Accelerated imaging

MRI Physics for SPM

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MRI Physics for SPM – May 2026

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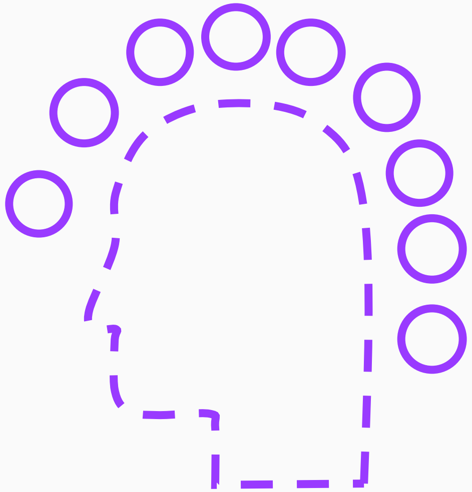
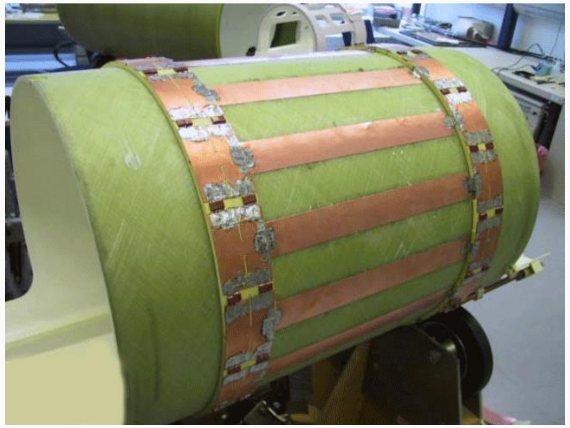


- **Understand what are RF coils**
  - Phased array coils
  - Coil sensitivity
  - Coil combination
- **Understand accelerated imaging**
  - Partial Fourier
  - *k*-space acceleration
  - Multi-band imaging
  - Compressed sensing
- **Understand the trade-offs of acceleration**
  - *g* Factor
  - Unfolding artefacts / Slice leakage
  - Temporal correlations

Receive coils

Birdcage – Transmission

Body coil



Head coil

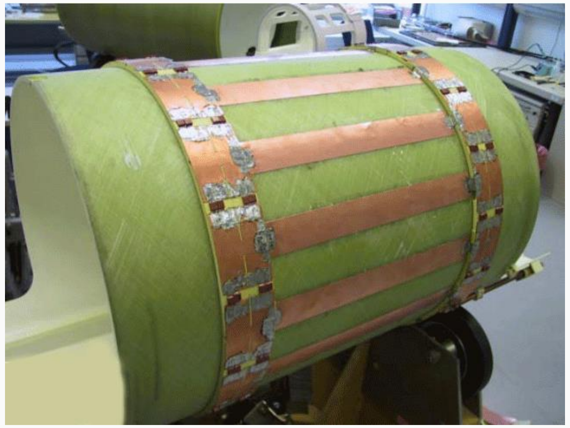
Transmit field: B_1^+

Primary role = homogeneous excitation

https://mriquestions.com/birdcage-coil.html
https://mriquestions.com/rf-safety-overview.html

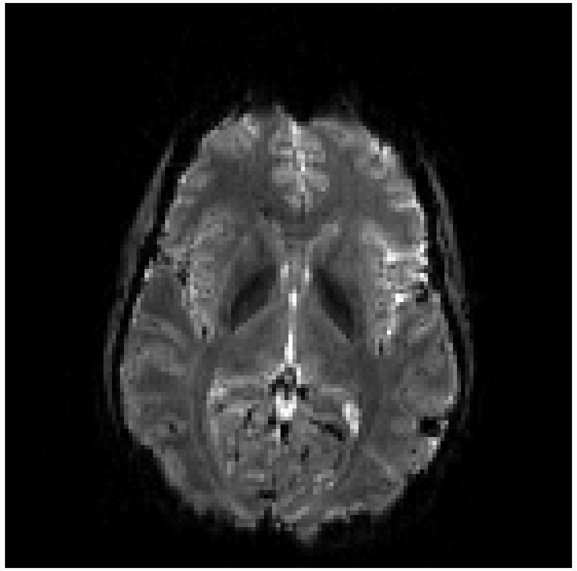
Birdcage – Reception

Body coil

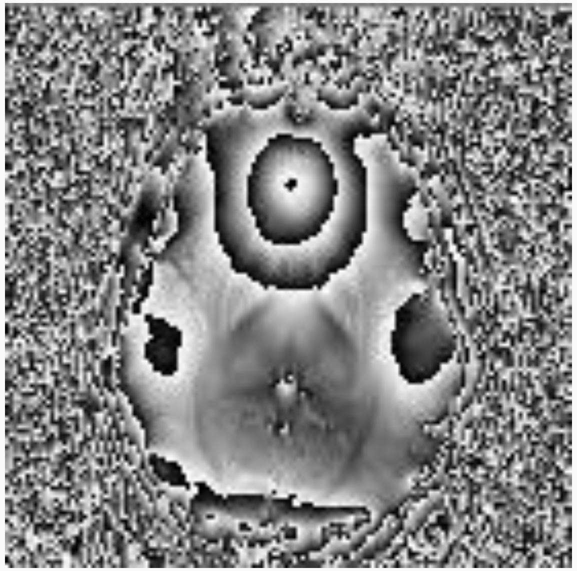


Head coil

Magnitude



Phase



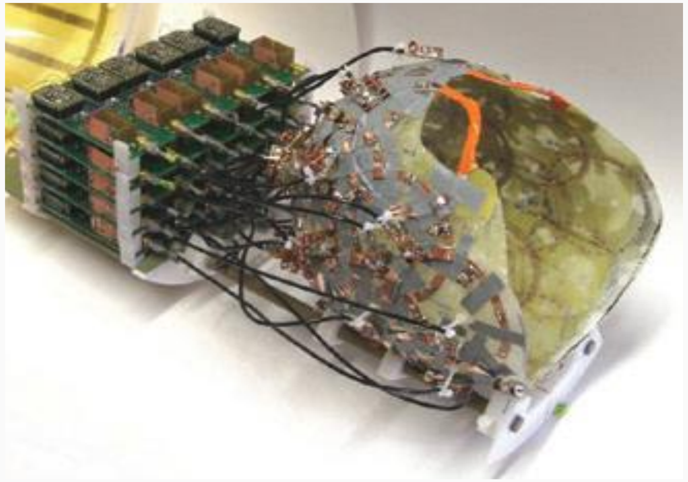
Two channels run in quadrature
= one complex channel

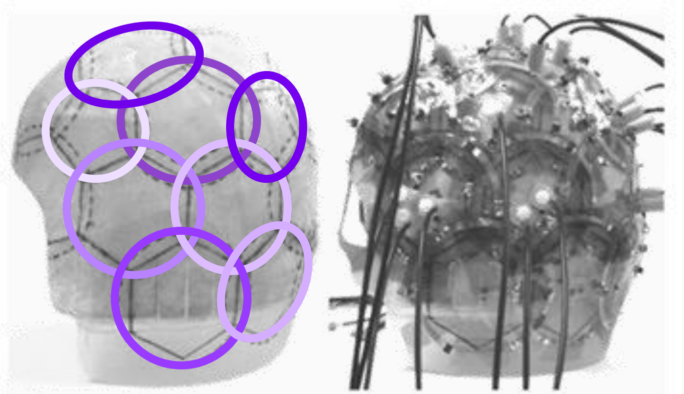
$$\rho = |\rho| \cdot \exp(i \cdot \psi)$$

https://mriquestions.com/birdcage-coil.html
https://mriquestions.com/rf-safety-overview.html

