To be Bayesian or Frequentist or Not: A Debate on Functional Imaging Analyses

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I. Inferences about Parameters

Model, m

Parameters, β

Data, y

For example, a GLM: $y = X\beta + e$

Bayes rule:

$$p(\beta \mid y, m) = \frac{p(y \mid \beta, m)\, p(\beta \mid m)}{p(y \mid m)}$$

Posterior: $p(\beta \mid y, m)$

Likelihood: $p(y \mid \beta, m)$

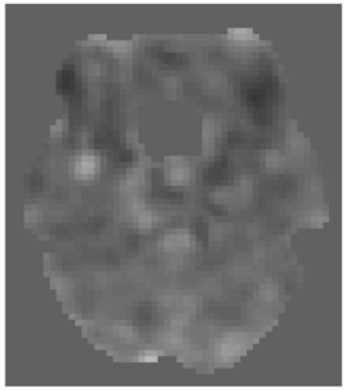
Prior: $p(\beta \mid m)$

Model Evidence: $p(y \mid m)$

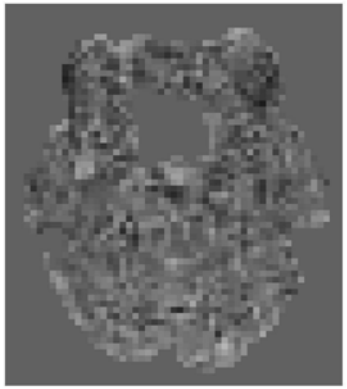
Efficient Computation using Approximate Inference methods (Bishop, 2007)

fMRI Analysis with Spatial Priors

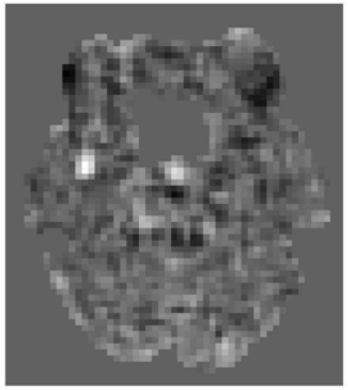
MRI



Smoothed data



ML estimate



Posterior Mean

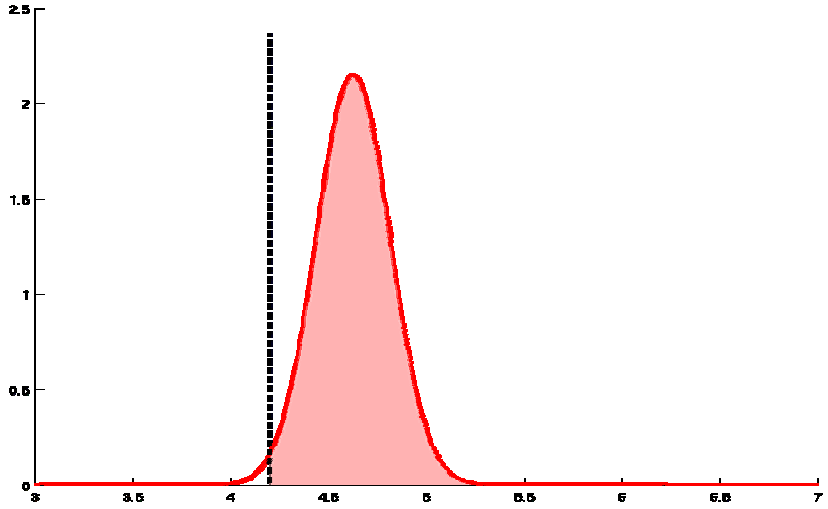
Spatial Priors:

1. Empirical Bayes: parameters of prior estimated from data
2. Spatial scale of effects can be automatically estimated
3. Can be different for eg. main effect versus interaction
4. Can be different in eg. visual cortex vs. amygdala

Increased sensitivity

Posterior Probability Maps

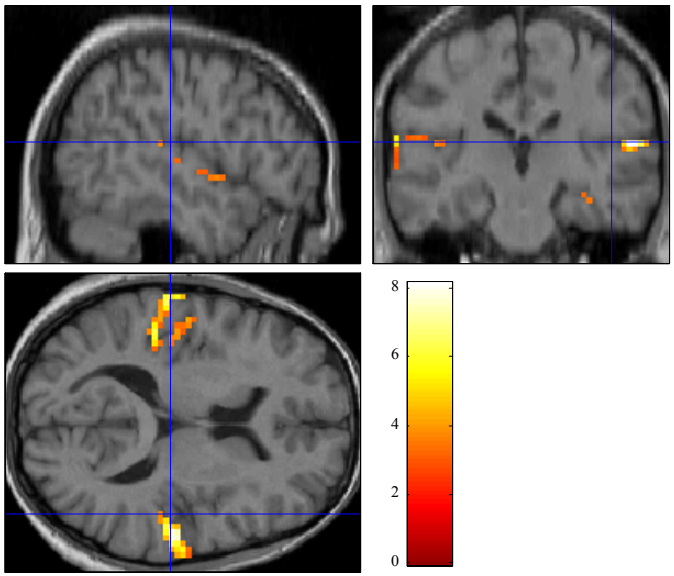
Effect size threshold s_{th}



Probability of getting an effect, given the data

$$p(w_n \mid y, m)$$

Active > Rest



Overlay of effect sizes at voxels where we are 99% sure that the effect size is greater than 2% of the global mean

L M Harrison, et al. *Neuroimage*, 41(2):408-23, 2008

II. Inferences about Models

- In model-based fMRI (O'Doherty etc.) signals derived from a computational model for a specific cognitive process are correlated against fMRI data from subjects performing a relevant task to determine brain regions showing a response profile consistent with that model.

