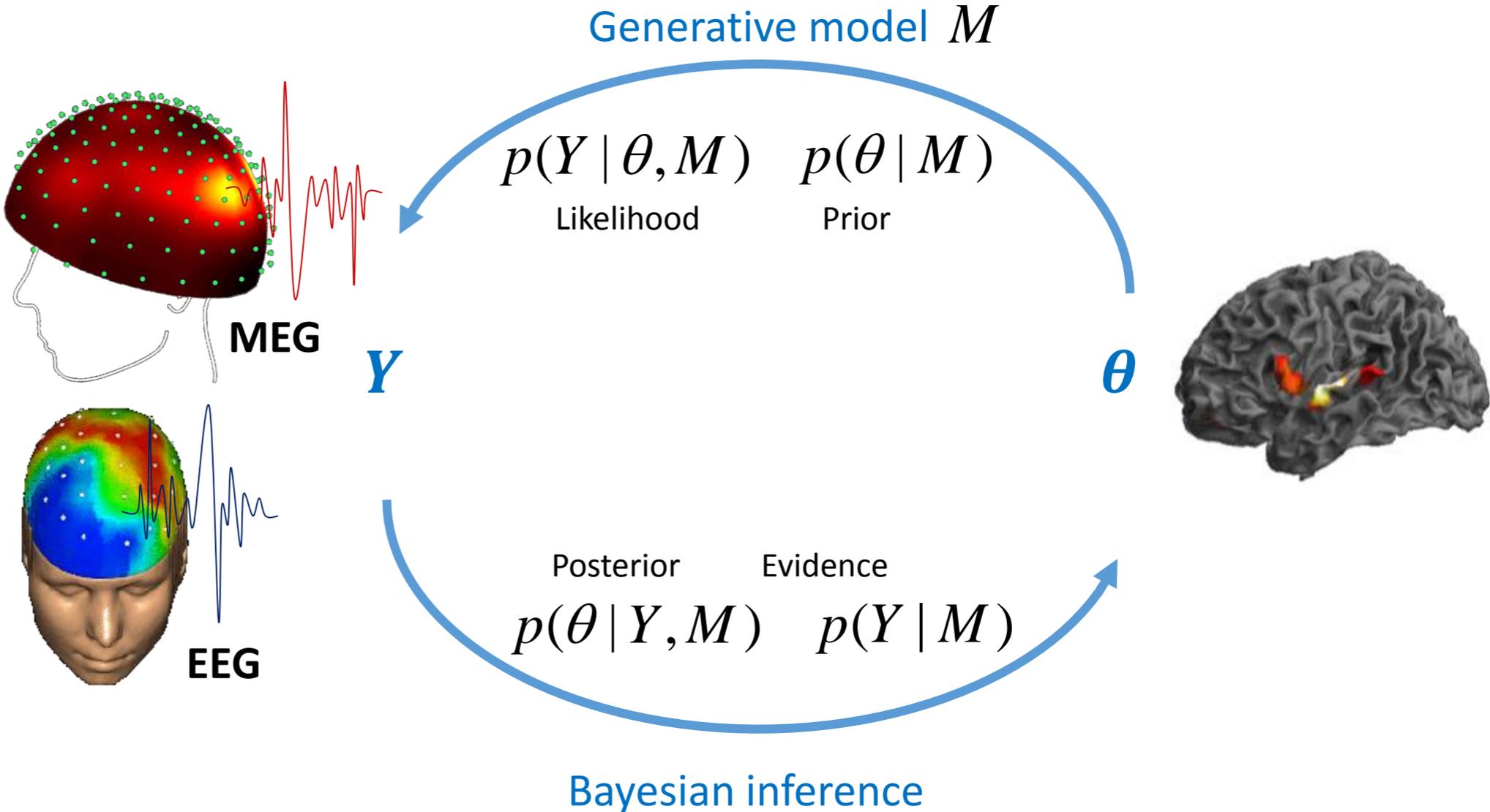
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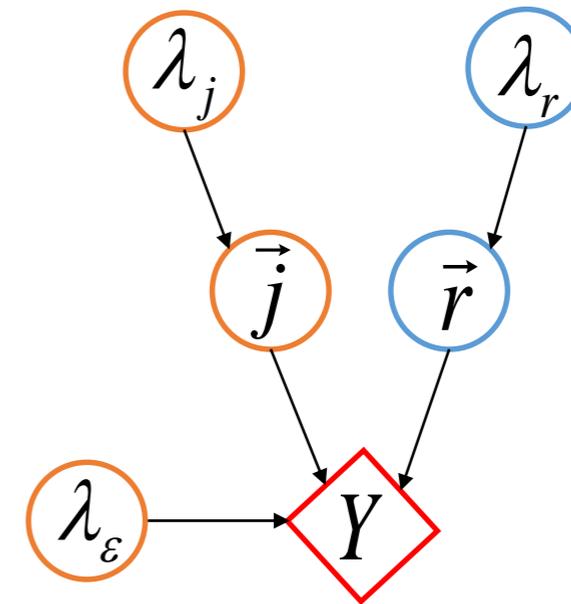
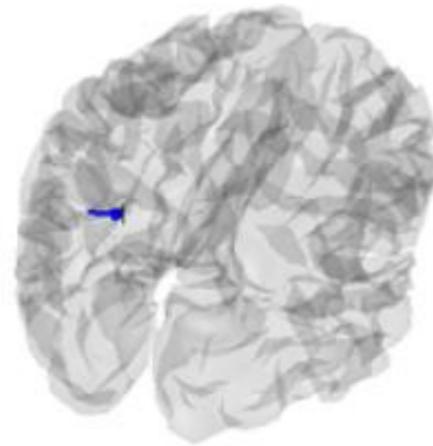
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### Example