Array coils – Reception only

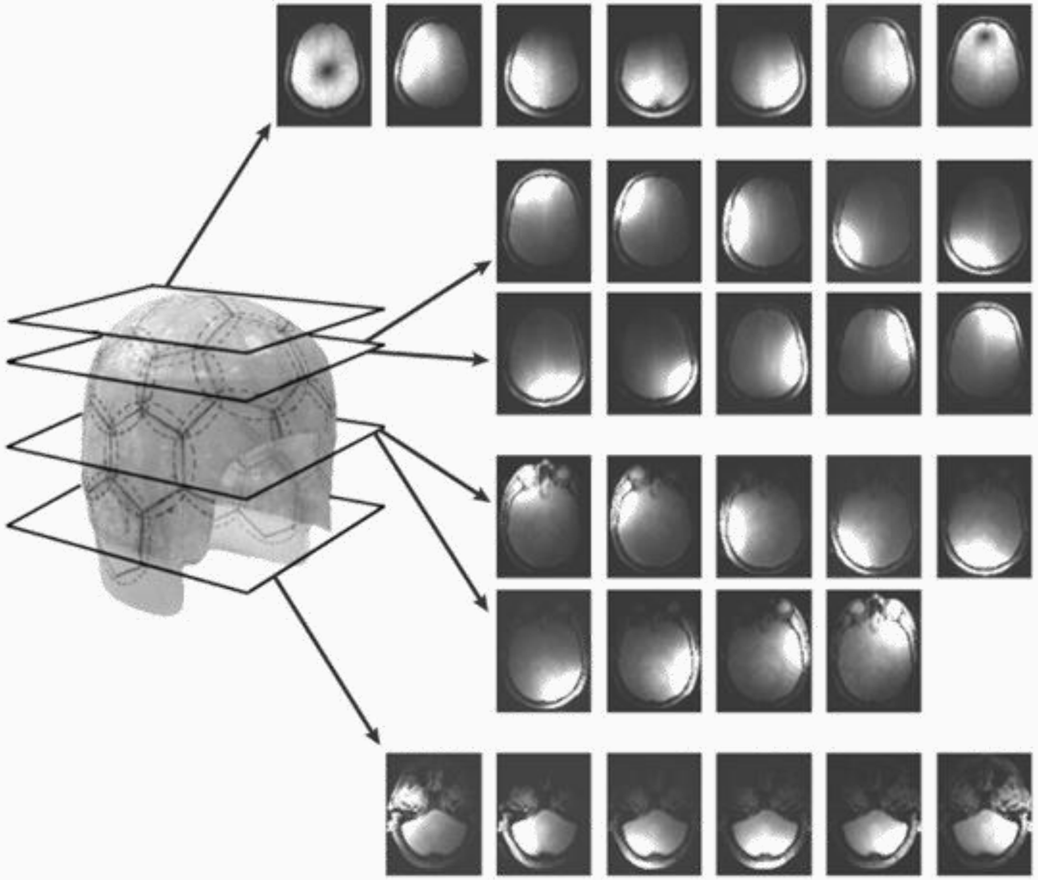
Phased-array head coil





Overlap for decoupling

One (complex) image per coil element



Anatomical signal

$$f_k = s_k \cdot \rho$$

Coil sensitivity

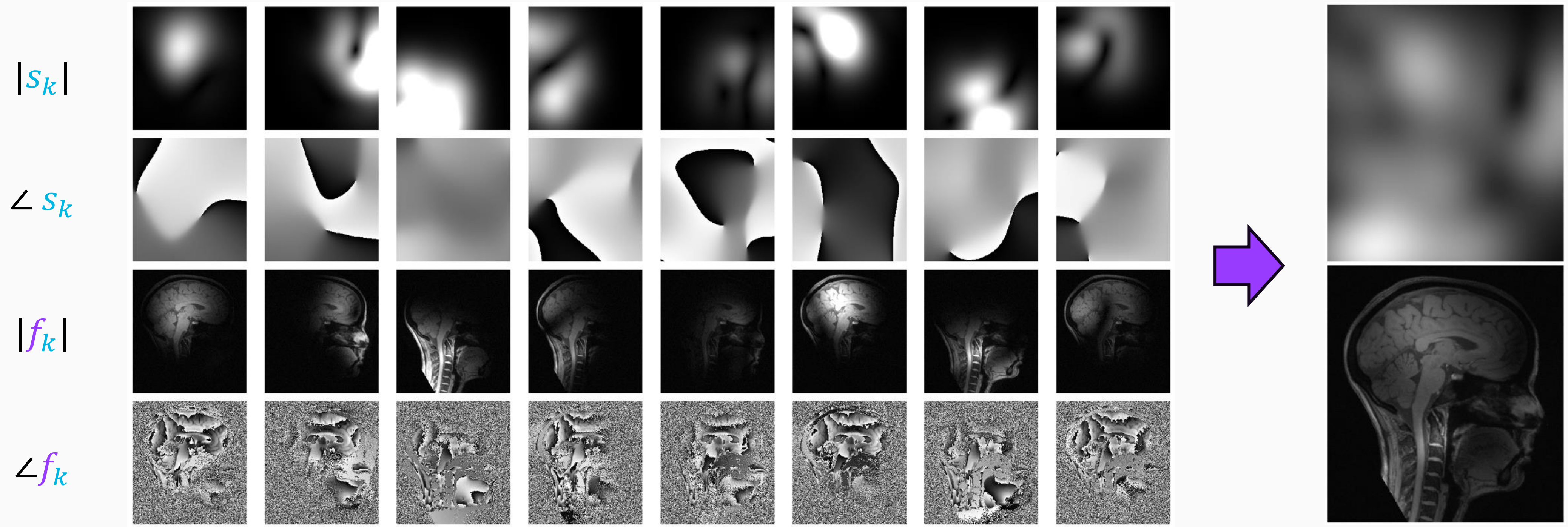
Image acquired by a coil

Wiggins & al. MRM (2006)

Root sum-of-square

-> Results in a real image (phase is lost)
-> Does not require the coil sensitivities to be known

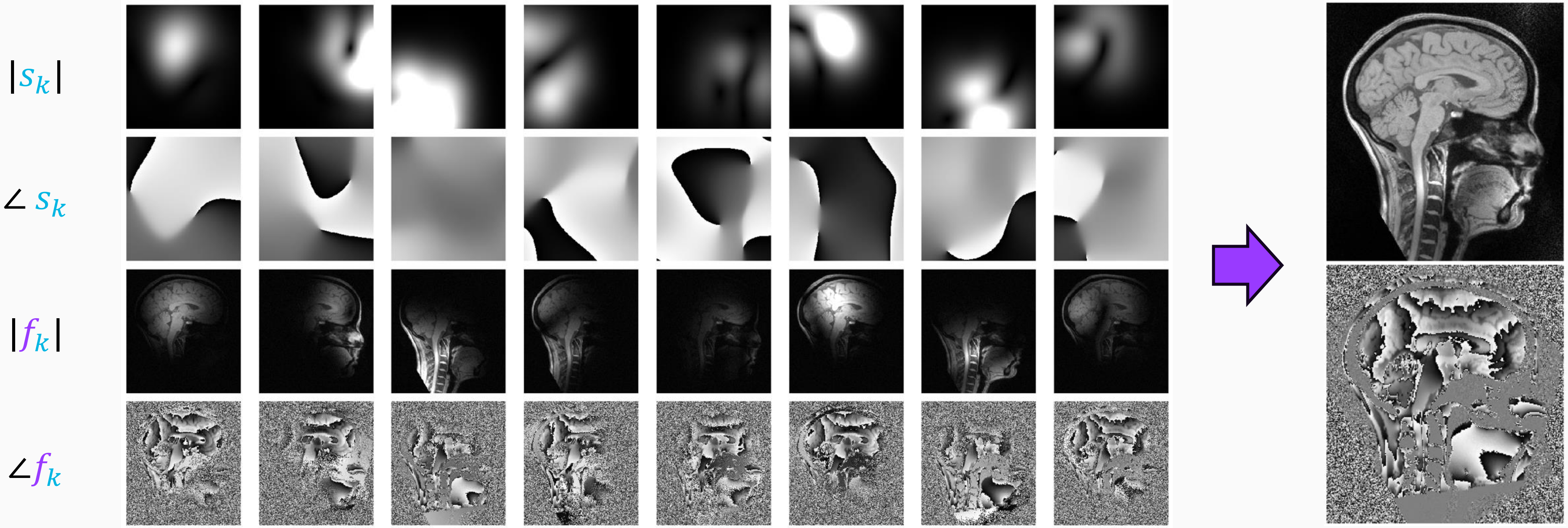
$$f_{\mathrm{RSS}} = \sqrt{\sum_k |f_k|^2} = \sqrt{\sum_k |s_k \cdot \rho|^2} = |\rho| \underbrace{\sqrt{\sum_k |s_k|^2}}_{\text{Net modulation}}$$



Optimal combination

-> Results in a complex image (phase is preserved)
-> Requires the coil sensitivities to be known

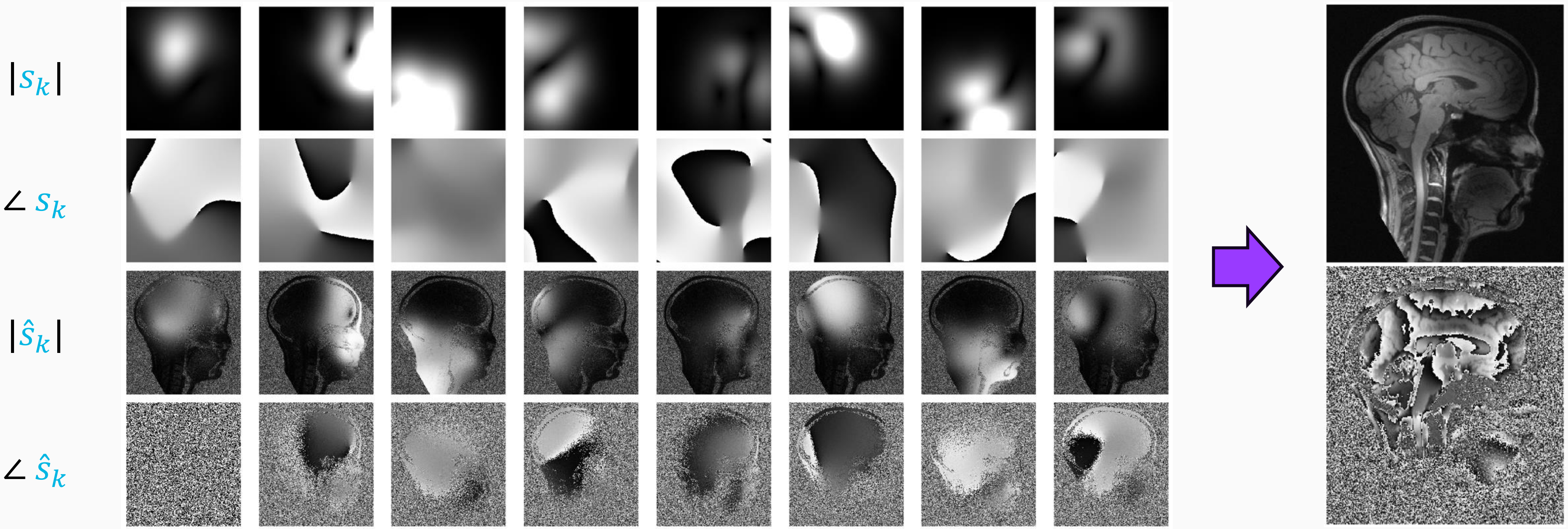
$$\underbrace{\begin{matrix} f_k = s_k \cdot \rho \\ \mathbf{f} = \mathbf{s} \cdot \rho \end{matrix}}_{\text{Linear system}} \quad \Rightarrow \quad f_{\text{opt}} = \underbrace{(\mathbf{s}^H \mathbf{s})^{-1} \, (\mathbf{s}^H \mathbf{f})}_{\text{Pseudo-inverse}} = \underbrace{(\mathbf{s}^H \mathbf{s})^{-1} \, (\mathbf{s}^H \mathbf{s}}_{\text{Net modulation = 1}} \cdot \rho) = \rho$$



Coil sensitivity mapping

❑ **Imaging = mapping** -> one sensitivity-weighted image per coil element

❑ **But** more unknowns (K coils + 1 anatomical signal) than observations (K sensitivity-weighted images)



Coil sensitivity mapping

- **Imaging = mapping** -> one sensitivity-weighted image per coil element

- **But** more unknowns (*K* coils + 1 anatomical signal) than observations (*K* sensitivity-weighted images)

- In practice, an additional constraint must be imposed.

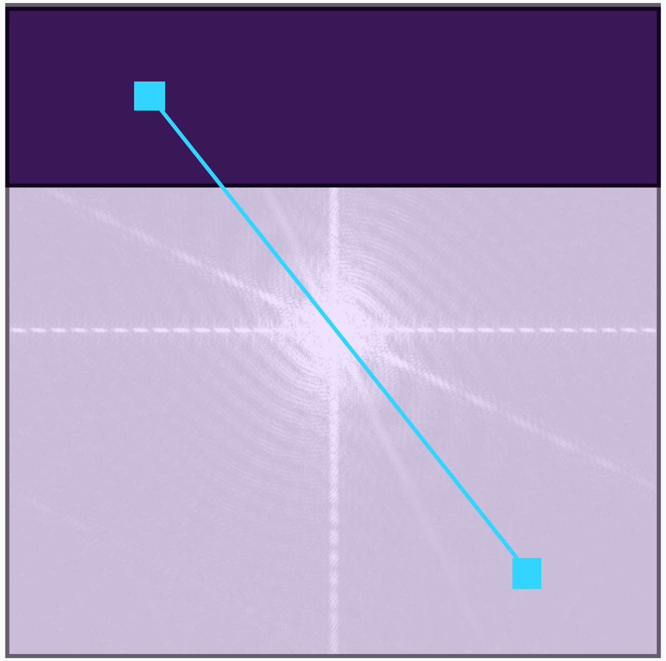
  - Often, this constraint causes the net modulation of the reconstructed image to be the same as in a RSS combination:

$$f_{\mathrm{opt}} = f_{\mathrm{RSS}} \cdot e^{i\psi}$$

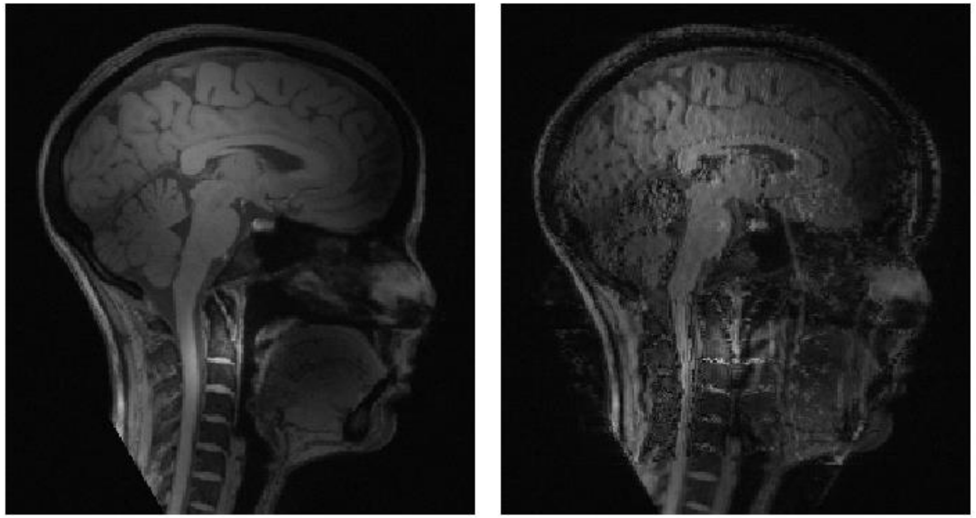
  - E.g.:
    - Adaptive combine Walsh & al. MRM (2000)
    - ESPIRIT: Uecker & al. MRM (2014)
    - MORSE: Josephs & al. arXiv (2025)

