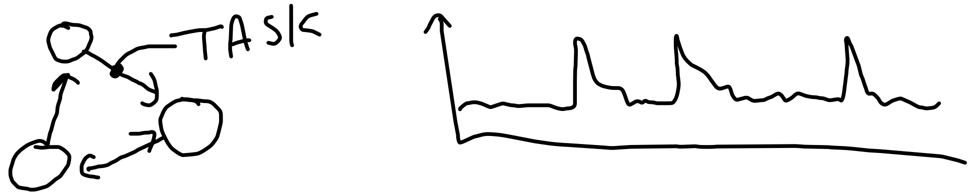
# Bayesian Model Comparison

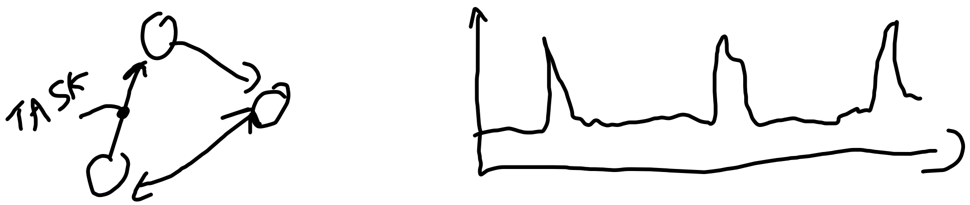
Peter Zeidman

# Overview

- Outputs of model estimation
  free energy, posteriors
- Bayes factors
- Converting
  to posterior probabilites




$m_1$




$m_2$

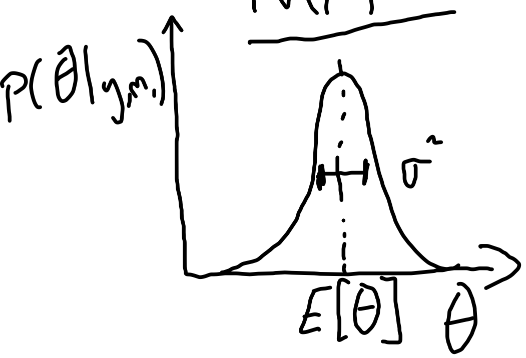
## ESTIMATION

① EVIDENCE $\ln p(y|m_1) \approx \boxed{F}$

$F$ = accuracy − complexity

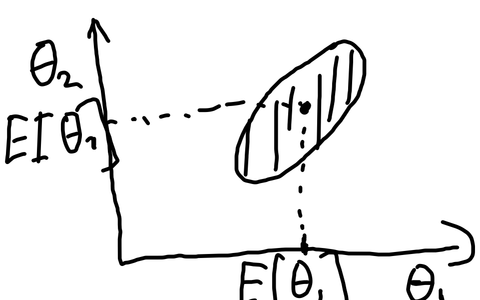
② POSTERIOR $p(\theta|y,m_1)$

$N(\mu, \sigma^2)$




DCM.Ep − $\mu$

DCM.Cp − $\sigma^2$




$N(\vec{\mu}, \Sigma)$

$$\vec{\mu} = \begin{bmatrix} E[\theta_1] \\ E[\theta_2] \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdot \\ \cdot & \sigma_2^2 \end{bmatrix}$$

$F_1 \simeq \ln p(y|m_1)$

$F_2 \simeq \ln p(y|m_2)$

Bayes factor

$$\frac{p(y|m_1)}{p(y|m_2)} \Rightarrow \ln BF_1 = \ln p(y|m_1) - \ln p(y|m_2)$$

$$\simeq F_1 - F_2$$

$BF = \boxed{20}\times \qquad \ln 20 \simeq \boxed{3}$

$$\ln BF_1 \approx F_1 - F_2$$
$$= \ln p(y|m_1) - \ln p(y|m_2)$$

$$\left.\begin{aligned} p(m_1|y) &= 90\% \\ p(m_2|y) &= 10\% \end{aligned}\right\}$$

Converting ln BF to posterior model prob.

$$p(m_1|y) = \frac{p(y|m_1)\,p(m_1)}{p(y)}$$

$$\boxed{p(a) = \sum_b p(a,b)}$$

$$p(y) = p(y|m_1)\,p(m_1) + p(y|m_2)\,p(m_2)$$

$$p(m_1|y) = \frac{1}{1 + \exp[-\ln BF_1]}$$

# Summary

## DCM Outputs

Log Evidence: $\ln p(y|m)$

Posteriors: $p(\theta|y,m)$

## Bayes factors

$$\frac{p(y|m_1)}{p(y|m_2)}$$

Free energy: $F \sim \ln p(y|m)$

log Bayes Factor: $F_1 - F_2 \geq 3$ "strong evidence"

