Contrasts & Statistical Inference

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Lecture outline

✓What is a t-contrast?

✓What is an f-contrast?

✓Orthogonality of regressors

What do we use contrasts for?

A contrast is **a linear combination of variables (parameters or statistics**) whose coefficients add up to zero, allowing comparison of different treatments.

Within fMRI we use contrasts to:

1. Examine whether task conditions produce brain activity that is significantly different (increase or decrease) compared to signal noise (t-contrast)

2. Examine whether brain activity between two conditions is significantly different (increase or decrease) for one compared to the other (t-contrast)

3. Compare multiple regressors (parts of your model) and/or between conditions to see if is any difference between the two (f-contrast)

The story so far…

Data acquisition

↓

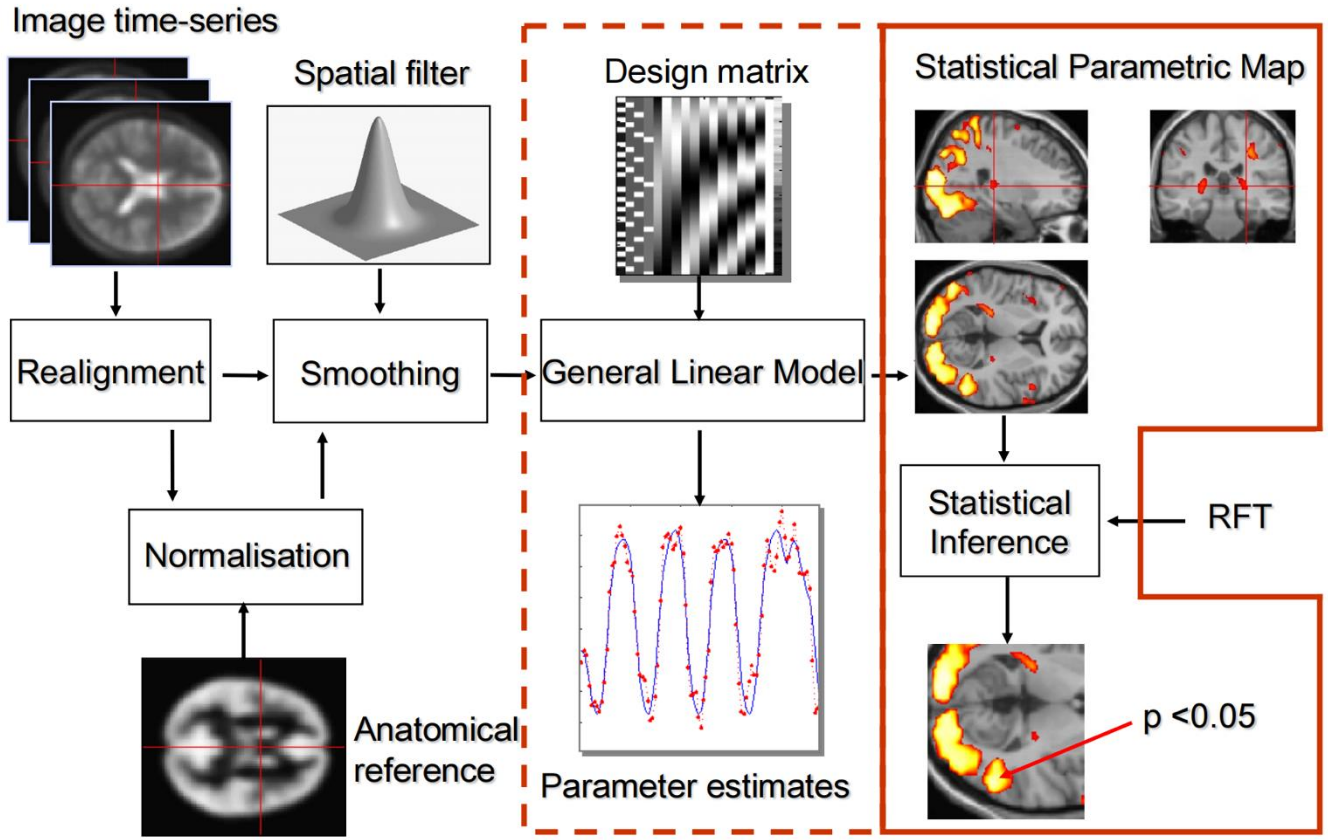
Data preprocessing

↓

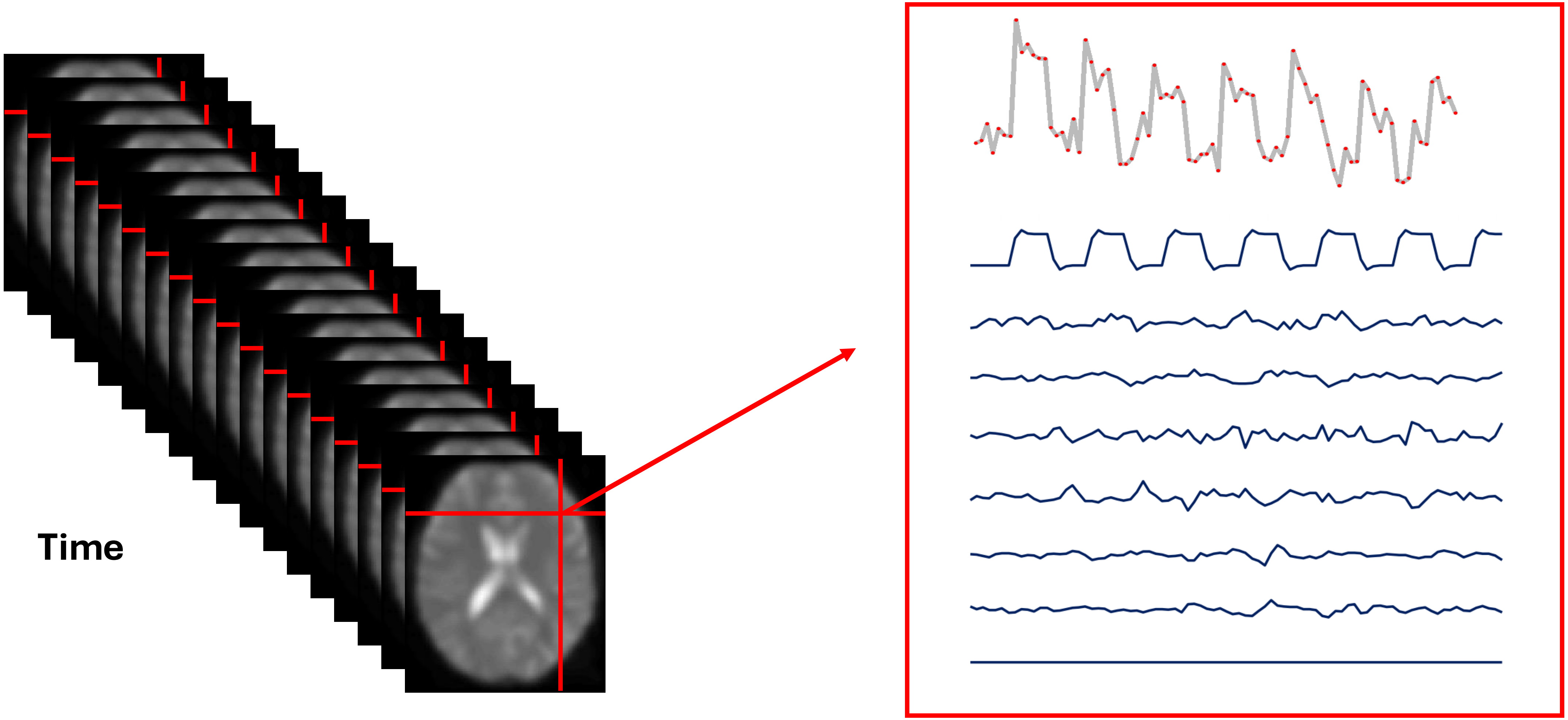
Task Design
task conditions + regressors of no interest

↓

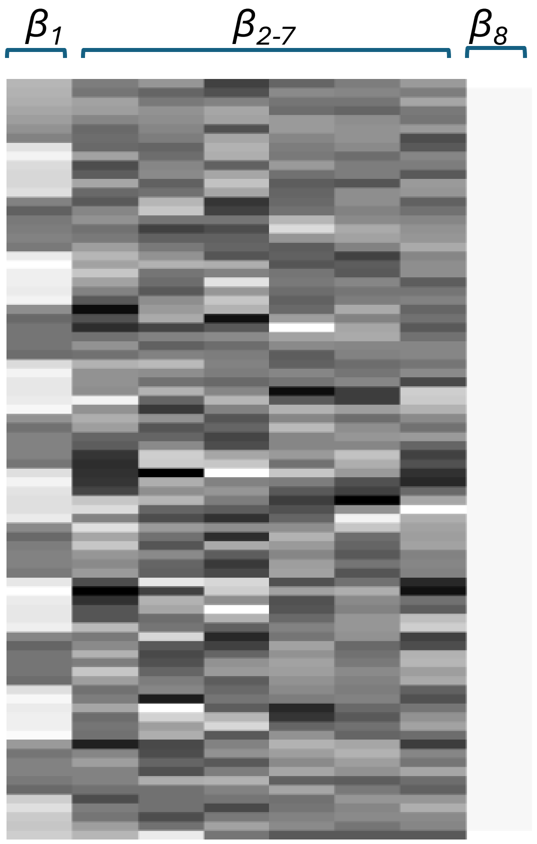
Contrasts – Statistical inference

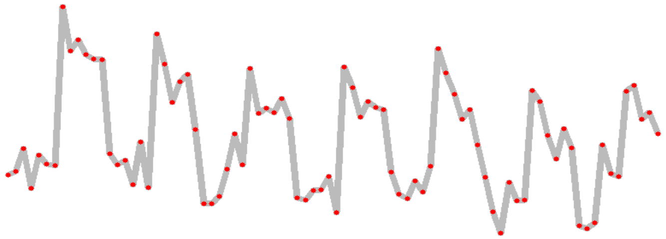


Mass-univariate approach



Your Design Matrix -Estimation of the parameters





$$Y = \beta_1 X + \beta_4 r_1 + \cdots \beta_{10} r_6 + \varepsilon$$

$$\beta \sim N(\beta, \sigma^2 (X^T X)^{-1}) \qquad \sigma^2 = \frac{\varepsilon^T \varepsilon}{N - p}$$

$\beta_1 = 3.9835$

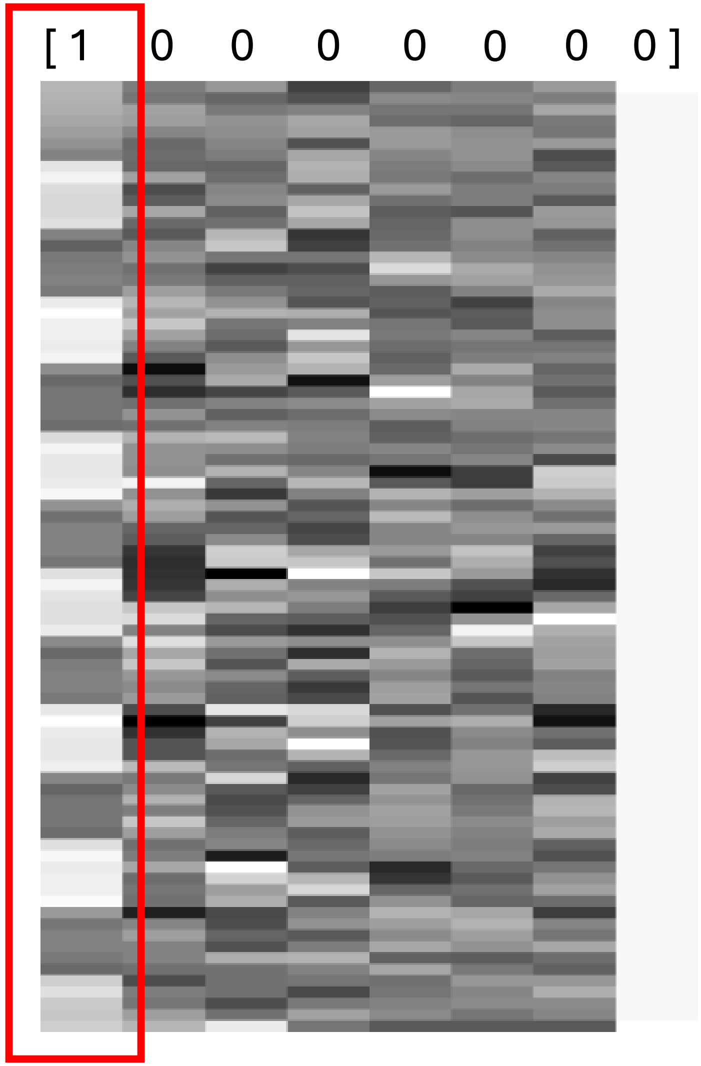
$\beta_{2\text{-}7} = \{0.678, 2.465, 1.309, 4.985, 6.784, 0.0021\}$