Accelerated imaging

Partial Fourier

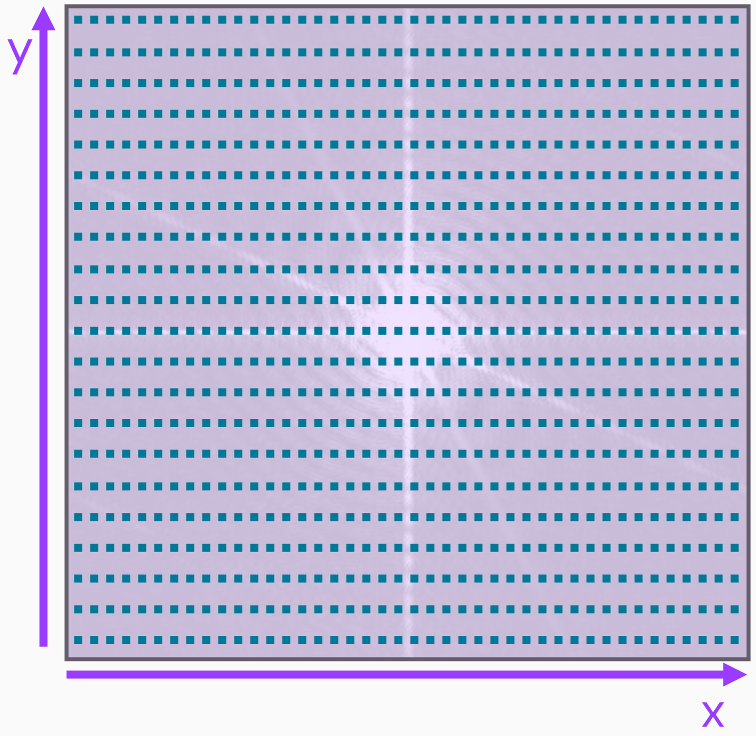


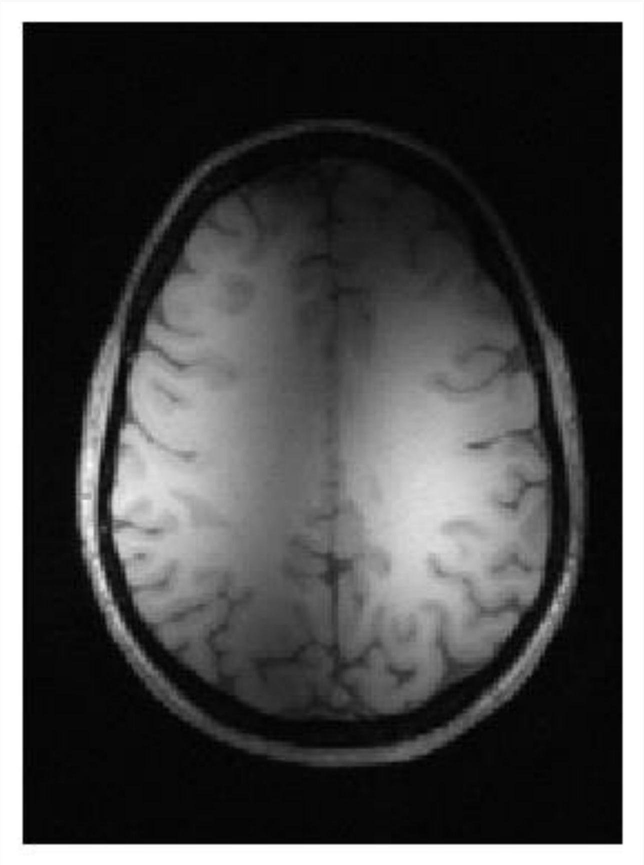
- Partial Fourier consists in not acquiring one side of k-space
- If the acquired signal was not complex (= no phase), we would only need one half of k-space.
  - The negative frequencies are equal to the **complex conjugate** of the positive frequencies: $F[-k] = \mathrm{Real}(F[k]) - \mathrm{i} \cdot \mathrm{Imag}(F[k])$



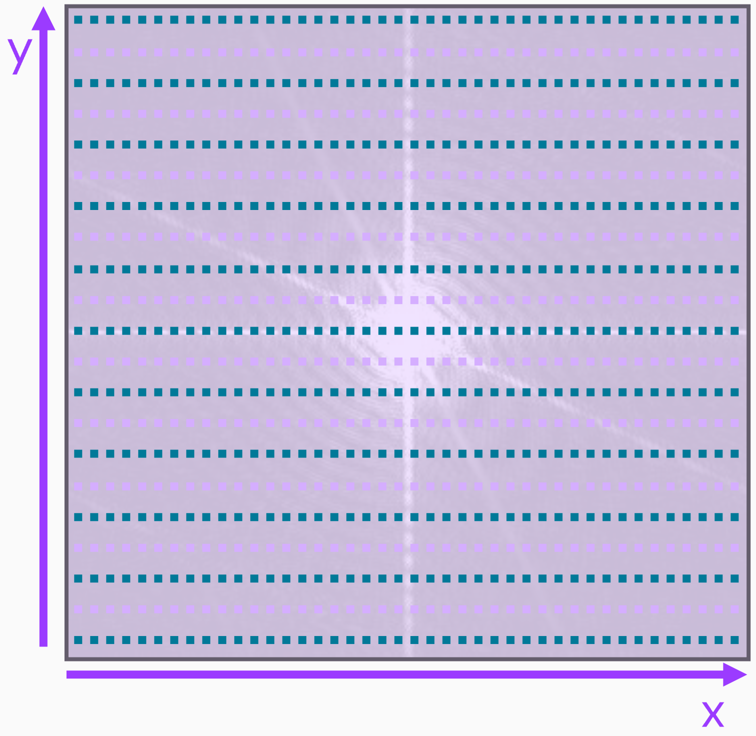
- However, the acquired data are noisy. Using partial Fourier means **acquiring less data**, so the reconstructed image has **lower SNR**:
  - $\mathrm{SNR}_{\mathrm{PF}} = \mathrm{SNR}_{\mathrm{Full}} \cdot \sqrt{f}$ where f is the proportion of data sampled.
- In practice, the signal is complex, so k-space is not conjugate symmetric. This results in some **smoothing** in the reconstructed image.