- For example, reinforcement learning models fitted to behavioural data producing subjective estimates of `value', 'prediction error' …

- But which models are correct ?

II. Inferences about Models

Bayes Rule (parameter level):

$$p(\beta|y,m) = \frac{p(y|\beta,m)\,p(\beta|m)}{p(y|m)}$$

Model evidence:

$$p(y|m) = \int p(y|\beta,m)\,p(\beta|m)\,d\beta$$

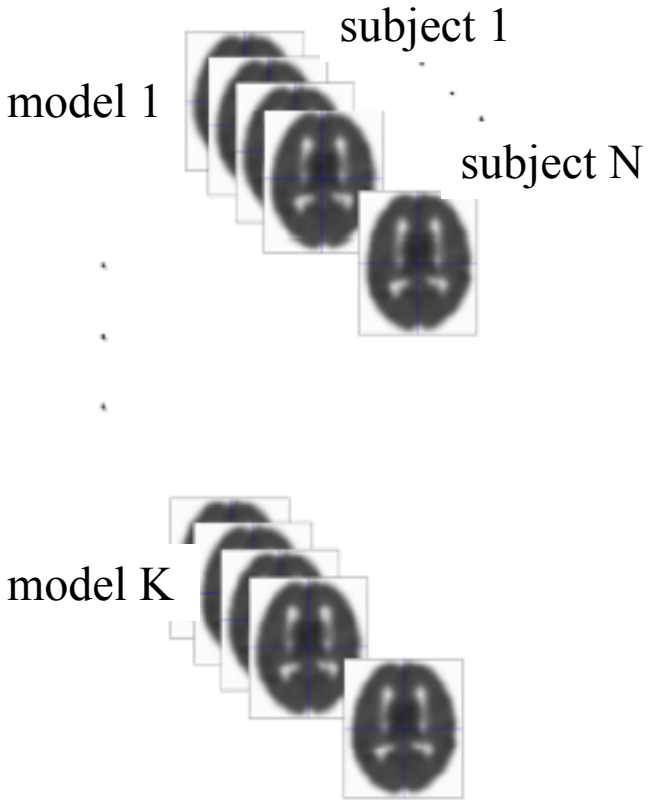
$$\log p(y|m) \quad = Accuracy(m) - Complexity(m)$$

Bayes Rule (model level):

$$p(m|y) = \frac{p(y|m)\,p(m)}{p(y)}$$

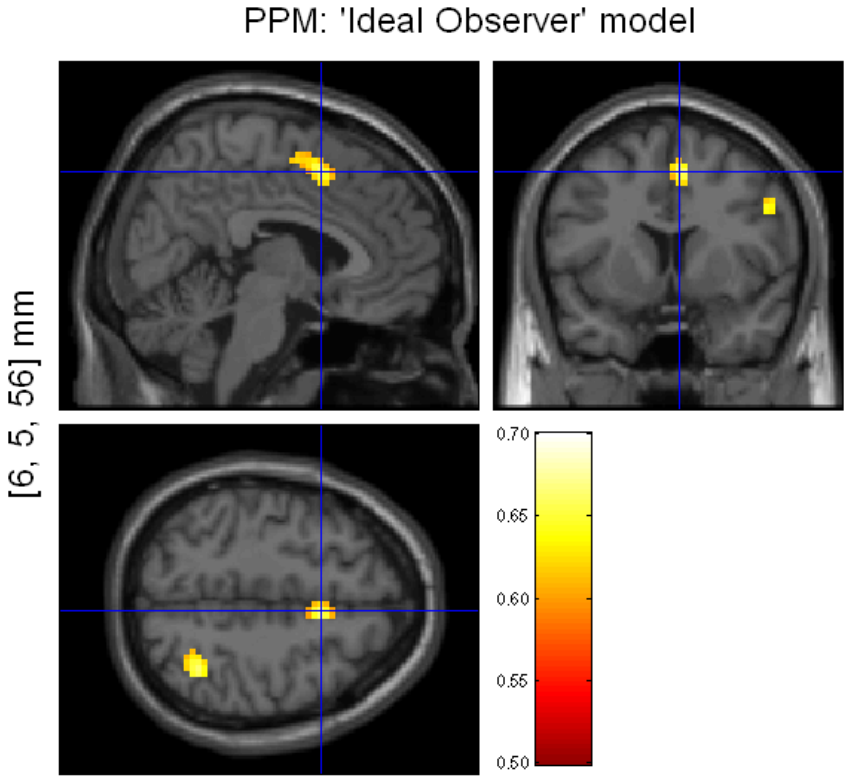
Bayesian Model Selection Maps for Group Studies

Evidence maps



Compute evidence for each model/subject

Activity best predicted by Ideal Observer model in 70% subjects in these regions



Maria Joao Rosa et al, 2009: Poster 357, Sunday AM

Summary

- Bayesian inference is computationally efficient
- There are many sources of prior information eg. spatial smoothness
- Empirical Bayes framework allows parameters of prior to be estimated
- PPMs for inference about effect size
- Model-based fMRI enhanced by model inference
- Same algorithms used for eg. M/EEG source reconstruction
- Neurobiological priors on connectivity models