$\beta_8 = 181.005$

ε

The beta value are random variables that are distributed around their true value and their estimate depends on the variability of our residuals (σ^2) .

Contrasts – t-contrasts



Contrasts allow you to isolate specific effects of interest

A contrast is a c vector of length p

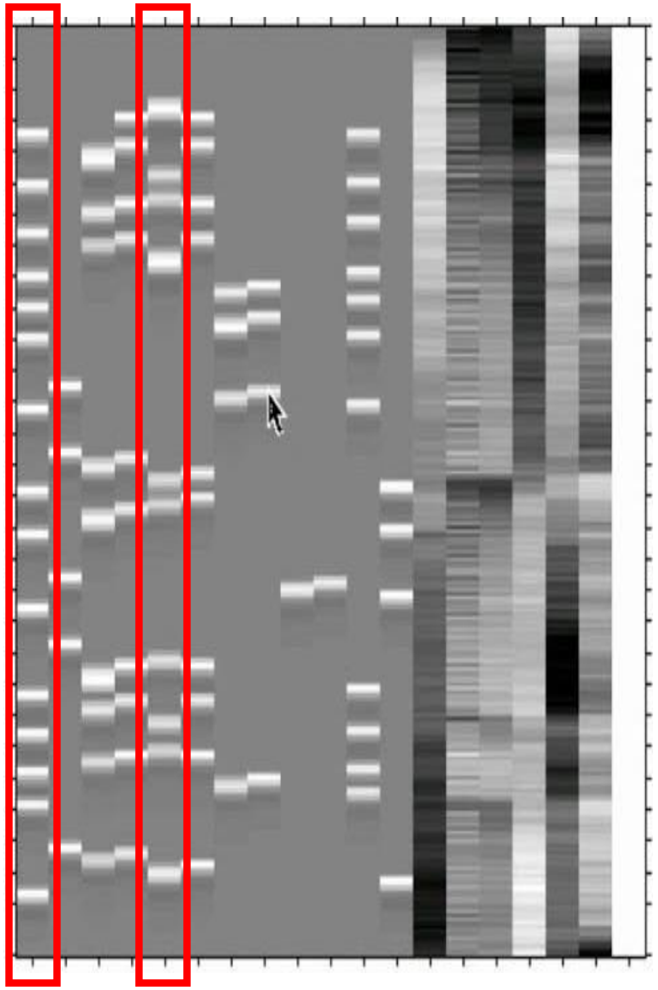
A contrast $\boldsymbol{c^T\beta}$ is a linear combination of your regressor coefficients, β

If we are interested in the effects of β_1

$$c^T\beta = 1x\beta_1 + 0\chi\beta_2 + 0\chi\beta_3 + \ldots + 0\chi\beta_8$$

Contrasts – t-contrasts

[1 0 0 0 -1 0 0 0 …



Contrasts allow you to isolate specific effects of interest

A contrast is a c vector of length p

A contrast $\boldsymbol{c^T\beta}$ is a linear combination of your regressor coefficients, β

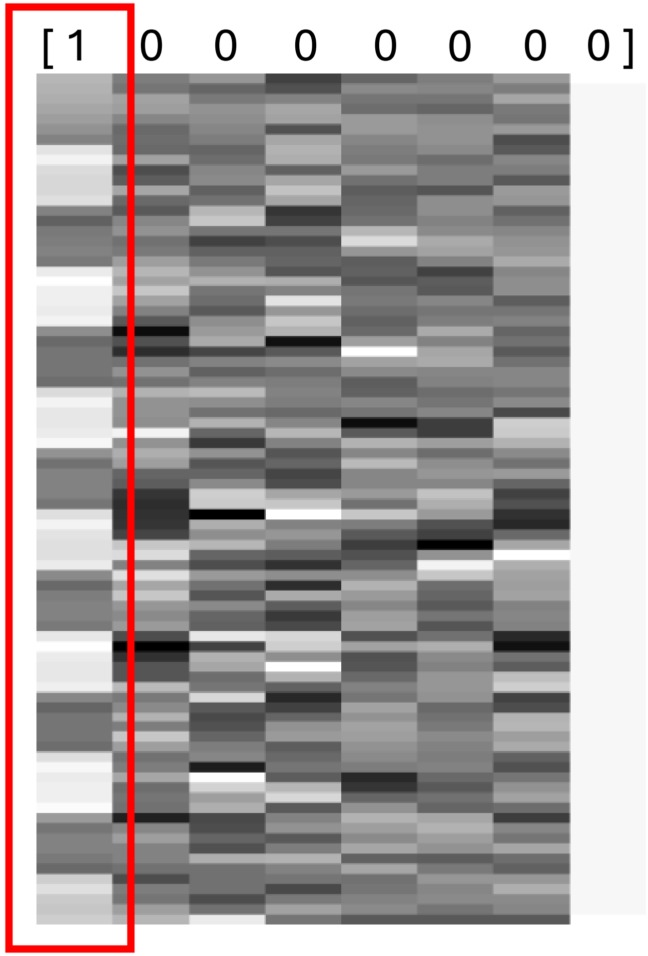
If we are interested in the effects of $\boldsymbol{\beta_1}$ compared to $\boldsymbol{\beta_5}$

$$\boldsymbol{c^T\beta = 1x\beta_1 + 0\chi\beta_2 + 0\chi\beta_3 + 0\chi\beta_4 - 1\chi\beta_5 + \ldots}$$

$$\boldsymbol{\beta_1 - \beta_5}$$

The million-dollar question…

Is the beta in my conditions of interest statistically significant?