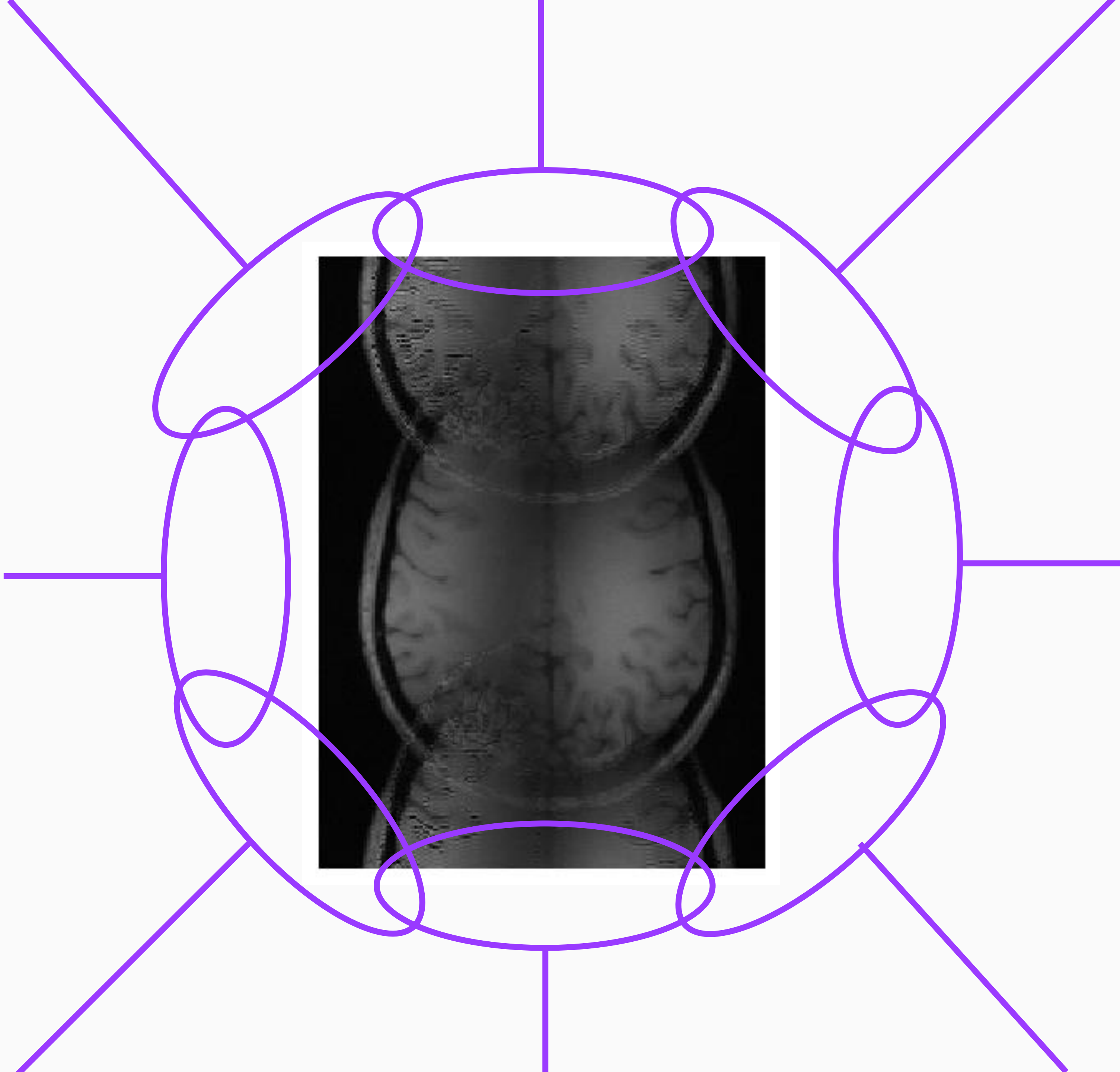
Parallel Imaging





Parallel Imaging

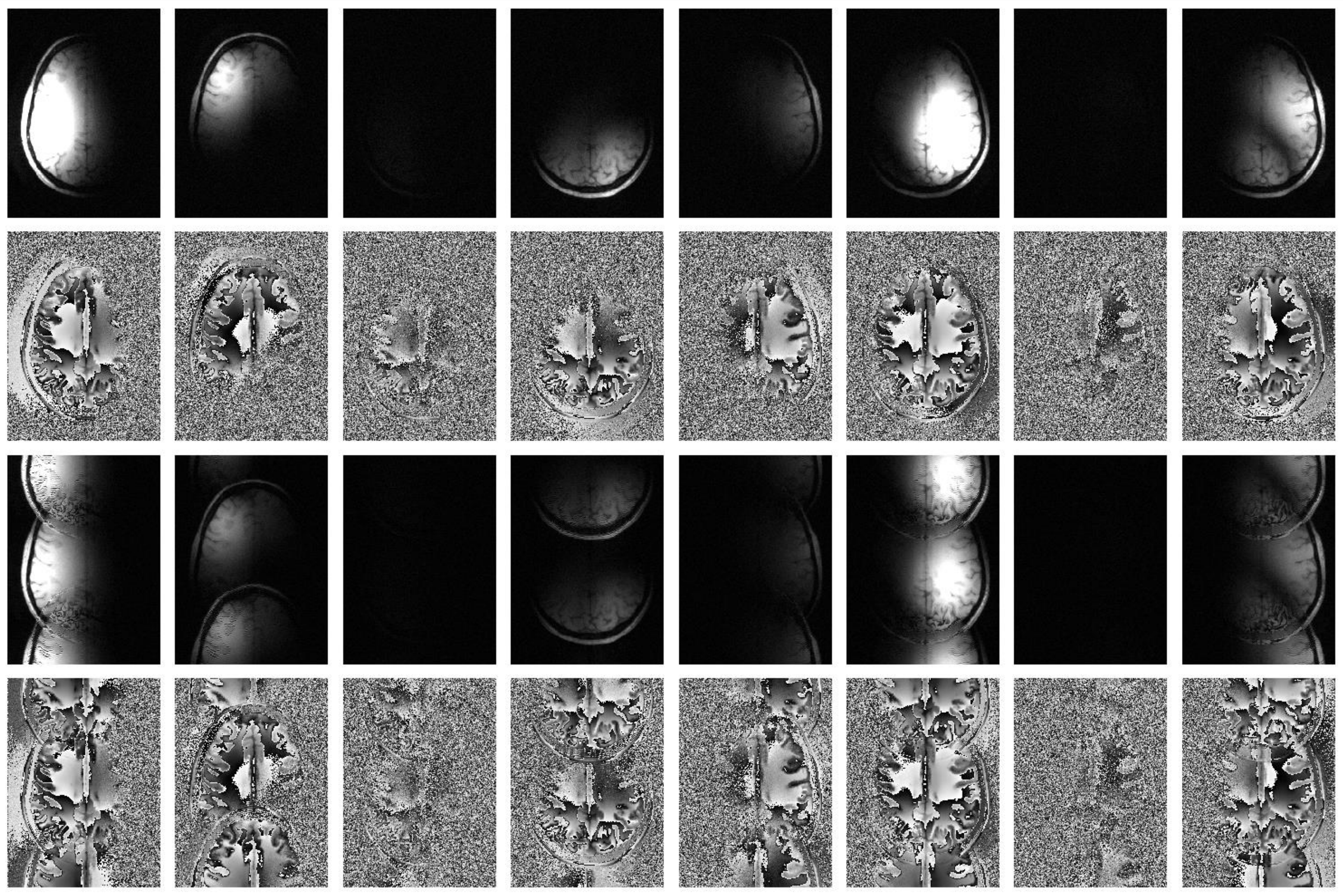


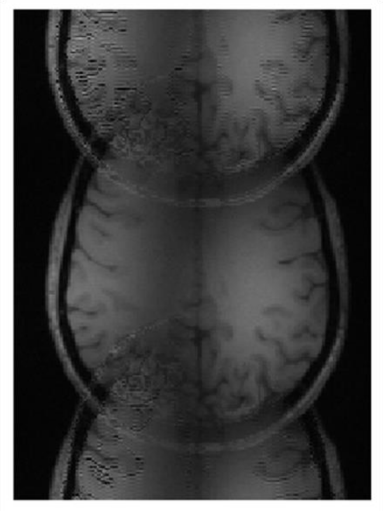


Parallel Imaging

No acceleration

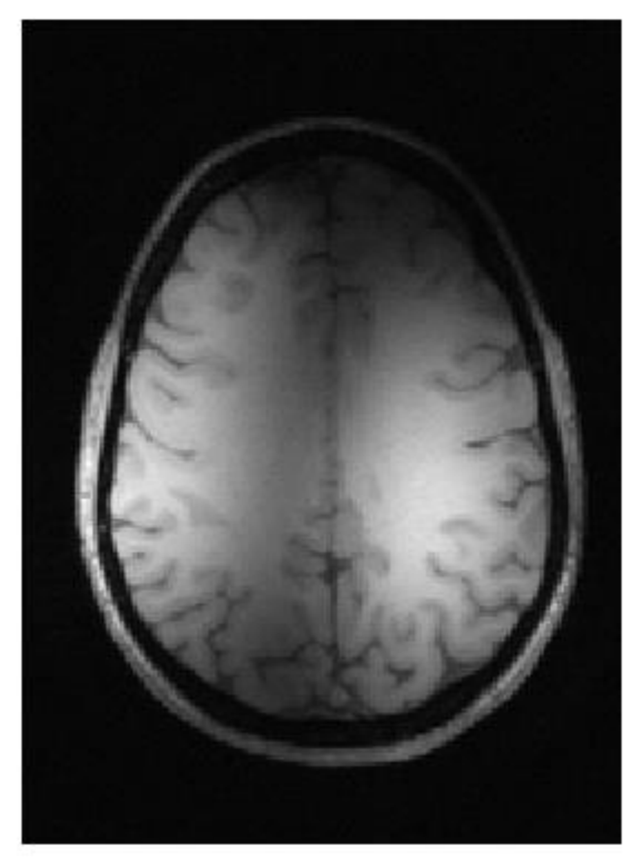
2x acceleration





Parallel Imaging

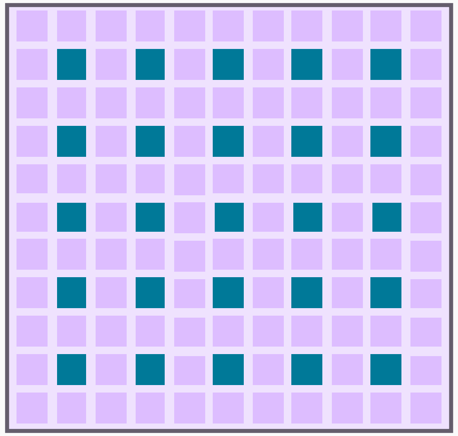
No acceleration



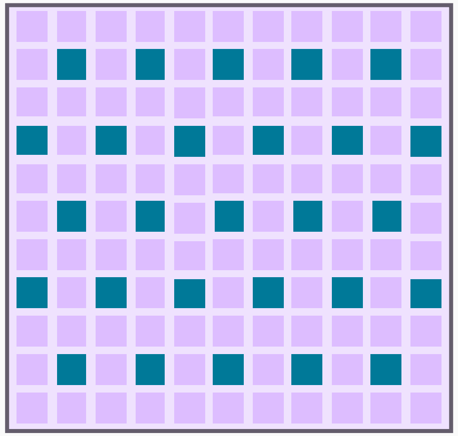
2x2 Cartesian

2x2 CAIPIRINHA

2x2 Cartesian



2x2 CAIPIRINHA

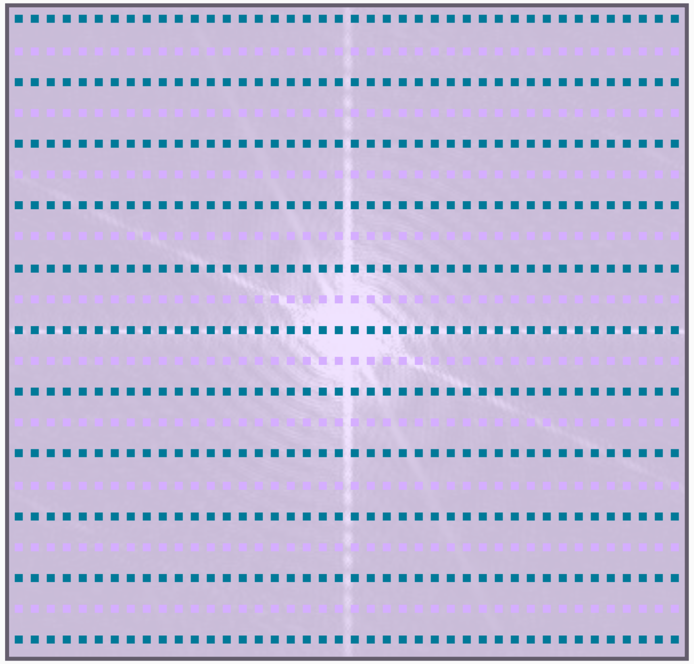


Parallel Imaging

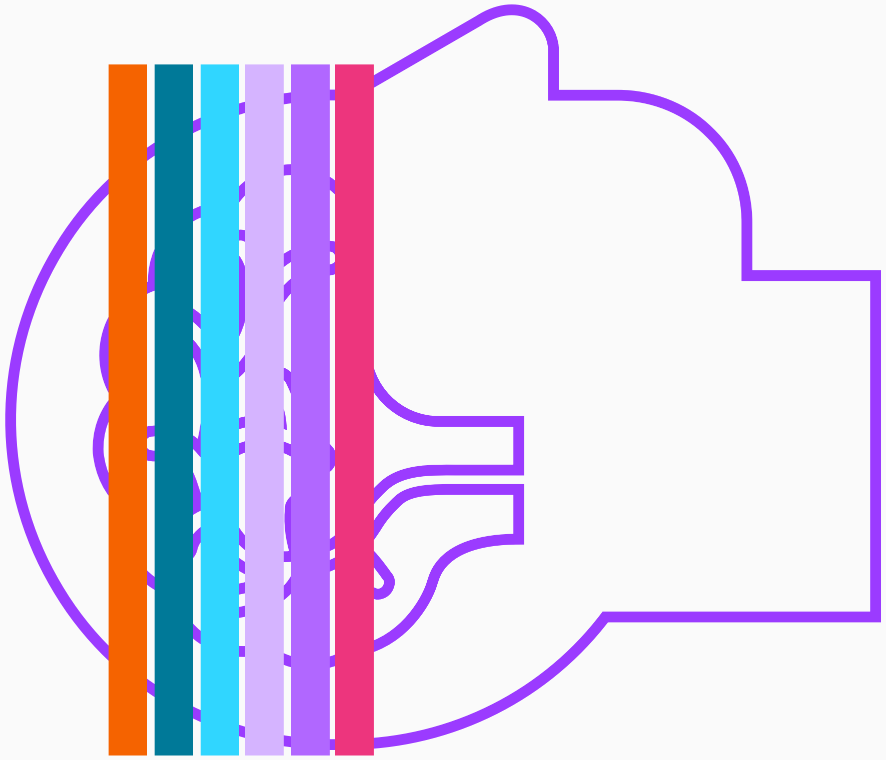
- Cartesian acceleration consists of only acquiring **one every few lines** of k-space.