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Group multimodal inference

- A Bayesian model for Equivalent Current Dipole (ECD) solutions
  - Enables to put priors on source parameters
  - Enables formal model comparison (e.g. on number of sources or initial conditions)



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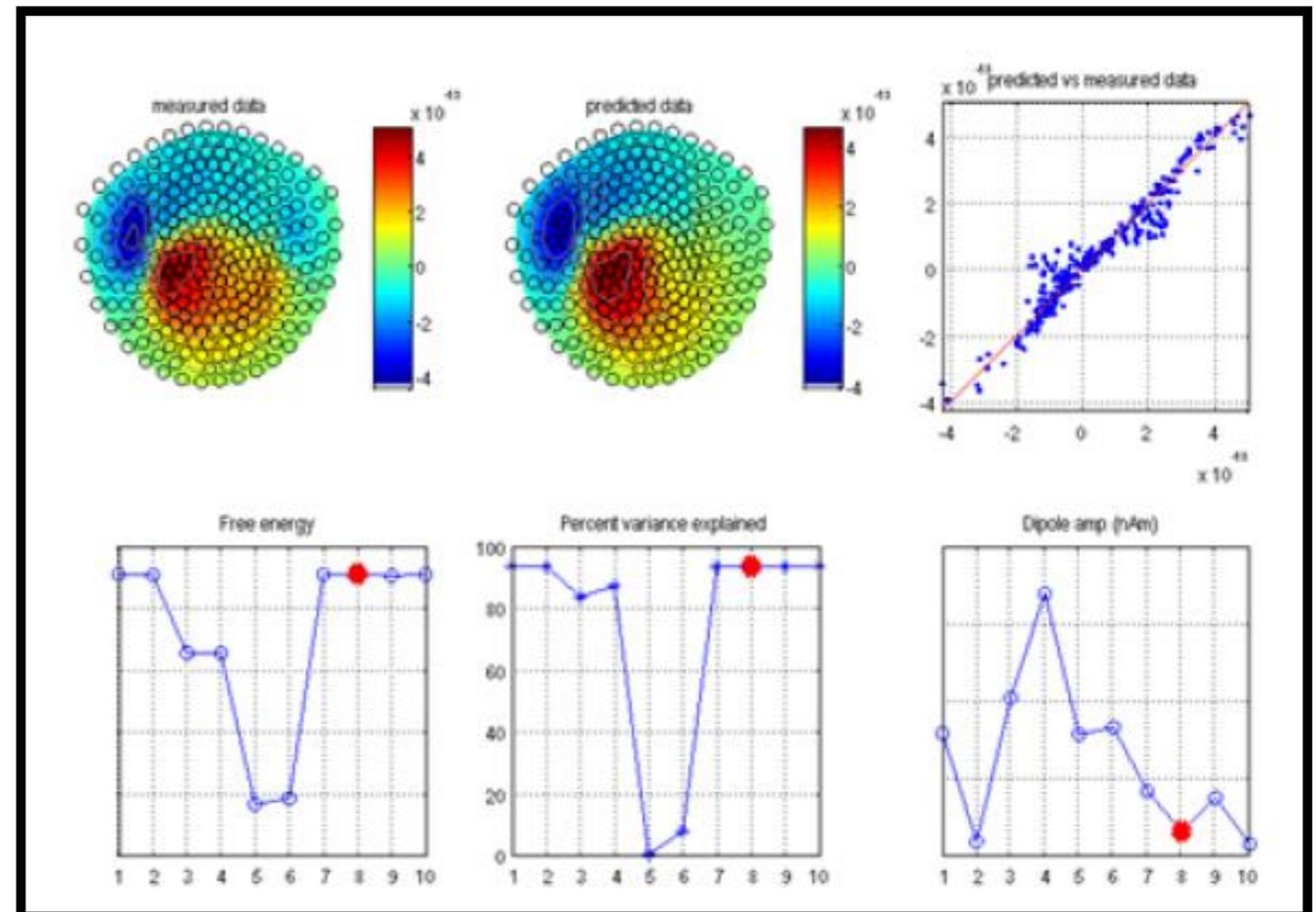
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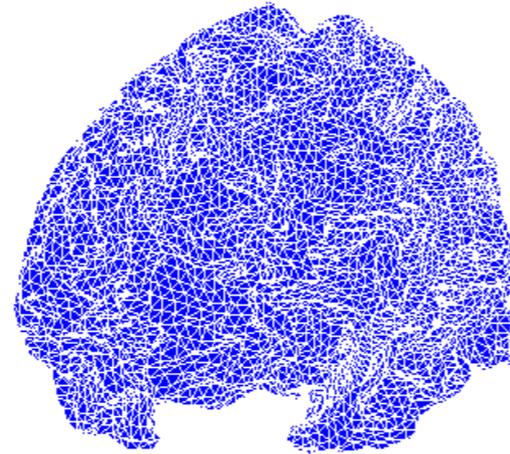
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- A Bayesian model for Distributed / Imaging solutions
  - Many dipoles with fixed location and orientation
  - Dipole strength ? -> linear model