Null Hypothesis – H_0

In this case, that our beta of interest, <u>does not</u> statistically significantly explain any variance in our model

We are trying to reject the H_0

$$H_0: cT\beta \ = 0$$

The alternative hypothesis is what we expresses the outcome of interest

$$H_1: cT\beta \ \neq 0$$

To test hypotheses, we need to calculate a **<u>t-statistic</u>**

What is the t-statistic?

T-statistic: the ratio of the difference in a number's estimated value form its assumed value over its standard error.

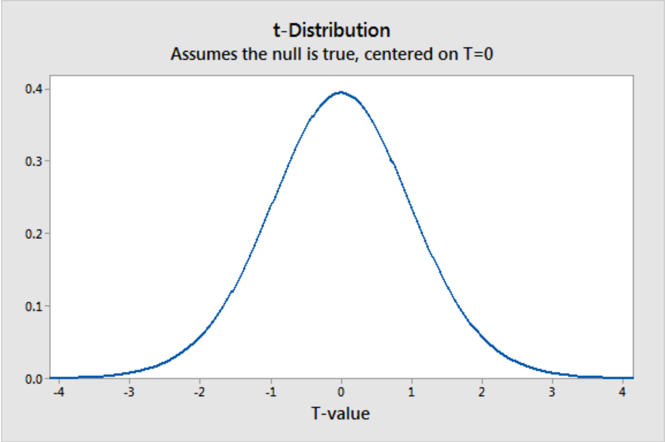
Test statistic:

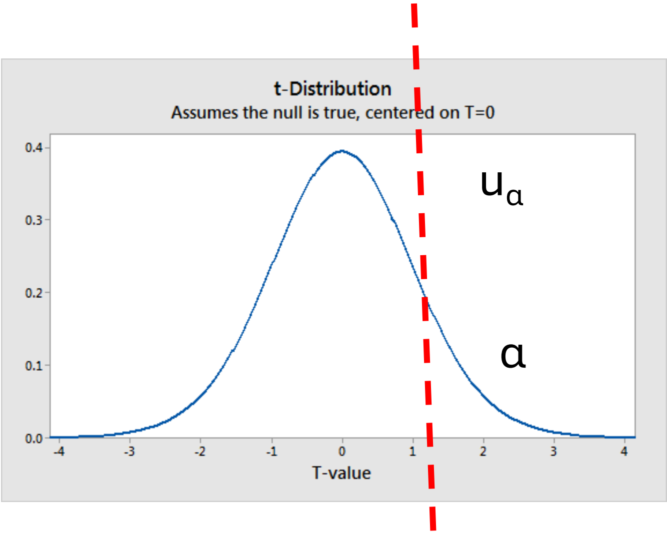
$$T = \frac{\textbf{\textit{contrast} of estimated parameters}}{\sqrt{\textbf{variance estimate}}}$$

Acceptable false positive rate α → threshold u_α

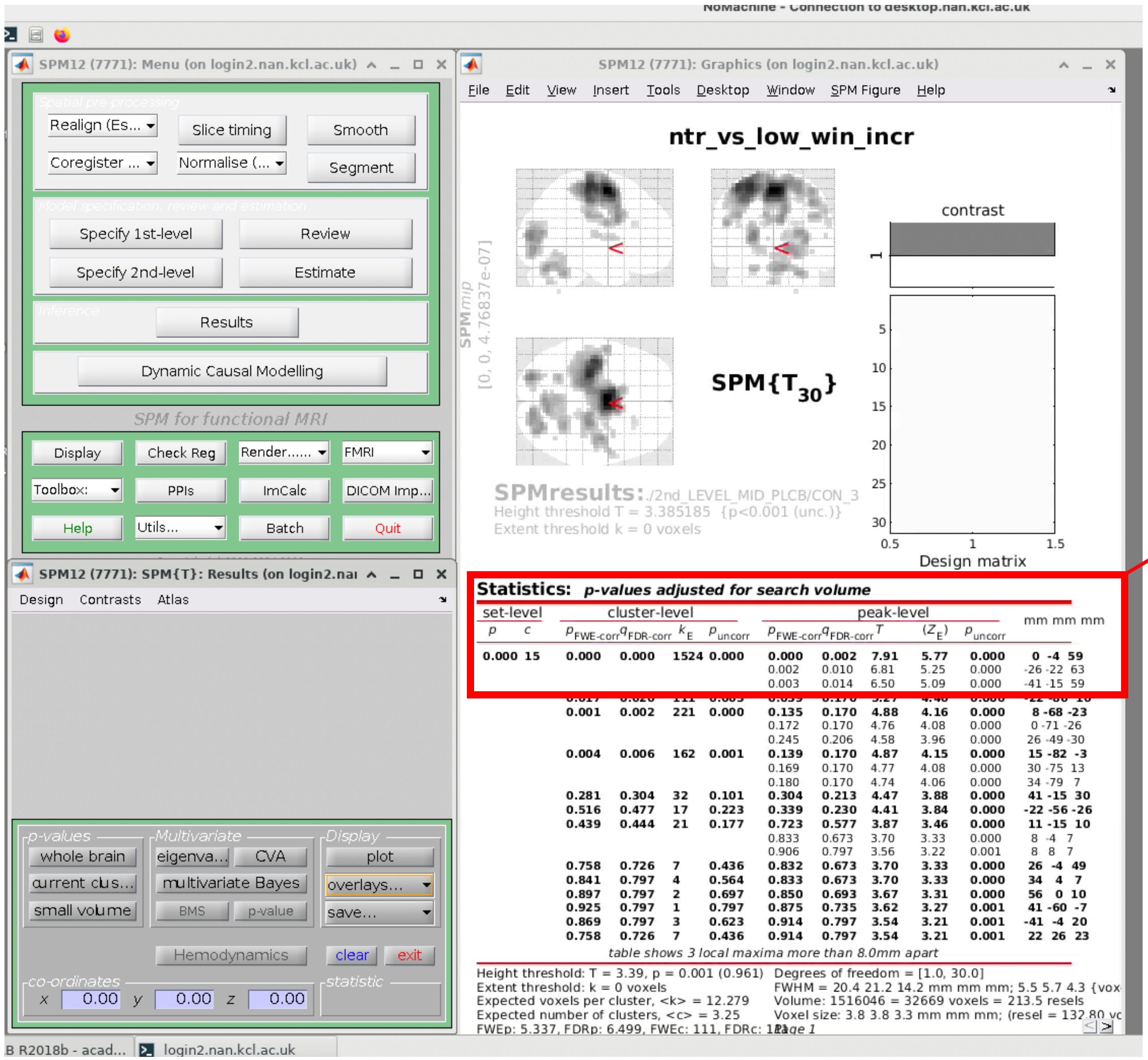
We reject the null hypothesis when t> u_α