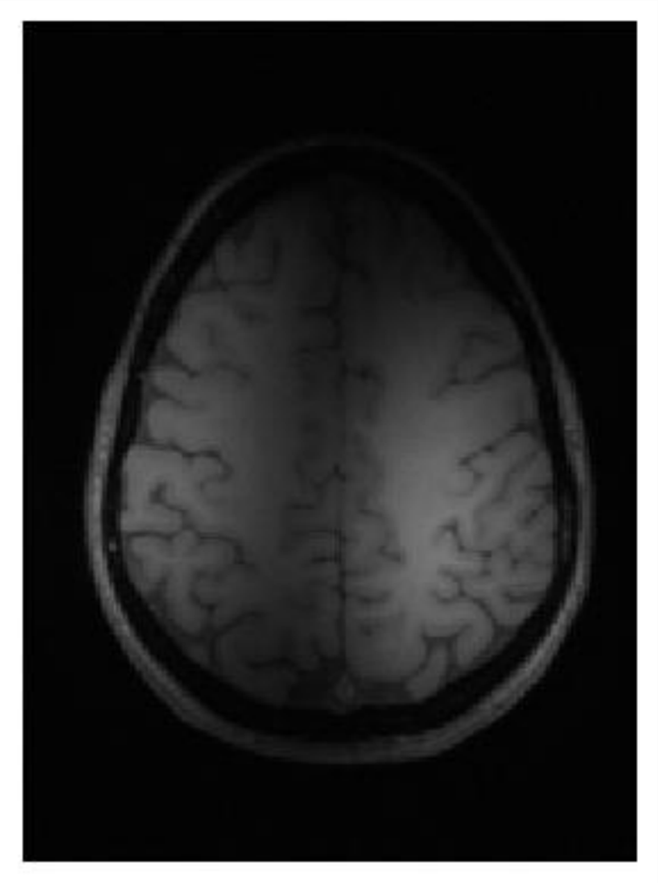
- Having multiple coil elements with **spatially varying sensitivities** ensures that we are acquiring more samples (Number coils) than values to reconstruct (Acceleration factor).
    - Reconstruction involves inverting a **well-posed linear system.**

- However, the acquired data are noisy. Because acceleration means **acquiring less data**, the reconstructed image has **lower SNR**:
    - $\mathrm{SNR}_{\mathrm{PI}} = \mathrm{SNR}_{\mathrm{Full}} \cdot \sqrt{1/R}$ where R is the acceleration factor.