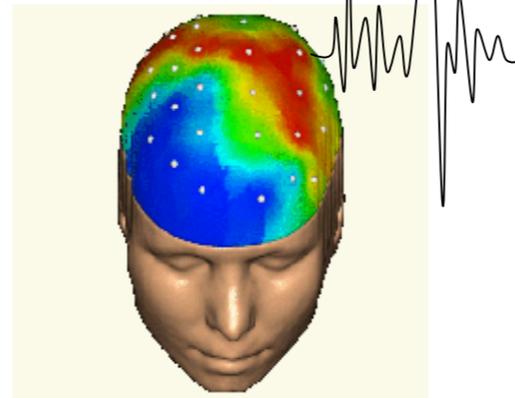


$$Y = K \cdot \theta + \varepsilon$$

noise

Unknown source dynamics:  $N_{\text{sources}} \times \text{Time}$

Forward operator or lead-field matrix:  $N_{\text{sens}} \times N_{\text{sources}}$



Evoked EEG response:  $N_{\text{sens}} \times \text{Time}$

$$p(Y|\theta) = N(K \cdot \theta, C_{\varepsilon})$$

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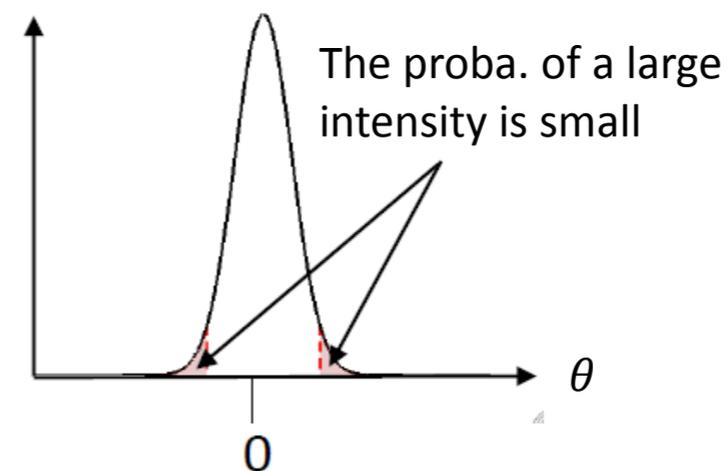
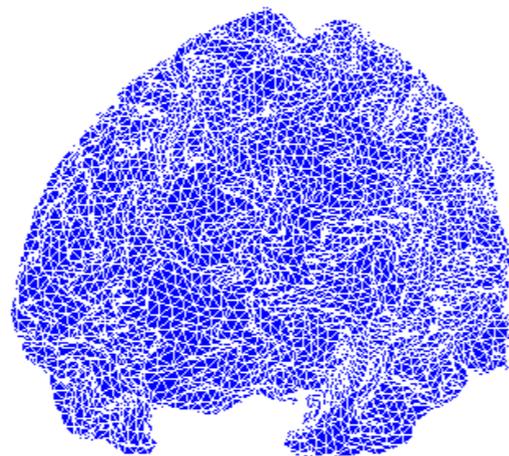
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Prior over dipole strength

$$p(\theta) = N(0, C_{\theta})$$

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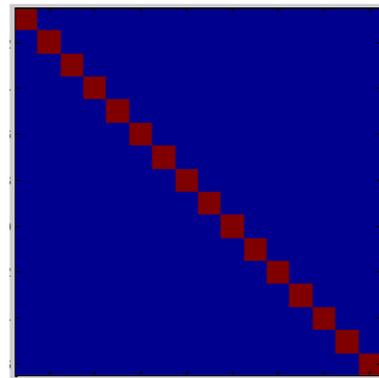
MMN study  
Group multimodal inference

- Alternative priors correspond to alternative prior covariance matrices

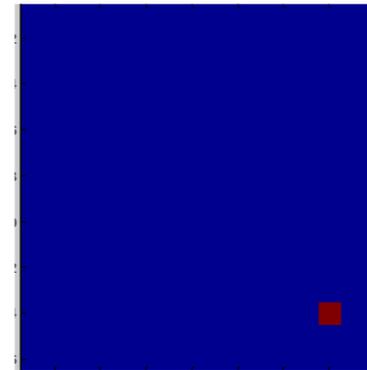
$$p(\theta) = N(0, C_{\theta})$$

Ndip x Ndip

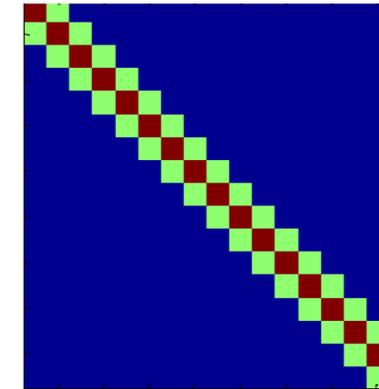
- Typical priors



i.i.d or Minimum norm



Single dipole



Smoothness (like LORETA)

