Reconstructing MEG Sources with Unknown Correlations

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Neural activity and magnetic fields

electric current
magnetic field
intracellular current (dendrite)
Magnetoencephalography (MEG)

First-Order Radial Flux Transformers

Sensing Coil Array
64 to 275 Sensing Locations

Vacuum plug

Vacuum space

Liquid Helium

SQUID Sensors

References Sensor Array

Whole-Cortex MEG System

MEG Data

Processed Data
(3rd order gradient)
The forward model relates the quantities

\[ b_t \] – magnetic field (gradient) measurements
\[ s_t \] – dipole strengths at a grid of possible source locations

However, the inverse problem is ill-posed. This means that any reconstruction depends crucially on prior assumptions about the nature of the source distribution. Our goal is to make this prior as compatible as possible, and thus to minimize bias.
An impossible problem

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linearly, through a lead-field matrix that is relatively easy to compute:

\[ b_t = Ls_t \]
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Many reconstruction methods are ultimately linear.

The Wiener filter

\[ \hat{s} = \langle sb^T \rangle \langle bb^T \rangle^{-1} b = \langle s(Ls + n)^T \rangle \langle bb^T \rangle^{-1} b = \langle ss^T \rangle L^T \langle bb^T \rangle^{-1} b \]

which requires knowledge of the measurement and source correlation matrices.

(This is the MAP estimate for Gaussian source priors.)
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The optimal linear reconstruction (in a least-squares sense) is given by the Wiener filter,

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(This is the MAP estimate for Gaussian source priors.)
Common assumptions

All methods for MEG source reconstruction rely on implicit or explicit assumptions regarding the source correlation matrix:

- Equivalent dipole fitting – very sparse source distribution.
- MUSIC-based source localization – sparse distribution with at most weak correlations.
- Minimum norm – identity correlation matrix.
- Minimum weighted norm – known correlation, usually independent.

By contrast, physiological measurements suggest strong and variable (stimulus-dependent) correlations.

Our approach is to retain the assumption of a sparse source distribution, but to learn the correlation matrix from the data themselves.
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A generative model for correlated sources

- Basic linear generative model.

\[ b \sim \mathcal{N}(Ls, \Psi) \]
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\[ z \sim \mathcal{N}(0, I) \]

\[ \langle ss^T \rangle = WW^T \quad s = Wz \]

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A generative model for correlated sources

- Basic linear generative model.
- Normal, linearly mixed, pre-source stage models correlated sources.
- ARD-type hyperprior leads to sparsity in $W$ (and therefore $s$).

\[
\begin{align*}
    z &\sim \mathcal{N}(0, I) \\
    \langle s s^T \rangle &= WW^T \quad s = Wz \\
    b &\sim \mathcal{N}(Ls, \Psi) \\
    A &= \text{diag} [\alpha_1, \alpha_2, \ldots] \\
    W_{ij} &\sim \mathcal{N}(0, \alpha_i^{-1})
\end{align*}
\]
Learning

Estimation is carried out in three stages:

1. Estimate $\alpha_i$ by maximizing the marginal likelihood:
   $$\hat{\alpha} = \text{argmax} \ P(B|\alpha) = \text{argmax} \int dZ \ dW \ P(W, Z, B|\alpha)$$
   (Actually maximize a variational bound.)

2. Estimate $\hat{W}$ by maximizing the posterior:
   $$\hat{W} = \text{argmax} \ P(W|B, \hat{\alpha}) = \text{argmax} \int dZ \ P(W, Z|B, \hat{\alpha})$$

3. Estimate $s$ by optimal linear filtering:
   $$s = \hat{\hat{W}} \hat{\alpha}^T L \langle b^T \rangle^{-1} b$$
Learning

Estimation is carried out in three stages:

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$$\hat{A} = \arg\max P(B \mid A) = \arg\max \int dZ \ dW \ P(W,Z,B \mid A)$$
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$$s = \hat{W} \hat{W}^T L^T \langle b b^T \rangle^{-1} b$$
The first two steps (obtaining $\hat{A}$ and $\hat{W}$) are approximated using a “variational Bayesian” approach.

$$\log P(B | A) = \log \int dZ dW Q(Z, W)$$

$$\geq \langle \log P(B, Z, W | A) \rangle_Q + H(Q_z) + H(Q_w)$$
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The standard Jensen bound on the likelihood...

$$\log P(B | A) = \log \int dZ dW \frac{Q(Z,W)}{Q(Z,W)} P(B,Z,W | A)$$

$$\geq \langle \log P(B,Z,W | A) \rangle_{Q(Z,W)} + H(Q)$$

with equality iff $Q(Z,W) = P(Z,W | B,A)$
Variational Bayes

The first two steps (obtaining $\hat{A}$ and $\hat{W}$) are approximated using a “variational Bayesian” approach.

The standard Jensen bound on the likelihood . . . is approximated using a factored posterior.

$$\log P(B | A) = \log \int dZ \, dW \frac{Q_Z(Z)Q_W(W)}{Q_Z(Z)Q_W(W)} \, P(B, Z, W | A)$$

$$\geq \langle \log P(B, Z, W | A) \rangle_{Q_Z(Z)Q_W(W)} + H(Q_Z) + H(Q_W)$$
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\geq \langle \log P(B, Z, W | A) \rangle_{Q_z(Z)Q_w(W)} + H(Q_z) + H(Q_w)
\]

The bound is then tightened by alternating optimizations with respect to $Q_z$ and $Q_w$.

\[
Q_z^{n+1}(Z) \propto \exp \langle \log P(B, Z, W | A) \rangle_{Q_w^n} \\
Q_w^{n+1}(W) \propto \exp \langle \log P(B, Z, W | A) \rangle_{Q_z^n}
\]

For our model, both $Q_z$ and $Q_w$ prove to be normal under the variational assumption.
Making one further simplification, we can derive straightforward update equations.

\[
\Sigma_z^{n+1} = \left( \hat{W}^{n\top} L^{\top} \Psi^{-1} L \hat{W}^n + \text{Tr} \left[ L^{\top} \Psi^{-1} L^{\top} \Sigma_w^n \right] I + I \right)^{-1}
\]

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\Sigma_w^{n+1} = (NL^{\top} \Psi^{-1} L + A^n)^{-1}
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\[
A^{n+1} = d_z \text{diag} \left[ \hat{W}^{n+1\top} \hat{W}^{n+1\top} \right]^{-1} \left( I - A^n \text{diag} \left[ \Sigma_w^{n+1} \right] \right),
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Q_z(z) = \mathcal{N}(\ldots, \Sigma_z), \quad Q_w(W) = \mathcal{N}\left( \hat{W}, \Sigma_w \right).
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• Iterated quantities are all compact (no time dependence).
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\]

- Iterated quantities are all compact (no time dependence).
- Measurements affect \( A \) and \( W \) only through the correlations \( BB^T \).
Simulated sources

**Power**

**Timecourses**

**Correlation coefficients**
Beamformer reconstruction

Power

Timecourses

Correlation coefficients
Probabilistic reconstruction: powers
Probabilistic reconstruction: correlation coefficients

simulation

beamformer

bayesian reconstruction
Real data
Future work

• Model non-stationary source correlations.
• Integrate over the hyperprior (by sampling) to obtain meaningful posterior distributions on $W$ and $s$.
• Incorporate external information (about brain structure and activity obtained by other imaging methods).
• Investigate ways to compare performance to other methods... difficult because of the ill-posed nature of the problem.
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