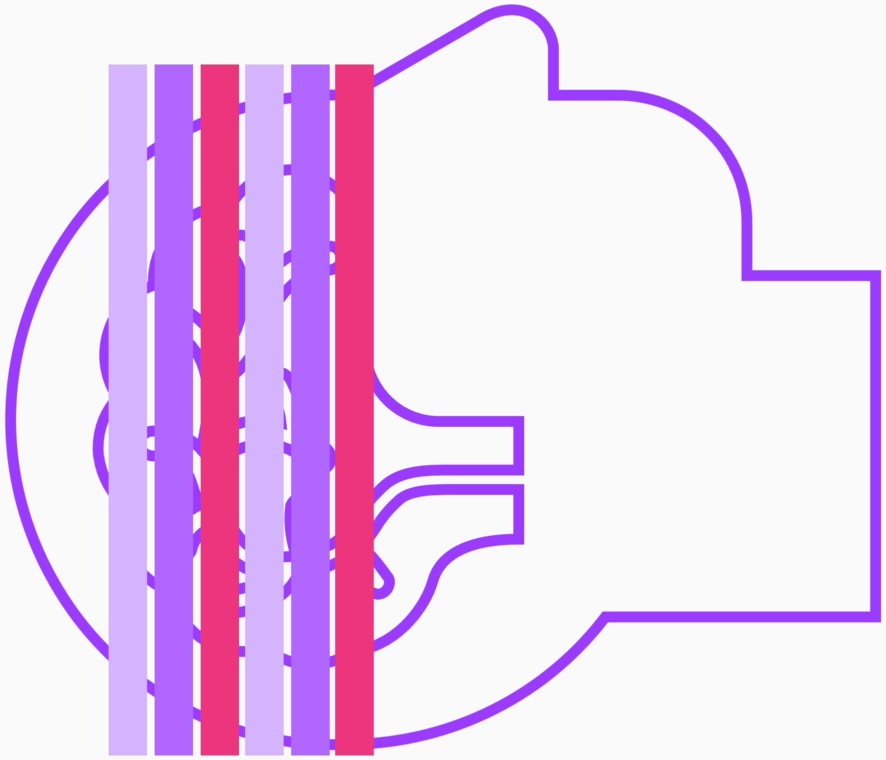


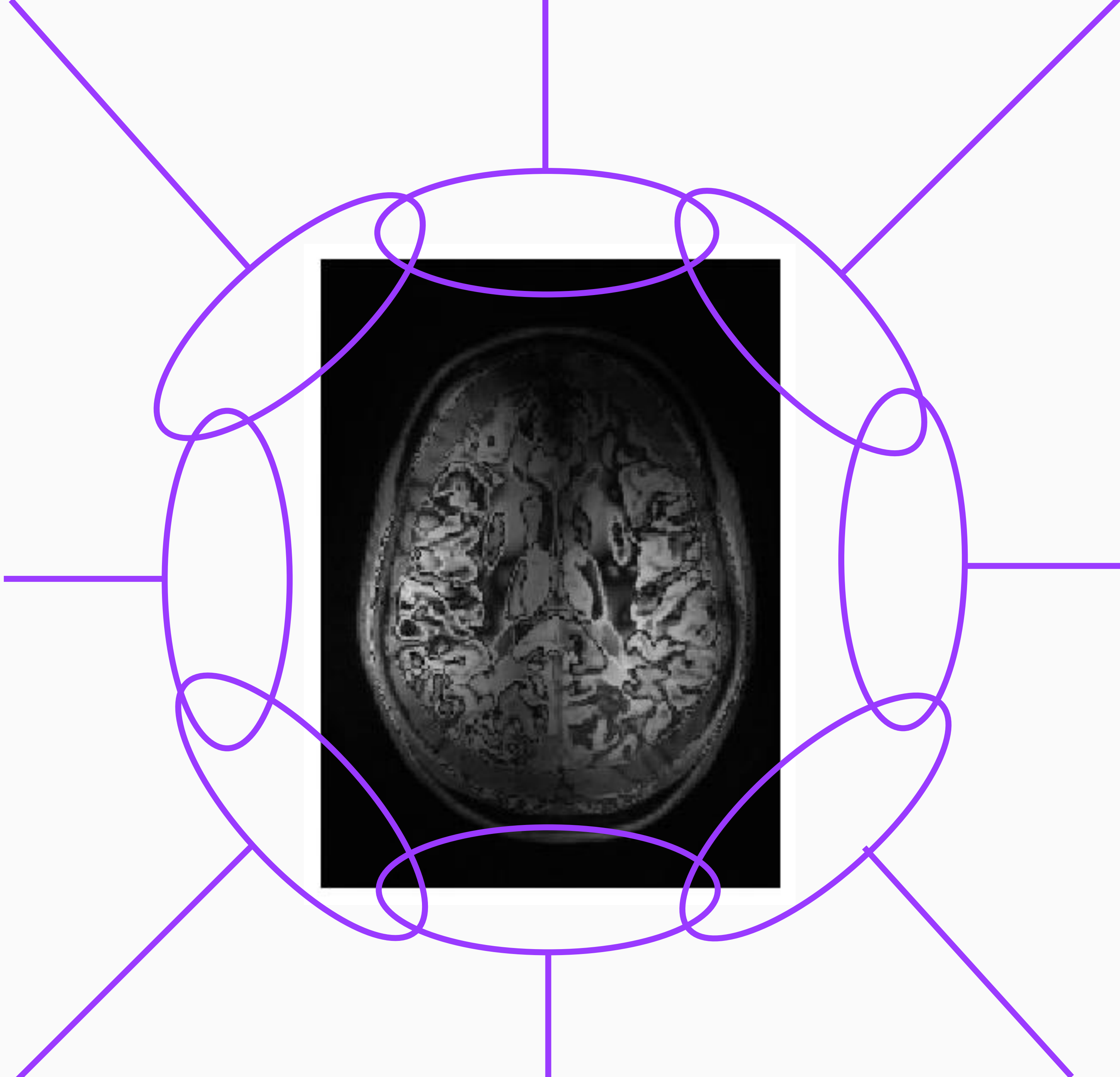
Single Band



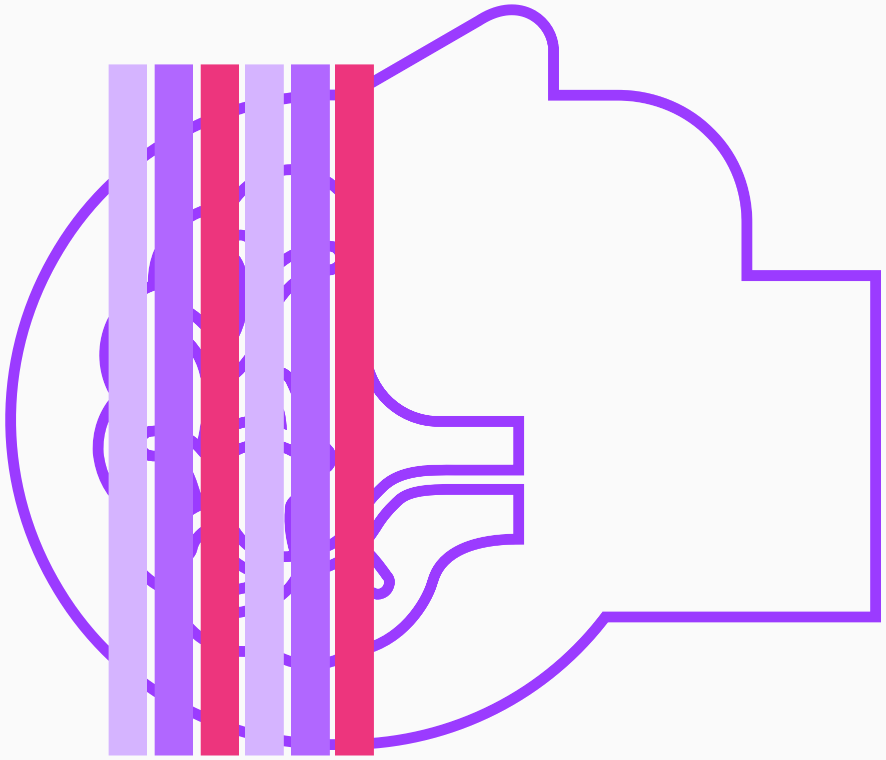


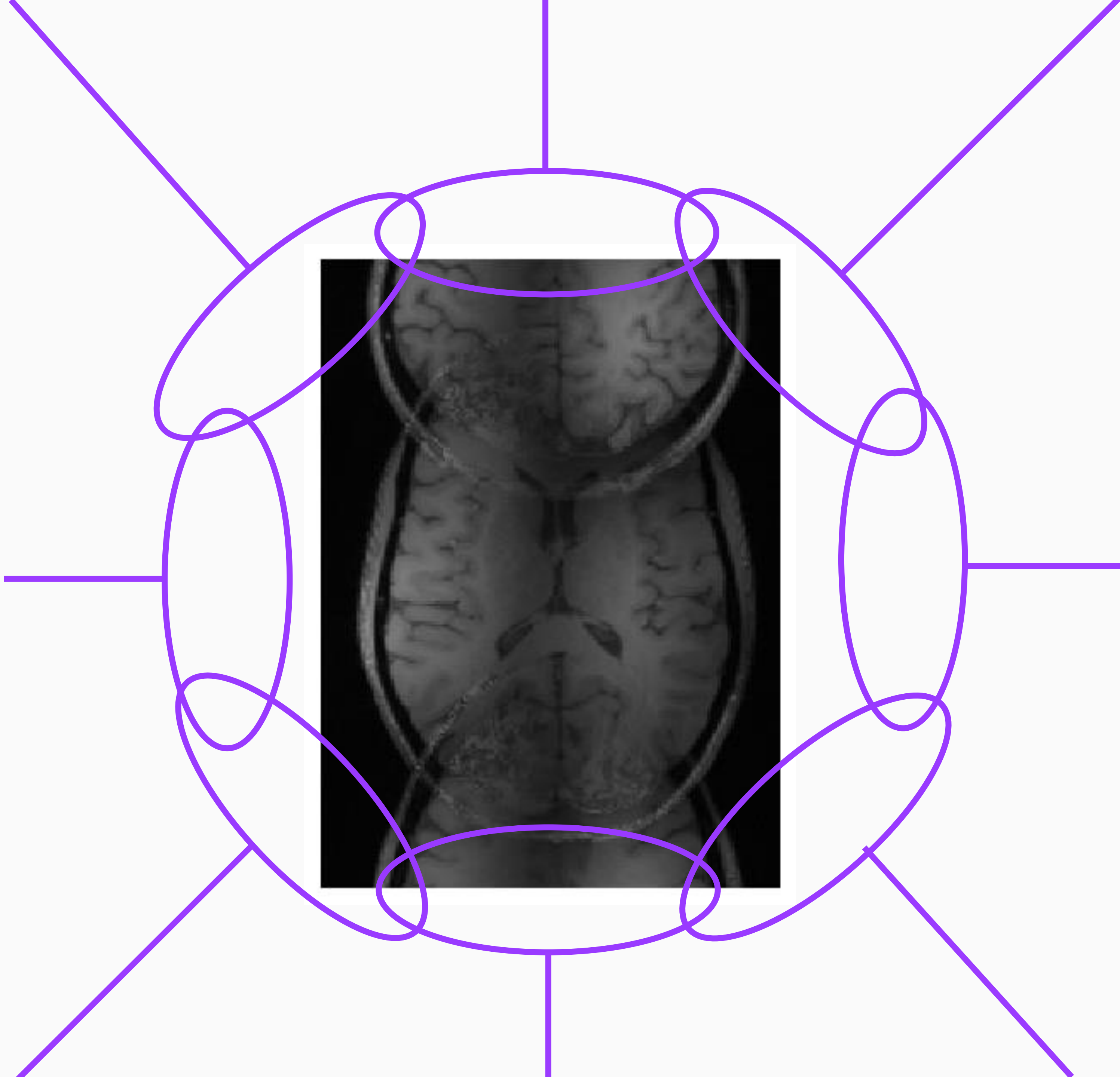
Multi Band



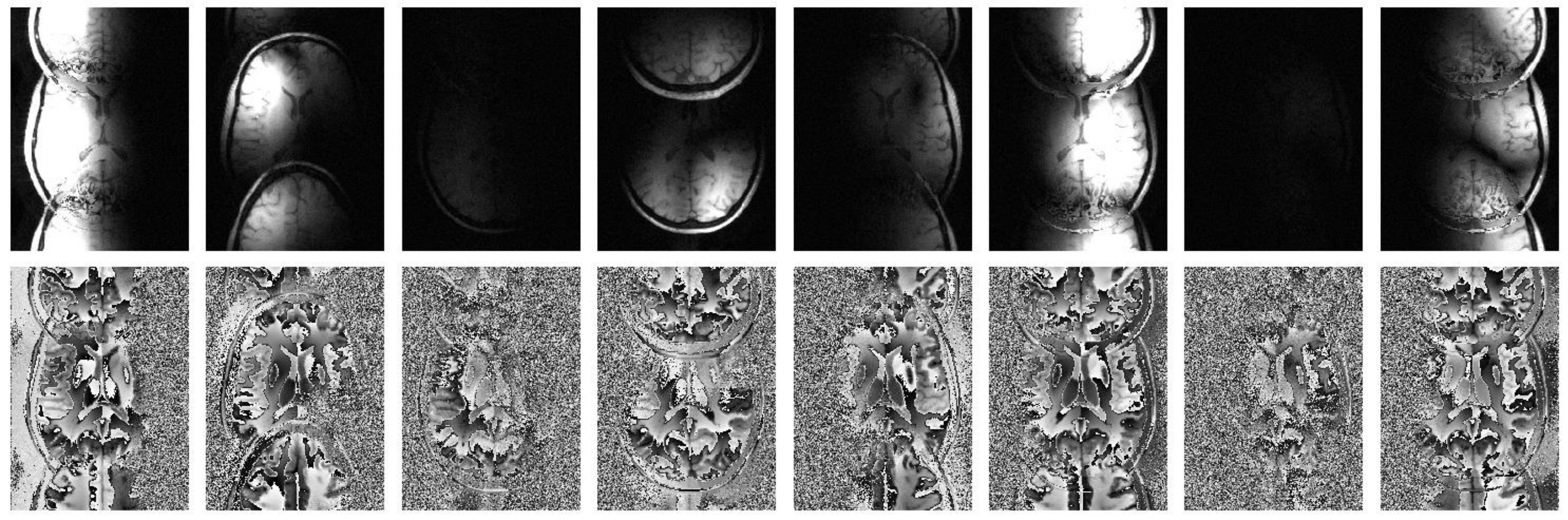


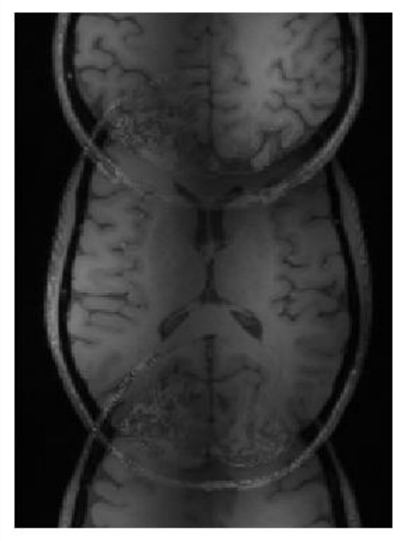
Multi Band



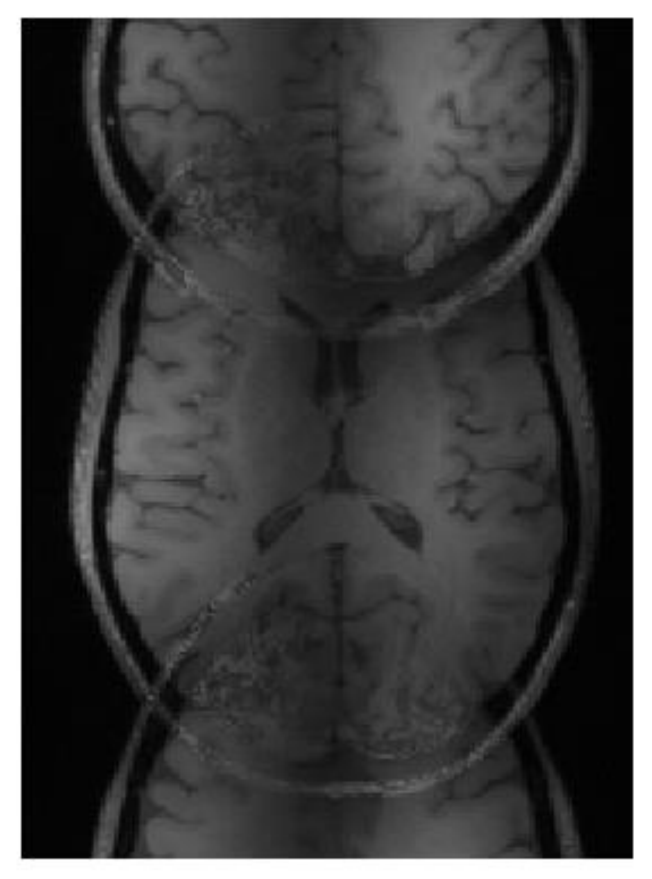


Multi Band





Multi Band



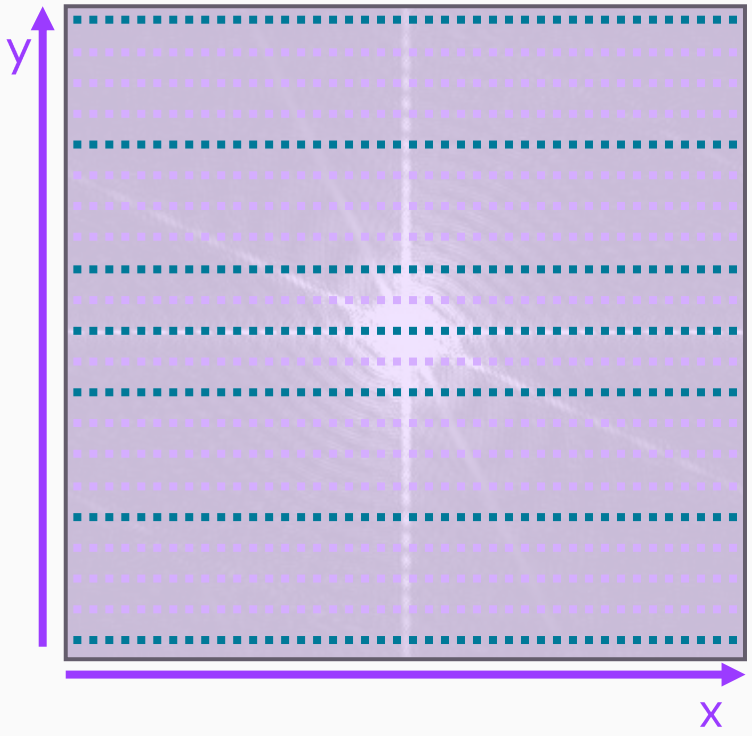
- Multi band imaging consists of exciting and acquiring **multiple slices simultaneously**. It is also known as simultaneous multi-slice (SMS).

- Having multiple coil elements with **spatially varying sensitivities** ensures that we are acquiring more samples (Number coils) than values to reconstruct (Acceleration factor).
  - Reconstruction involves inverting a well-posed linear system.