A p-value contains information around the evidence against the H_0 . It gives the probability of observing a value more extreme than the t under the null hypothesis





An example



$P_{FWE-CORR}$	$P_{FDR-CORR}$	T	mm	mm	mm
0.000	0.002	7.91	0	-4	59
0.002	0.010	6.81	-26	-22	63
0.003	0.014	6.50	-41	-15	59

How about scaling?

- The T-statistic is the signal to noise ration
- The T-statistic does not depend on the scaling of your regressors
- The T-statistic does not depend on the scaling of your contrasts

However, there are scenarios where scaling is important..

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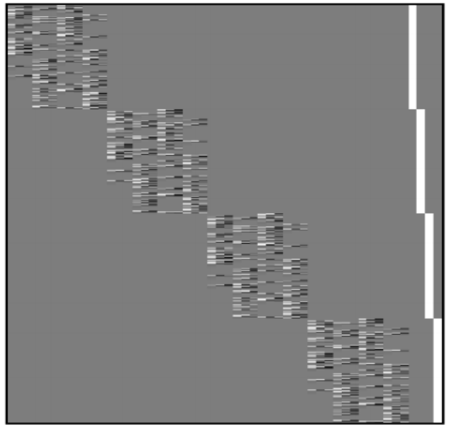

Subject 1

$Sum \neq average$

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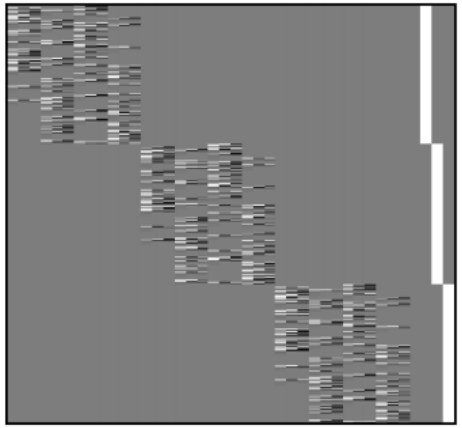
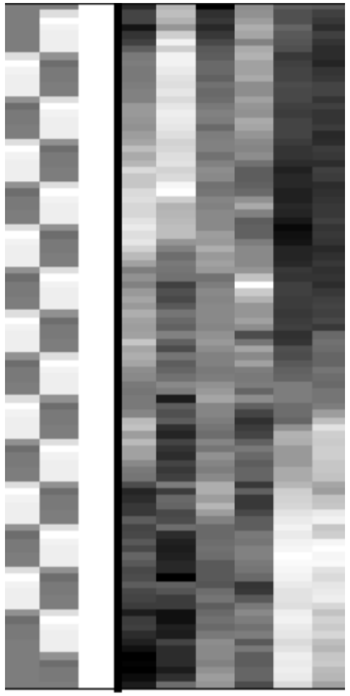


Subject 2

Contrasts - F-contrast

- When we want to compare models

X_0 X_1



Full model

→ RSS

X_0

Partial model

→ RSS_0

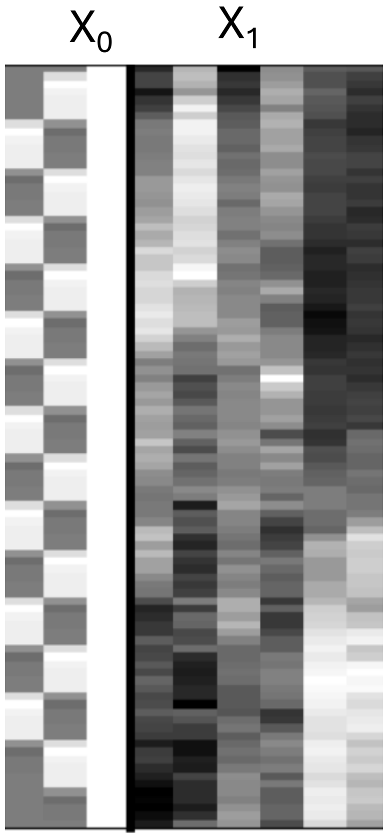
F-statistic:
explained variance/unexplained variance

$$F = \frac{RSS_0 - RSS}{RSS}$$

*RSS: Residual sum of Squares
Measures the level of variance in the error term, or residuals in a regression model