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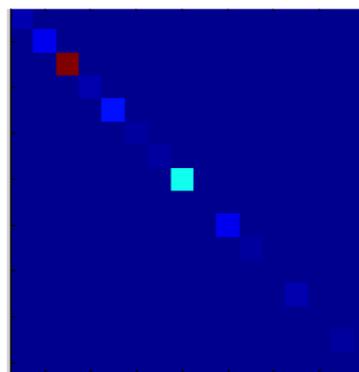
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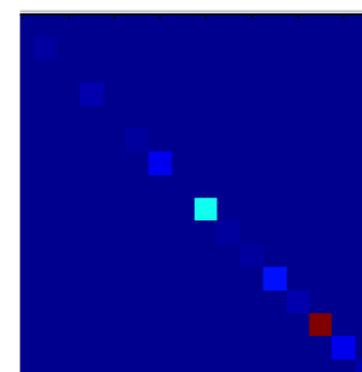
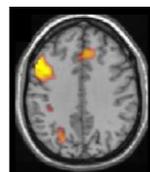
$$p(\theta) = N(0, C_{\theta})$$

Ndip x Ndip

- More advanced priors



fMRI based



Data or Lead-field based  
e.g. (Beamformer or MSP)

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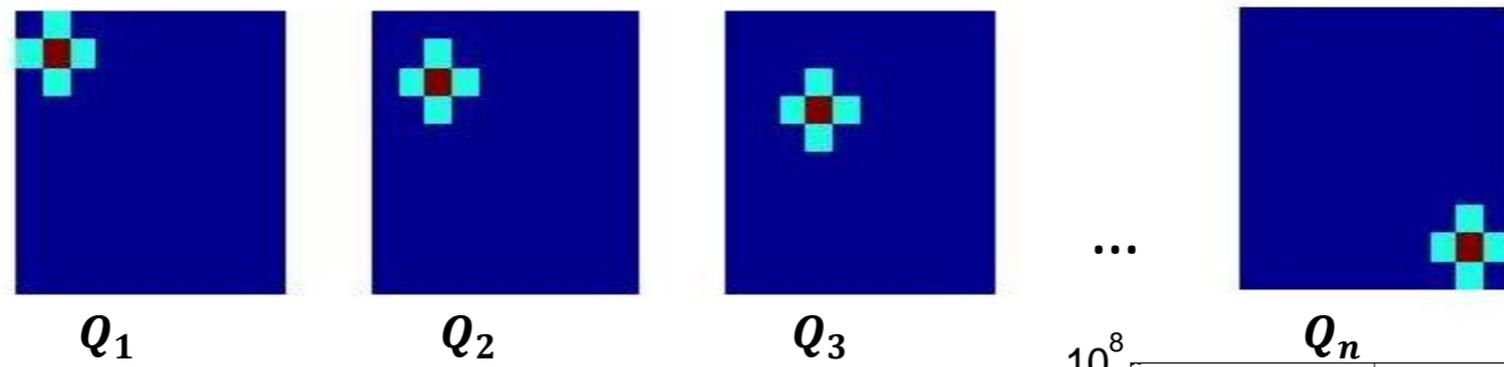
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### Multivariate Sparse Priors (MSP)

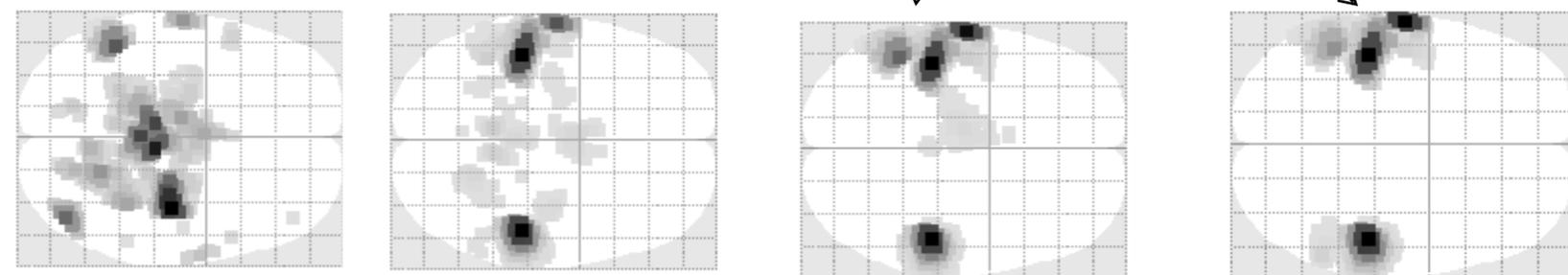
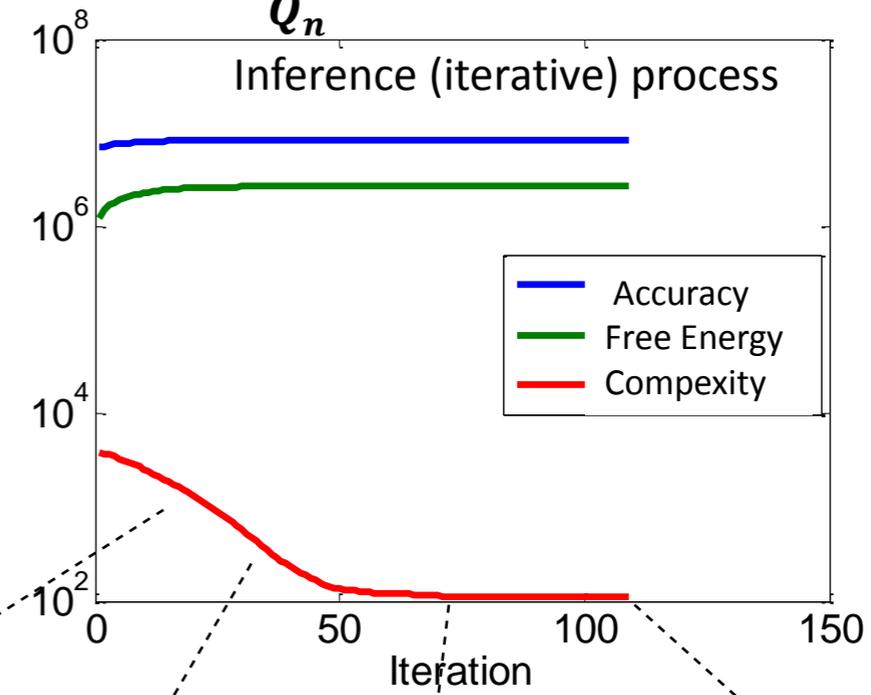