- Multi-band sequences **acquire as much signal** as single-band sequences; there is **no** $\sqrt{1/R}$ **SNR penalty**.

- Often multi-band is **combined with parallel imaging** in the phase—encoding direction.

Compress sensing



- ❑ When the number of samples becomes less than the number of values to reconstruct, the problem becomes **ill-posed**.
- ❑ This is called **compressed sensing**.
- ❑ It requires the use of a strong prior / regulariser:
  - ➢ Total variation
  - ➢ Low rank
  - ➢ …

Image reconstruction

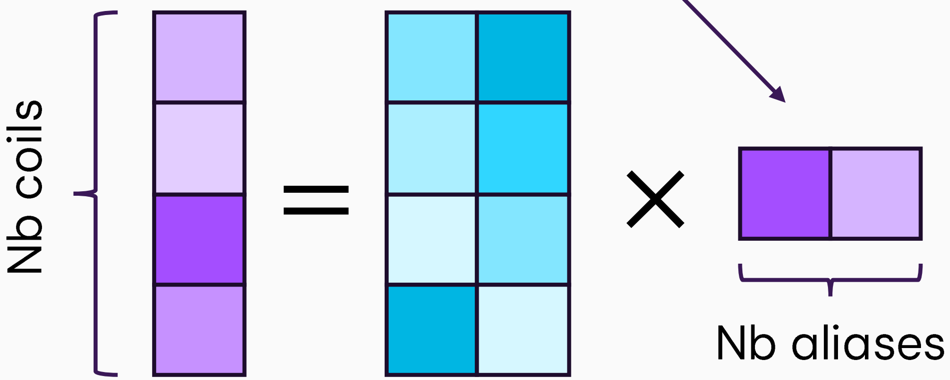
SENSE

No acceleration = adaptive combine:

$$\mathbf{f} = \mathbf{s} \cdot \rho \quad \Rightarrow \quad f_{\mathrm{opt}} = \left(\mathbf{s}^H \mathbf{s}\right)^{-1} \left(\mathbf{s}^H \mathbf{f}\right)$$

Acceleration:

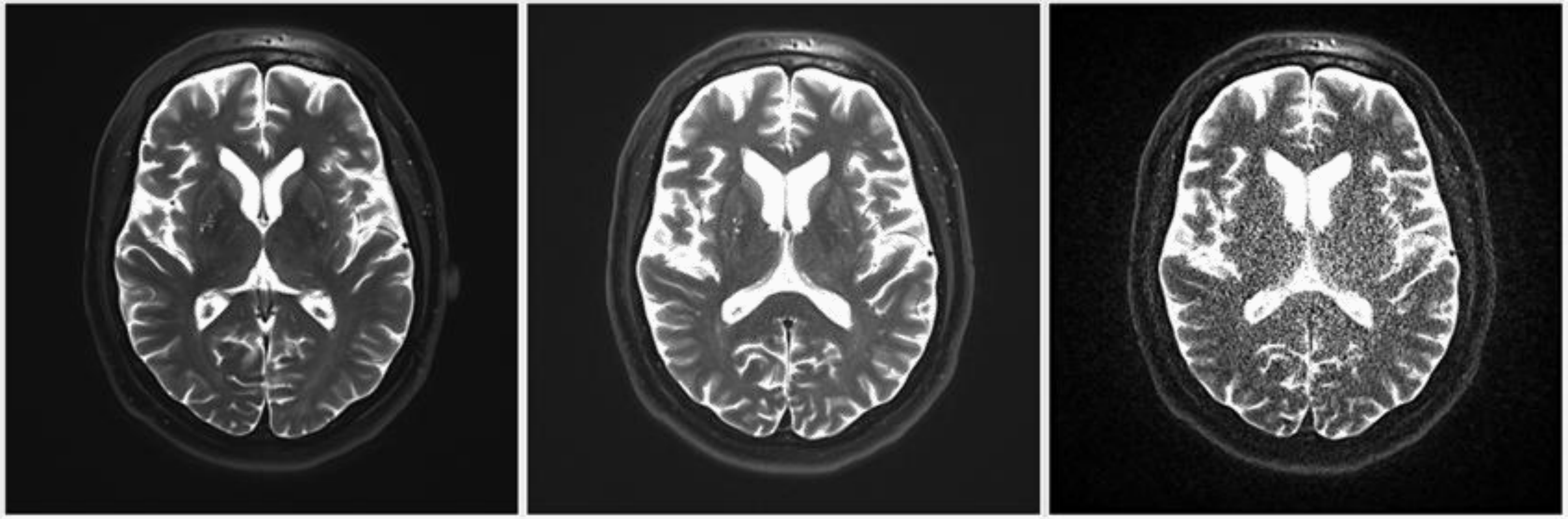
$$\mathbf{f} = \mathbf{S} \times \boldsymbol{\rho} \quad \Rightarrow \quad \mathbf{f}_{\mathrm{opt}} = \underbrace{\left(\mathbf{S}^H \mathbf{S}\right)^{-1}\left(\mathbf{S}^H\right.}_{\text{Pseudo-inverse}}\left.\mathbf{f}\right)$$



Noise amplification

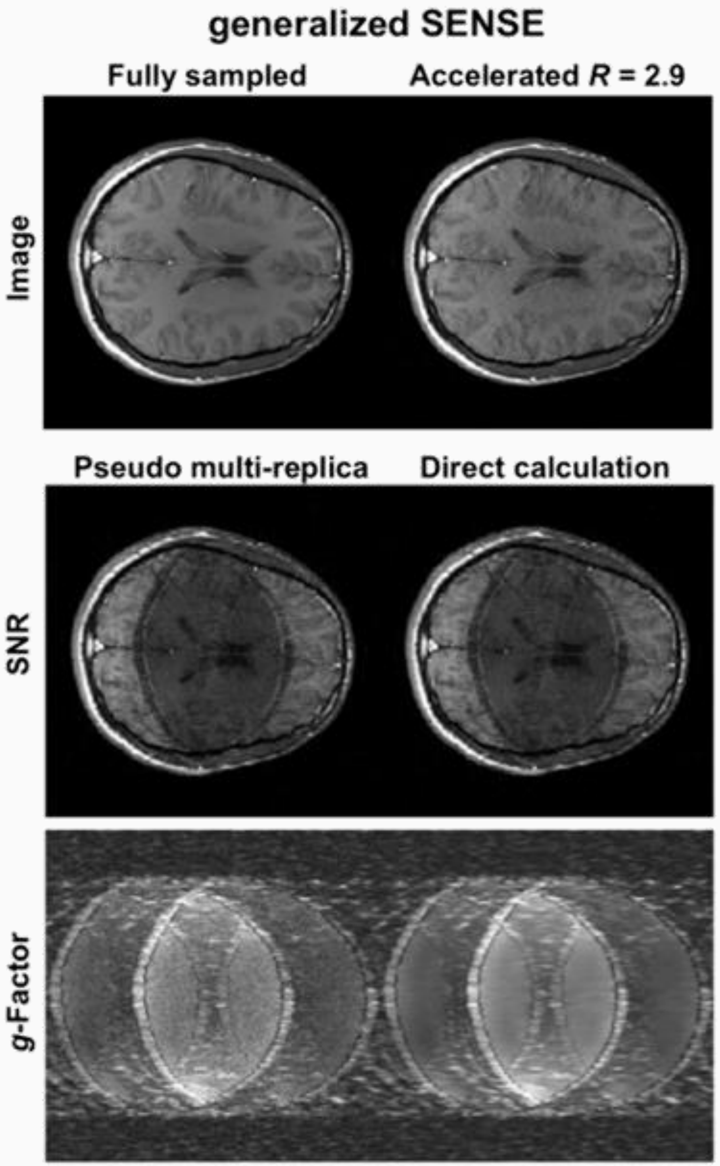
- ❑ Parallel imaging -> less total signal -> $\sqrt{1/R}$ SNR penalty
- ❑ Multi Band -> same total signal -> no such penalty

- ❑ **Geometry (*g*) factor** = noise standard deviation map
  - ➢ $\mathbf{f}_{\mathrm{opt}} = (\mathbf{S}^H\mathbf{S})^{-1}(\mathbf{S}^H\mathbf{F})$
  - ➢ $\mathrm{cov}[\mathbf{f}_{\mathrm{opt}}] = (\mathbf{S}^H\mathbf{S})^{-1}$



No acceleration (R=1) Acceleration factor R=3 Acceleration factor R=6

https://mriquestions.com/noise-in-pi.html

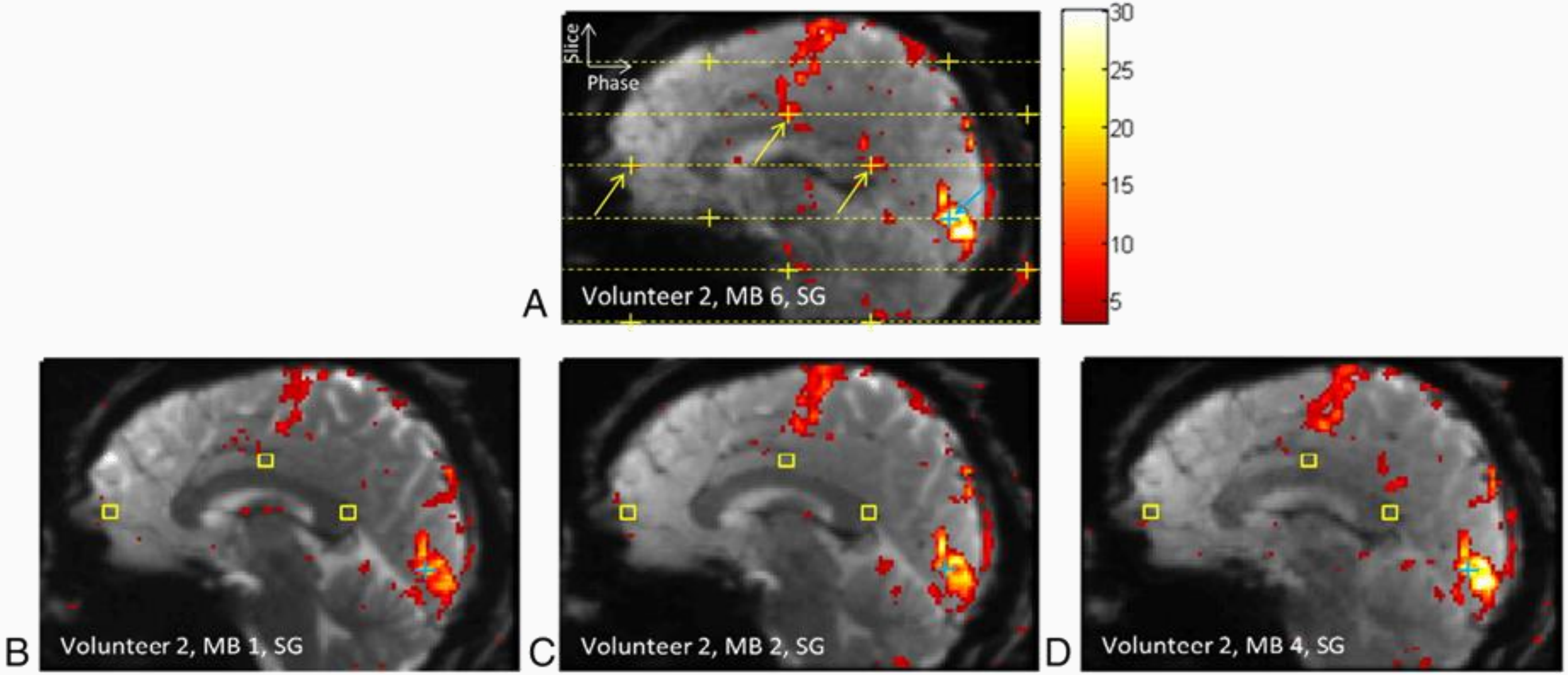


Robson & al. MRM (2008)