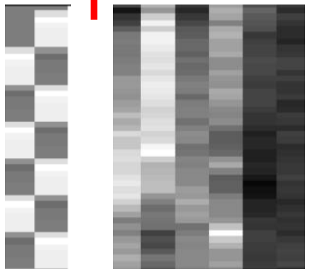
Contrasts - F-contrast

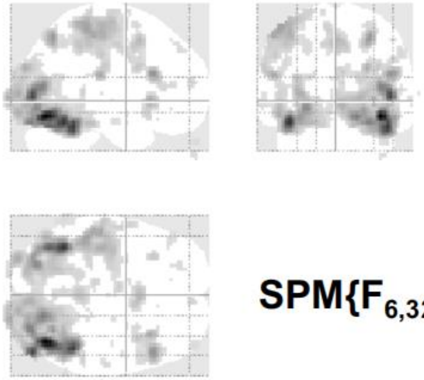
H_0: True model is the X_0 → $H_0: \beta_4 = \beta_5 = \ldots = \beta_9$



Full model

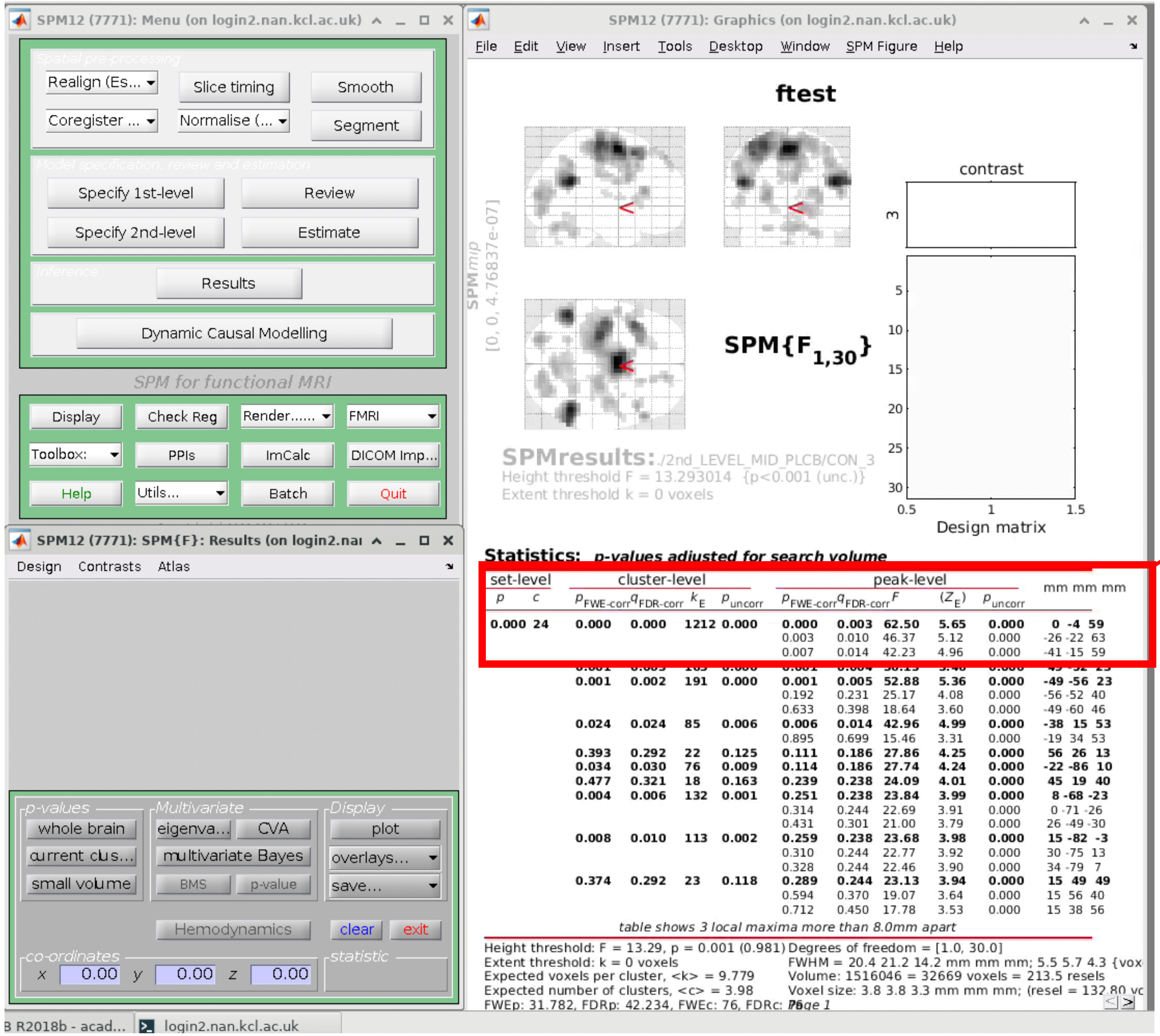
$$c^T = \left[\begin{array}{ccc|cccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$