Model (Hyper)parameters

$$C_{\theta} = \lambda_1 \cdot Q_1 + \lambda_2 \cdot Q_2 + \lambda_3 \cdot Q_3 + \dots + \lambda_n \cdot Q_n$$

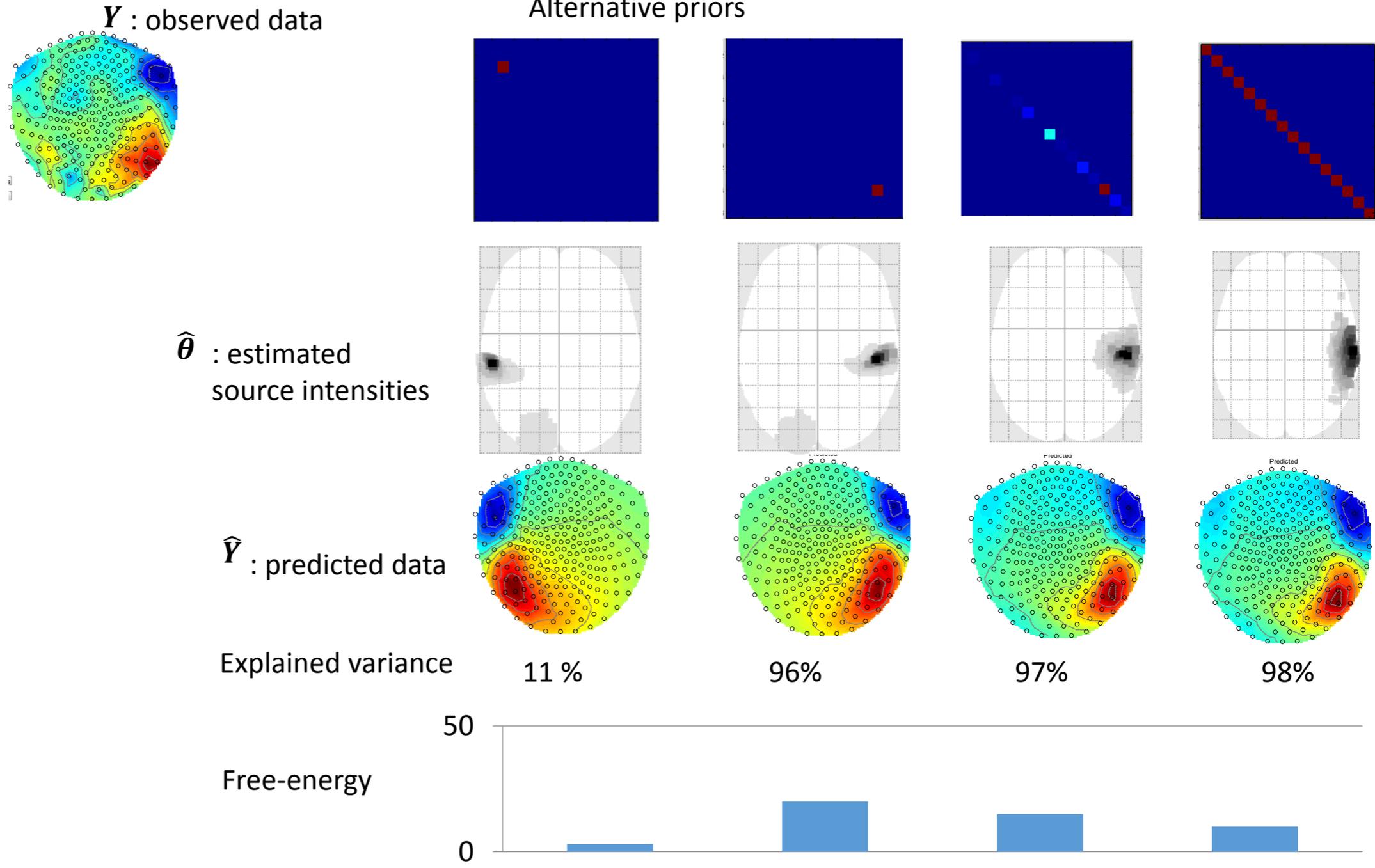
*Philips et al., NeuroImage, 2005*  
*Mattout et al., NeuroImage, 2006*  
*Friston et al., NeuroImage, 2008*  
*Lopez et al., NeuroImage, 2014*