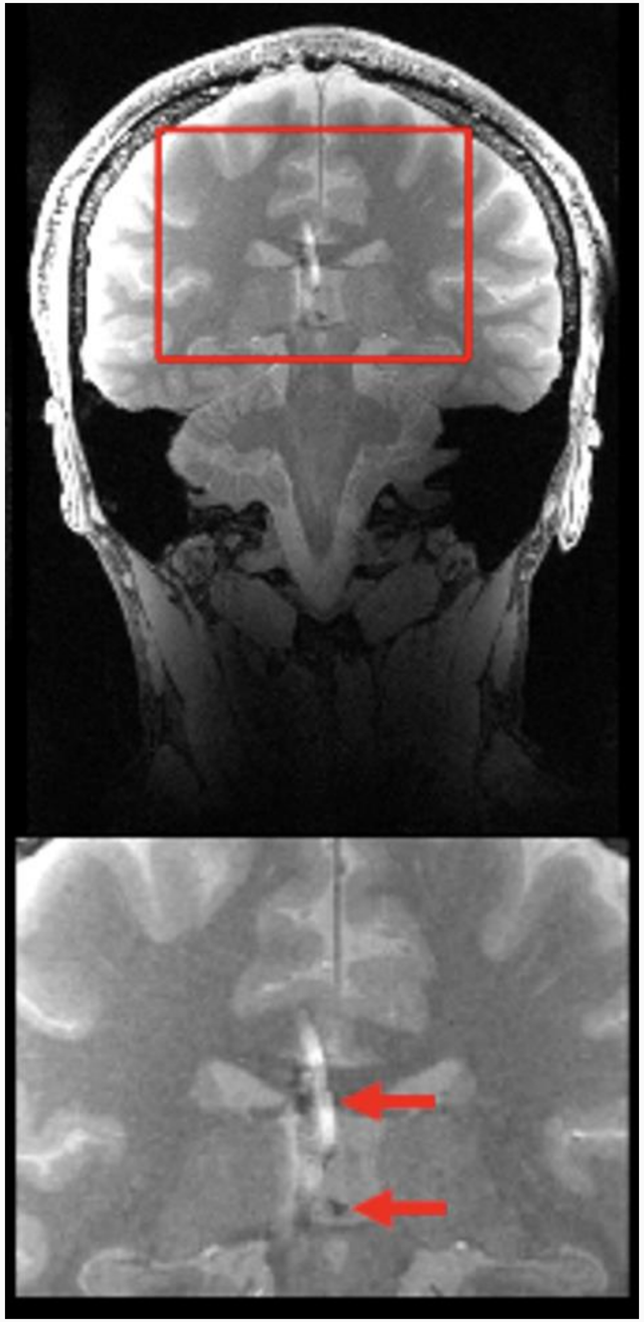
Understanding the trade-offs of acceleration

Imperfect unfolding

Multi Band



Parallel Imaging

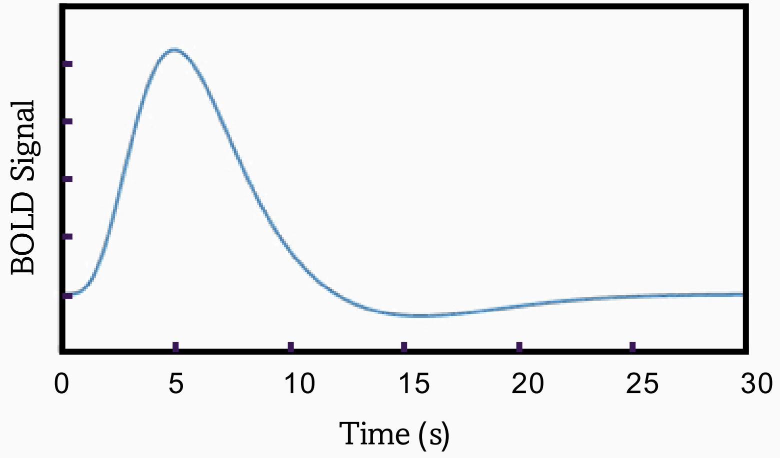


Todd & al. NeuroImage (2016)

Temporal auto-correlation

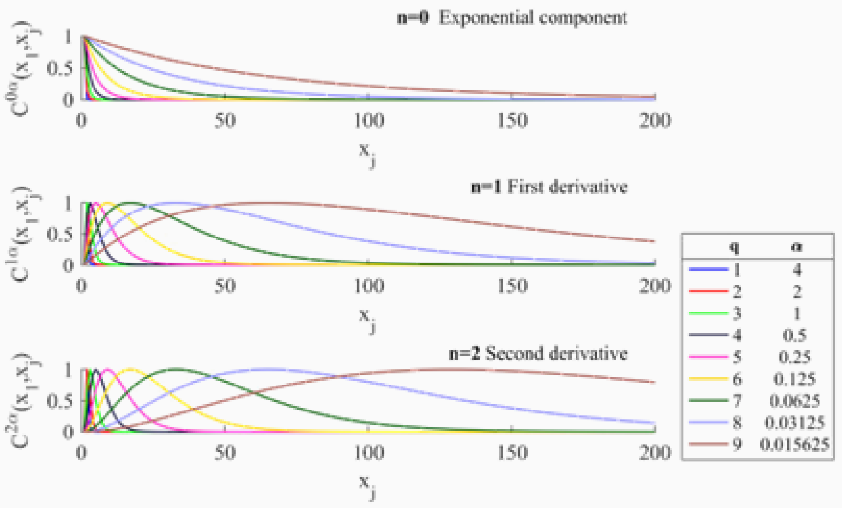
- The signal of interest has a temporal point-spread function (PSF), due to the shape of the hemodynamic response function (HRF)

$$\mathbf{y} = \mathbf{S}\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}, \qquad \boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$$



- The noise *also* has a temporal point-spread function:
  - Physiological effects (respiratory and cardiac cycles)
  - Scanner effects (scanner and gradient drift)
  - Residual motion and distortions

$$\mathbf{y} = \mathbf{S}\mathbf{X}\boldsymbol{\beta} + \mathbf{K}\boldsymbol{\varepsilon}$$



- The width of the PSF is ~ constant in "real time" (seconds), which means it increases in "series time" (frames) with lower TR.

Corbin et al. HBM (2018)

2D Functional MRI

<u>Advantages</u>

- Shorter TR:
  - More volumes per run time, or
  - Larger coverage, or
  - Higher resolution
- Reduced in-plane distortions (PI).
- Flexibility for shorter TE.
- Flexibility for higher resolution.

<u>Caveats</u>

- Multi Band
  - Shorten flip angle because TR is shorter -> SNR is down.
- Noise amplification (g factor) -> relates to the difficulty of unfolding data
- Leakage across aliased locations
- Correlated noise (spatially and temporally)
- Greater sensitivity to motion (discrepancy with reference)

3D Anatomical MRI

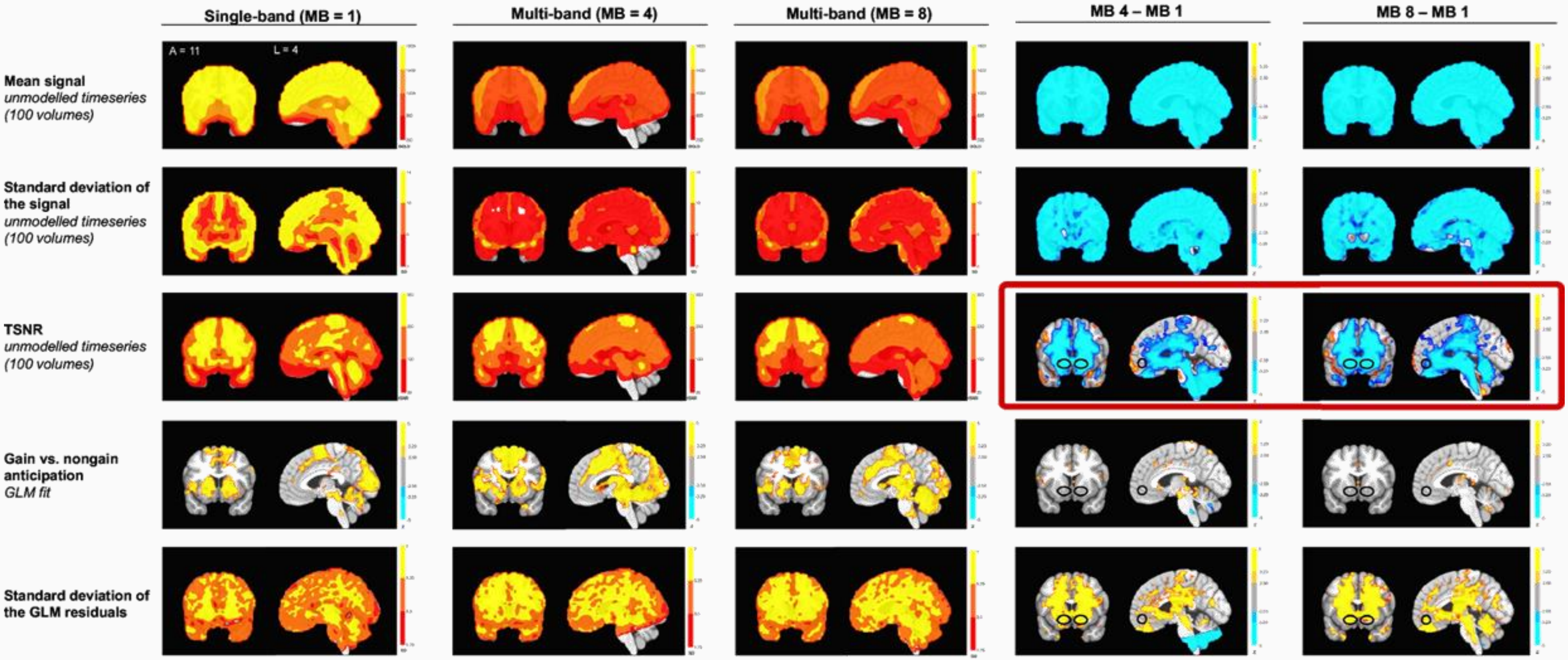
Advantages

- ❑ Fewer phase encodes = time saving (PI)
- ❑ Less time for within-scan motion.

Caveats

- ❑ SNR hit due to undersampling
- ❑ But, if motion -> more artefacts (small)
- ❑ Noise amplification (g factor)
  -> relates to the difficulty of unfolding data
- ❑ Unfolding artefacts

Loss of statistical power



Srirangarajan & al. NeuroImage (2021)