$SPM\{F_{6,322}\}$

An example



$P_{FWE-CORR}$	$P_{FDR-CORR}$	F	mm	mm	mm
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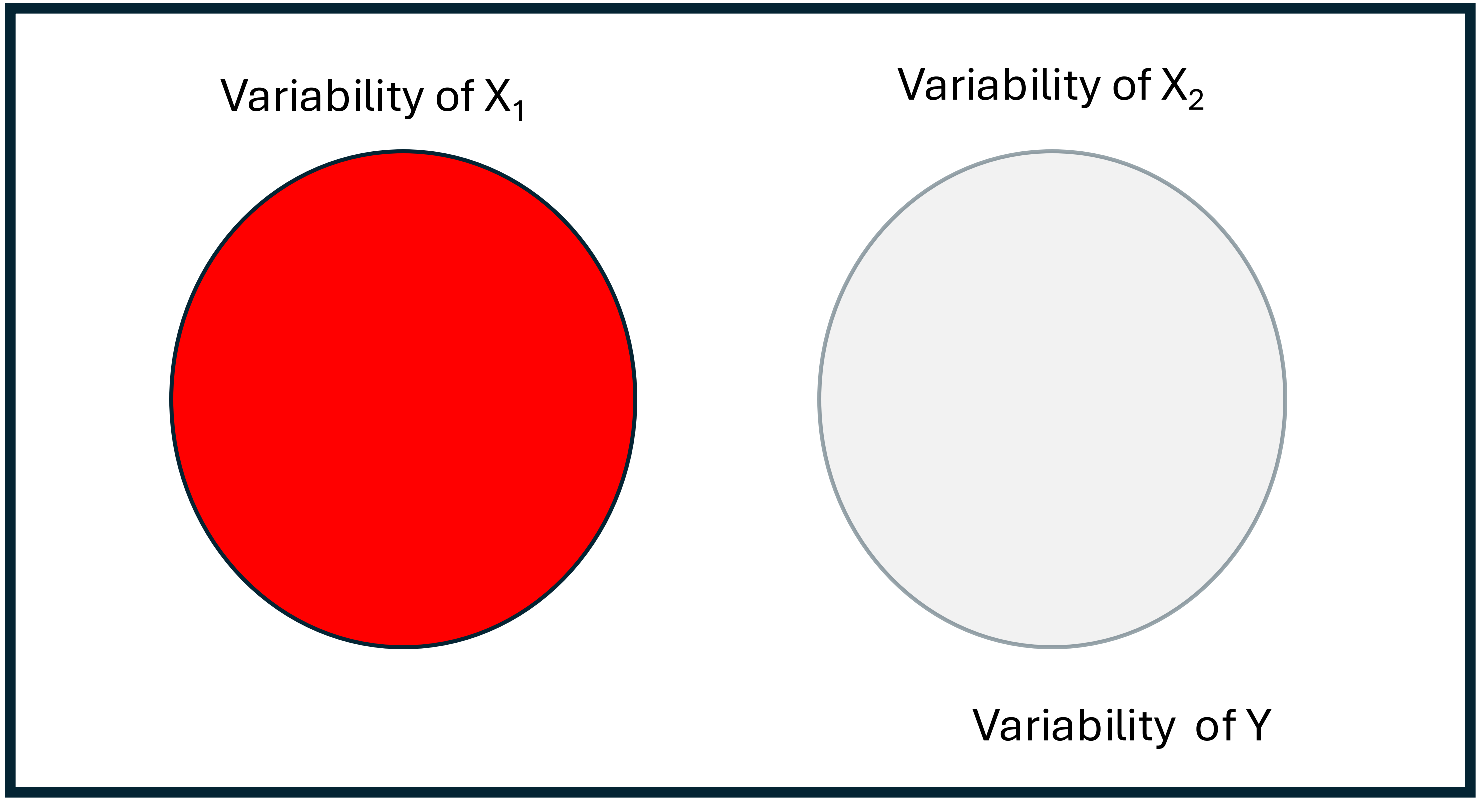
0.003	0.010	46.37	-26	-22	63
0.007	0.014	42.23	-41	-15	59

A final note…

Orthogonality: reflects how much your regressors correlate with one another



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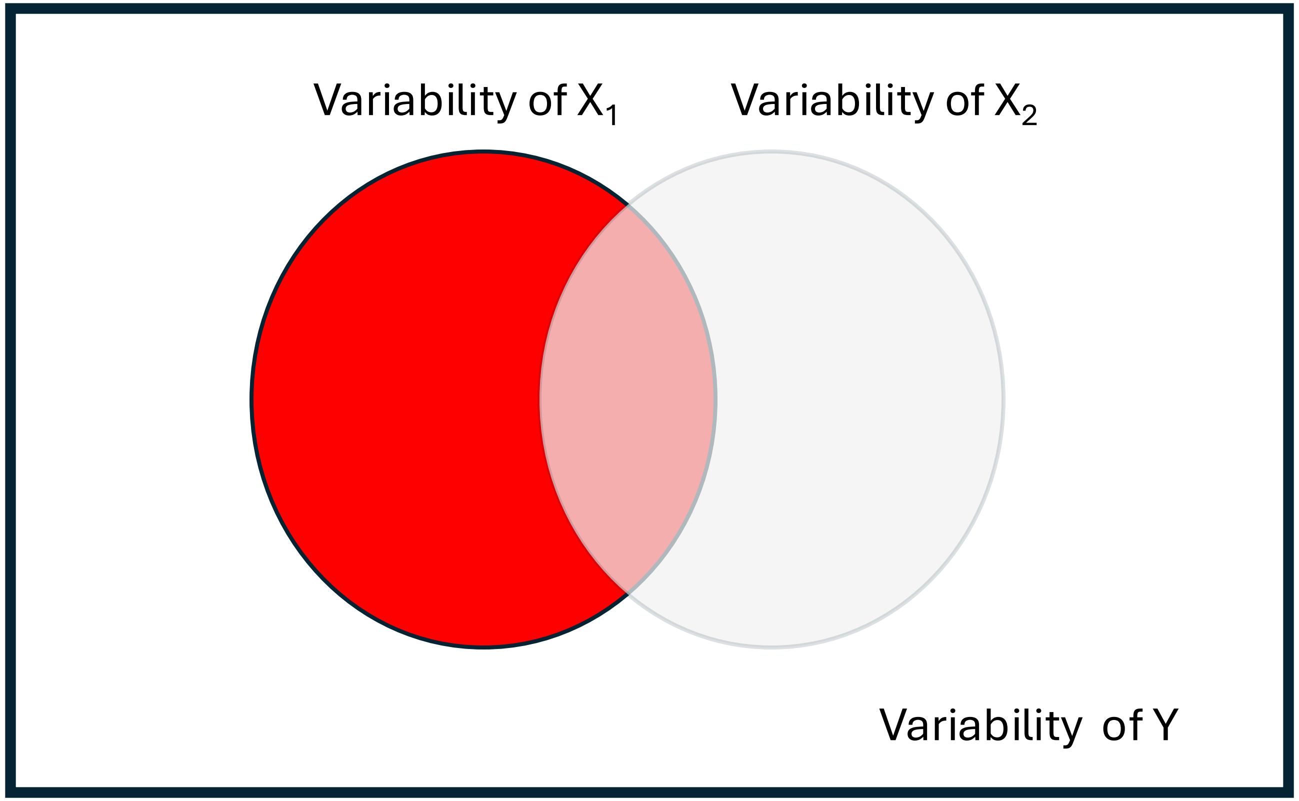