# Empirical Bayes



# Comparing models

- Comparing priors using log-evidence (free energy)  $F \cong p(Y|M)$



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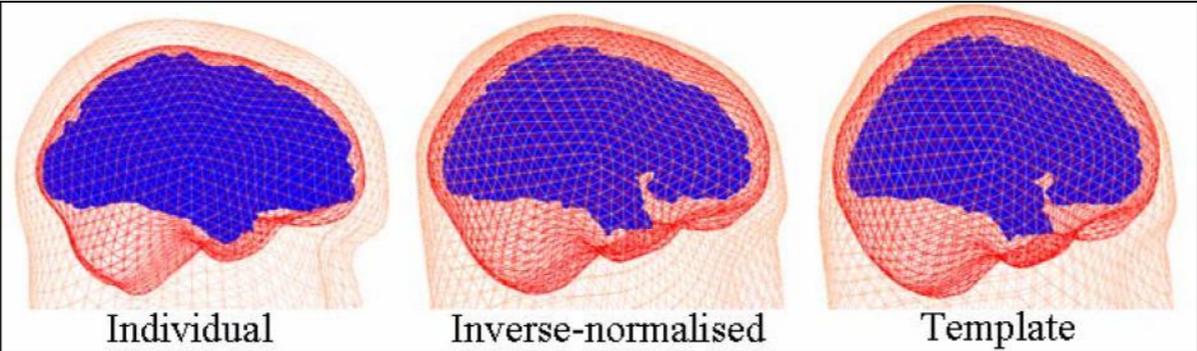
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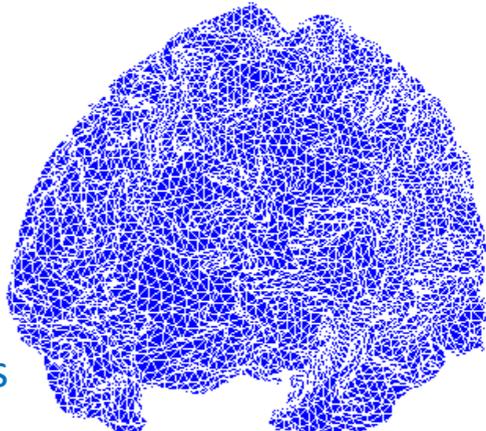
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- Any assumption (part of model M) can be formally tested on real data using Bayesian model comparison



*Mattout et al., NeuroImage, 2007*

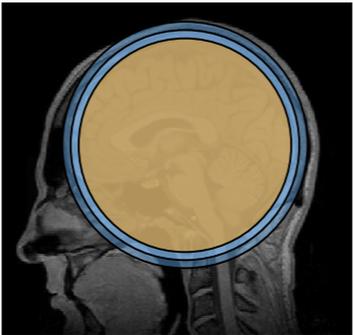
Anatomy



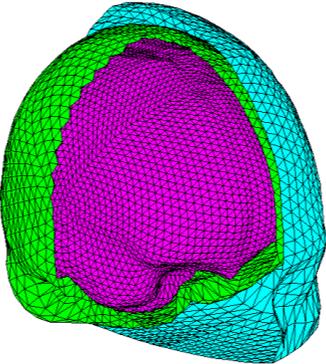
Sources

Mesh resolution (High / Low)  
Dipole orientation (fixed / free)