$p(m_1|y)$ :  $\dfrac{1}{1+\exp(-LBF_1)}$

Posterior model probabilities

# Group Analysis with Parametric Empirical Bayes

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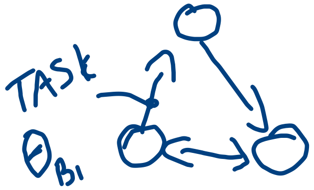
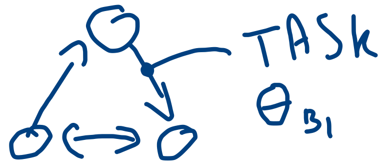
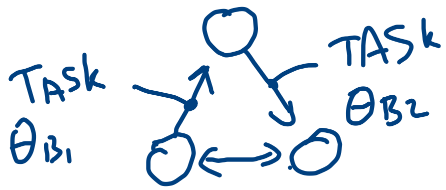



$M_1$




$M_2$

① "FULL DCM"




② $$\theta = \begin{bmatrix} \left.\begin{matrix} \theta_{B1} \\ \theta_{B2} \end{matrix}\right\} S1 \\ \vdots \\ \left.\begin{matrix} \theta_{B1} \\ \theta_{B2} \end{matrix}\right\} S2 \end{bmatrix}$$

③ spm_dcm_peb

$$\theta = X\beta + \varepsilon$$

unexplained variability (RFX)

$$y_i = f_i(\theta_i) + \varepsilon_i$$

$i = 1 \ldots N$

DCM

noise

# Design Matrices

$$\Theta = \begin{bmatrix}\Theta_1\\ \Theta_2\\ \Theta_3\\ \Theta_4\end{bmatrix} \begin{matrix}P\\ P\\ C\\ C\end{matrix}$$

$$\Theta = X\beta + \varepsilon$$

$$\begin{bmatrix}1 & 1\\ 1 & 1\\ 1 & 0\\ 1 & 0\end{bmatrix} \times \boxed{\begin{matrix}\beta_1\\ \beta_2\end{matrix}}$$

← baseline (controls)

← patients

$$X$$

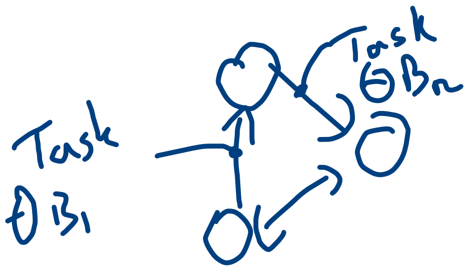
$$\Theta = X\beta + \varepsilon$$

$$\begin{bmatrix}1 & 1\\ 1 & 1\\ 1 & -1\\ 1 & -1\end{bmatrix} \times \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$$

← average

← patient vs control

$$\underline{X} = \underline{X_B \otimes X_w} \qquad X_w = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$




$\underline{X}$ covariates:

$\Theta_{B_1}$:
- average
- patient vs control

$\Theta_{B_2}$:
- average
- patient vs control

## Variability

$$\theta = X\beta + \varepsilon$$

$$\varepsilon \sim N(\vec{0}, \Sigma)$$

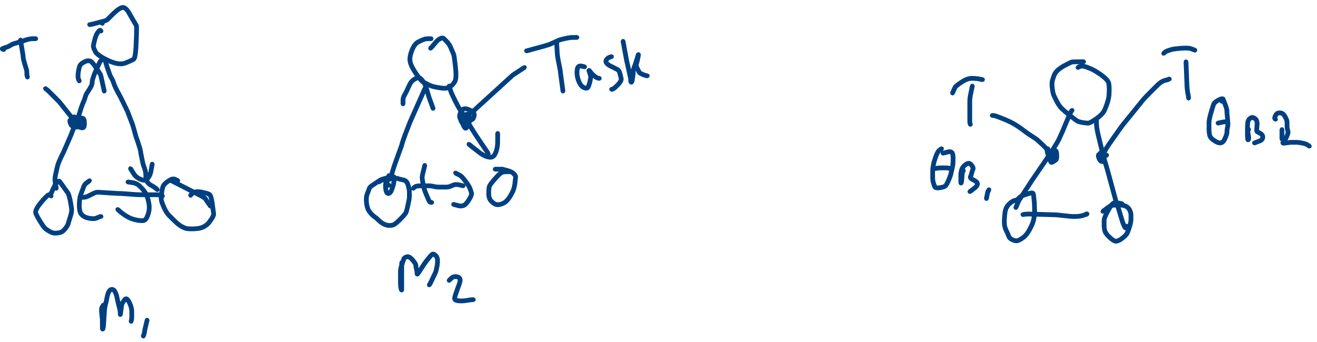
## Options

M.Q = 'single' - all DCM connections vary equally subjects

M.Q = 'all' - all DCM connections vary independently

PEB.F

# Bayesian Model Comparison




$$\theta = X\beta + \epsilon$$

| | | |
|---|---|---|
| $P_1$ | ~~avg~~ | ~~$\theta_{B_1}$~~ |
| $P_2$ | avg | $\theta_{B_2}$ |
| $P_3$ | patient | $\theta_{B_1}$ |
| $P_4$ | patient | $\theta_{B_2}$ |

full PEB

does disabling p1 increase or decrease free energy?

Options for specifying candidate PEB models:

1. Hand-craft  $P_1, P_2, P_3, P_4$
   $P_2, P_4$
2. Automatic search using BMR

# PEB

1. Full DCM per subject
2. Specify PEB model
3. Compare full PEB against reduced PEBs

- "empirical Bayes"
- prediction