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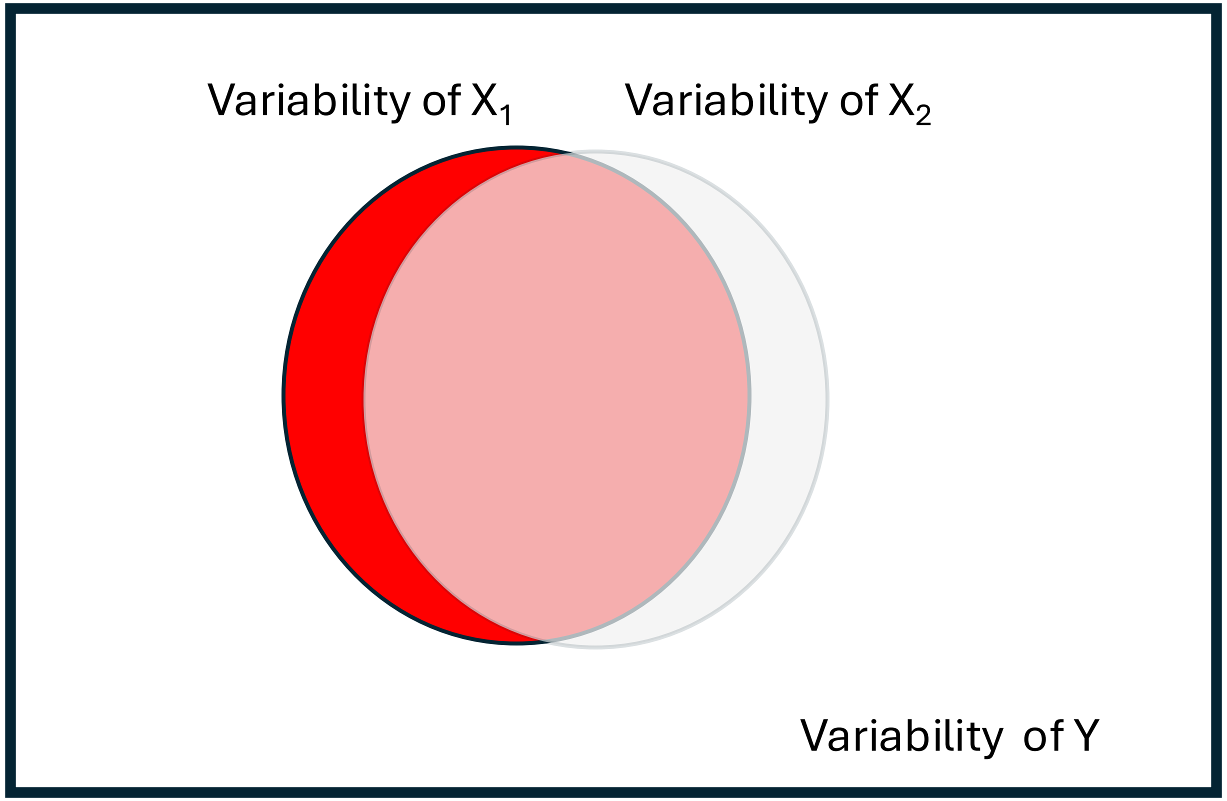
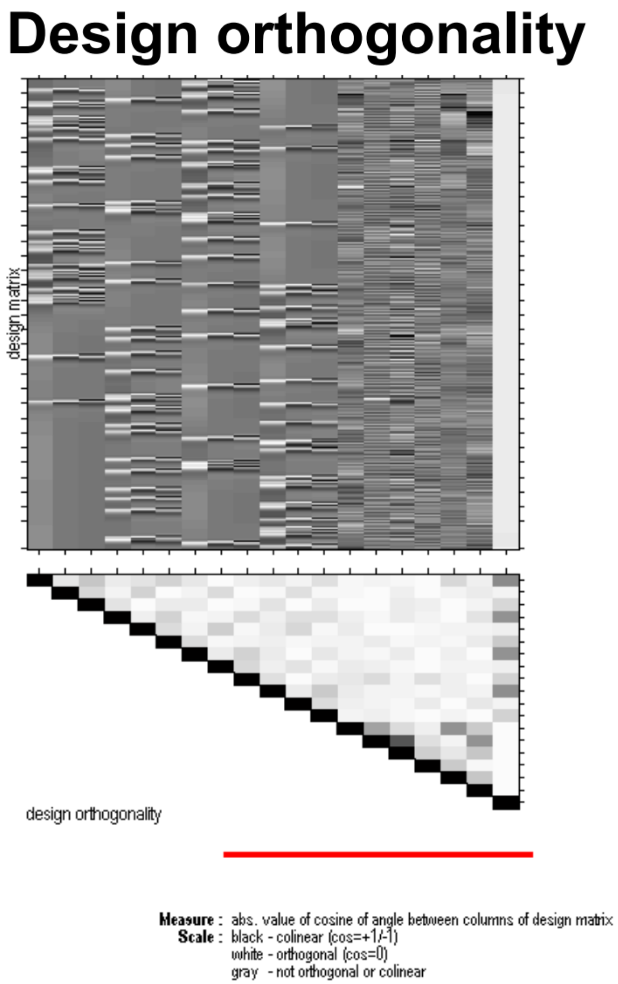


Orthogonality



- ✓ Orthogonal regressors are regressors that do not share any variability
- ✓ To account for a degree of shared variability we regress out the effect that is shared between regressors
- ✓ A lot of share variability, however, does not leave enough signal that is attributed to single regressors
- ✓ You can design your orthogonality matrix and check it before you do your experiment.

Thank you!