Biophysics



Spherical head model



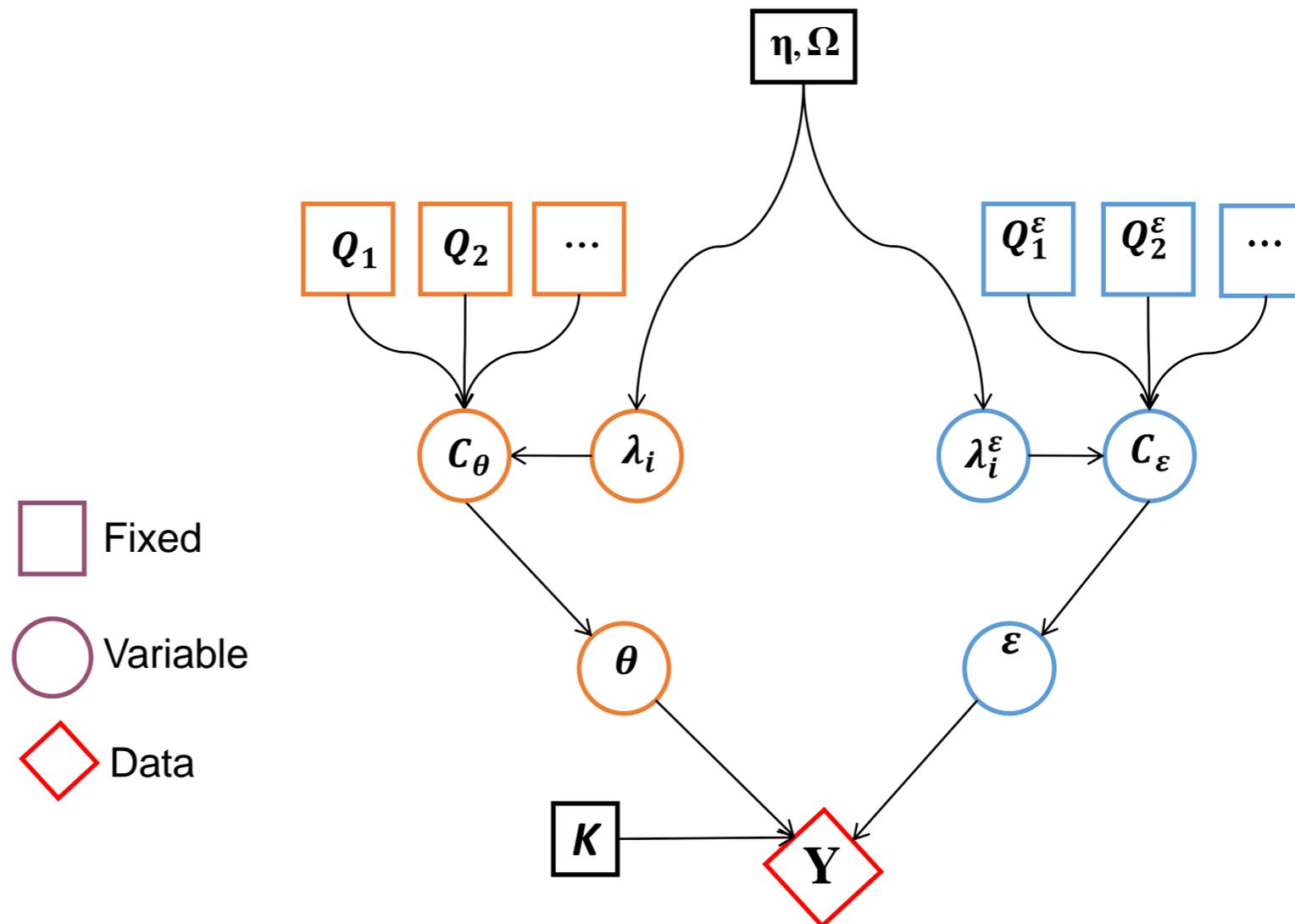
Realistic surfacic model (BEM)

*Henson et al., NeuroImage, 2007*  
*Henson et al., NeuroImage, 2009*

- **No evidence in favor of individual vs. Inverse-norm mesh**
- **Evidence in favor of BEM head model**
- **Evidence in favor of high + fixed vs. low + free**

# Group inference

- MSP based source reconstruction for a single subject



*Friston et al., NeuroImage, 2008*

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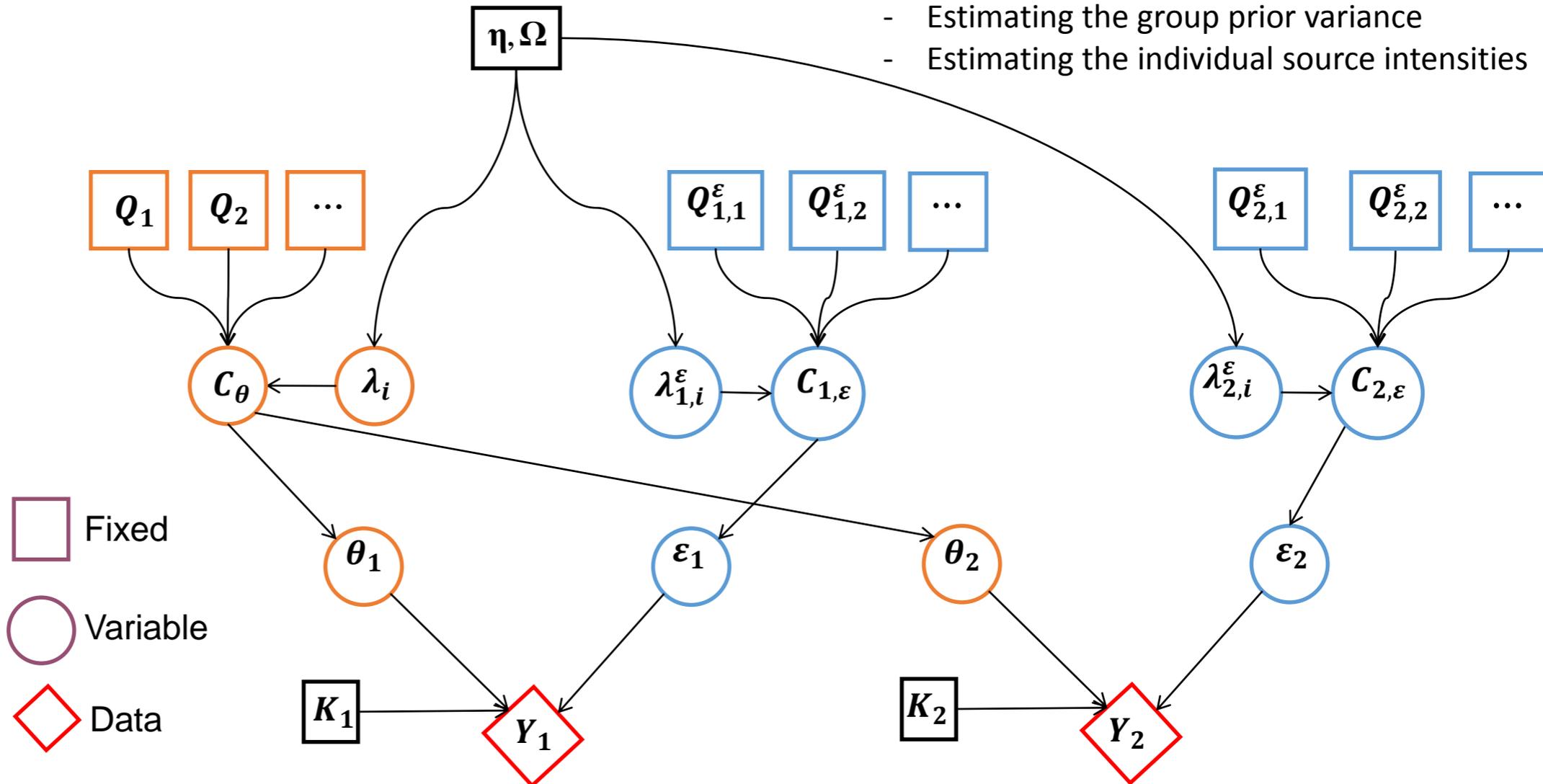
### Example

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Group multimodal inference

- MSP based source reconstruction for multiple subjects

A two-step procedure:

- Estimating the group prior variance
- Estimating the individual source intensities



# EEG/MEG fusion

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From sources to sensors  
Sensors

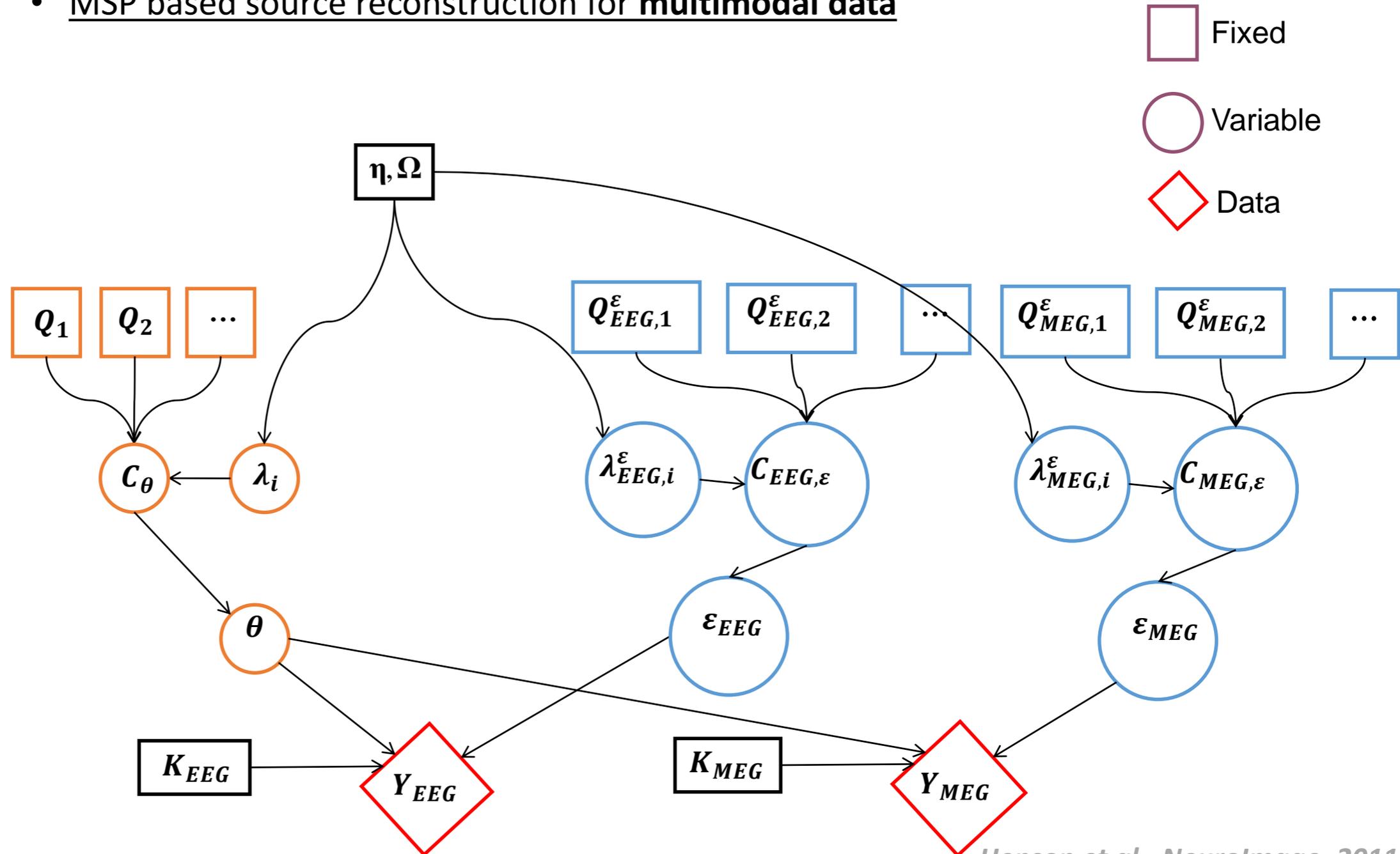
### Bayesian inference

ECD model  
Imaging models  
Setting priors  
Empirical Bayes  
Comparing models  
Group inference  
EEG/MEG fusion

### Example

MMN study  
Group multimodal inference

- MSP based source reconstruction for multimodal data



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